**Deanship of Graduate Studies**
**Alquds University**

# Optimal Vector Control of Three Phase Induction Machine

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**M.Sc. Thesis**

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# Thesis Approval

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**Master thesis submitted and accepted, Date: 02/01/2006**
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**Jerusalem- Palestine**

**1426/ 2005**

## Declaration

I certify that this thesis submitted for the degree of Master in the result of my own research, except where otherwise acknowledged, and that this thesis (or any part of the same) has not be submitted for higher degree to any other university or institution

Signed . . . . . . . . . . . . . . . . . .

Jasem M. Kh. Abu Rajab Tamimi

13 October 2005

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# ملخص

ان من اكثر الطرق المتبعة في التحكم بالمحرك الحثي الثلاثي الاطوار هي التحكم الحثي او التحكم بالمجال الموجهه.

ان قيادة المحرك الحثي باستخدام المجال الموجهه لتقليل مقياس الطاقة في المحرك الحثي باستخدام نظرية التحكم الامثل هو هدف هذه الرسالة.

فقد تم استخدام مسألة التحكم التربيعية المثلى الخطية واللاخطية مع عدة نماذج للمحرك الحثي.

في هذه الرسالة، تم استخراج حالات ومدخلات (متحكمات) المحرك الحثي التي تقلل الطاقة المقاسة في المحرك، حيث تم حل هذه المسألة باستخدام معادلة "راكاتي" الجبرية لحل المسألة الخطية.

اما المسألة اللخطية فقد تم استخدام طريقة التخطيط (Quazilinearization) التي تحول مسالة التحكم الامثل الى مجموعة من المسائل الخطية التربيعية المثلى.

و من الجدير بالذكر انه تم استخدام برامج الحاسوب الشخصي الرقمية مثل "ماتلاب" و "سميولنك" لتمثيل متحكمات و حالات المحرك الحثي عوضا عن بناء للنظام.

## Abstract

The most famous control method used to control the induction machine (IM) is vector control or field oriented control method (FOC). The purpose of this thesis is to drive an induction machine using FOC by minimizing the total energy measure in the IM based on optimal control theory.

In this thesis, the linear and non linear quadratic optimal control problems using second and third order model of IM are treated.

A second order model based on vector control approach relating motor fluxes (states) and currents (controls) is considered to obtain an optimal state and control trajectories of IM.

Linear quadratic optimal control problem is solved by solving algebraic Racciti equation (ARE).

Moreover, a third order nonlinear model described in arbitrary rotating frame of IM is used with a quadratic performance index. This problem is solved using the quasilineraization method which converts the nonlinear optimal control problem into a sequence of linear quadratic optimal control problems. The optimal trajectories of fluxes, speed, currents, and torque that represent the model states and controls of IM are obtained

Rather than building the system, digital simulation program (Matlab and Simulink) is used to show the final result of IM controls and states.

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# Chapter 1

# Introduction

The most commonly used machines of one horsepower and more are three-phase induction machines. Induction machines consist of two main parts: stator and rotor. These machines require no electrical excitation for the rotor winding (squirrel cage). The rotor windings are short circuited. Magnetic flux flows through air gap links, this closes the rotor circuit, and then voltages are induced in the rotor; causing of currents to flow.

The rotor current arises from induction, and the operating speed of the rotor is slightly less than the synchronous speed in the motor mode and slightly greater than synchronous speed in generator mode (Mcpherson *et al,* 1990).

## 1.1 Induction Machine

Induction machines are usually induction motors (operated in motor mode), and they have many advantages. They are rugged, have small size, relatively inexpensive, require very little maintenance, and found in wide power ranges. Their speed is nearly, but not quite, constant, dropping a few percent in going from no load to full load. They have greater efficiency and lower torque than other motors (Mcpherson *et al,* 1990, Novotny *et al*, 1995). The main disadvantages of the induction motors are:

1. The speed is not easily controlled.

2. The starting current may be five to eight times full load current.

3. The power factor is low and lagging when the machine is lightly loaded.

When the three-phase voltages are applied to the stator winding terminals, a balanced three phase-currents flows in the phase windings, so the rotating Electro-Magnetmotive Force (MMF) field is produced in the air gap of the machine. The speed of the rotating MMF field is given by equation:

$$\omega_s = \frac{4\pi f}{z}$$

where $\omega_s$: synchronous speed, $f$: frequency of the stator voltages and currents, and $z$ is the number of poles of the windings.

The magnitude and frequency of the rotor voltages depend on the speed of the relative motion between the rotor and flux crossing in the air gap $(\psi)$. The slip speed expresses speed of the rotor speed ($\omega_m$) relative to the field speed ($\omega_s$), and it is given by:

$$slip\ speed = \omega_s - \omega_m$$

where the per unit slip, usually called slip ($s$), is defined as follows:

$$s = \frac{\omega_s - \omega_m}{\omega_s} \tag{1.1}$$

## 1.2 Induction Machine Circuit

The induction machine, in certain aspects, is a rotating transformer. However the three-phase induction motors are of two types: squirrel cage and wound rotor (Mcpherson *et al,* 1990, Novotny *et al*, 1995, Dubey, 1995).

In squirrel cage motors, which are the types mostly used, the rotor consists of the longitudinal conductor-bars shorted by circular connectors at the two ends. Both squirrel cage motor and wound rotor motor have a per phase equivalent circuit of three-phase induction motor as shown in Figure (1.1a) with $\grave{R}_r$ and $\grave{L}_r$, rotor resistance and inductance, respectively referred to the stator. $R_s$ and $L_s$ are stator resistance and inductance respectively. $L_m$ is a magnetizing inductance.

Since the stator impedance drop is generally negligible compared to terminal voltage V , the equivalent circuit can be simplified to the one shown in Figure (1.1b).

From figure (1.1b) the following equation is got:

$$I_r = \frac{V}{\left(R_s + \frac{\grave{R}_r}{s}\right) + j\left(X_s + \grave{X}_r\right)} \tag{1.2}$$

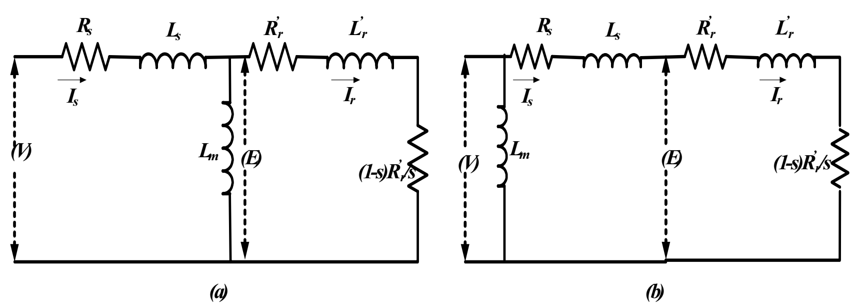



Figure 1.1: Per phase stator referred equivalent circuit of induction motor

And the developed torque is as shown in (Dubey, 1995, Rashid, 2004):

$$T_d = \frac{1}{s\omega_s}\left[\frac{3\grave{R}_r V^2}{\left(R_s + \frac{\grave{R}_r}{s}\right)^2 + \left(X_s + \grave{X}_r\right)^2}\right] \tag{1.3}$$

Analyzing the mechanical characteristic equation (1.3), the developed torque is a function of slip. A typical plot of developed torque as a function of slip or speed is shown in figure (1.2) (Rashid, 2004).

There are several methods to control an induction motor torque, speed, or position. These methods can be categorized in two groups: the scalar and vector control (Texas Instrument, 1996).

## 1.3 Induction Machine Control Literature Review

### 1.3.1 Scalor Control:

Scalar control means that variables are controlled only in magnitude, and the feedback and command signals are proportional to DC quantities. This technique drives the stator voltage or current as a command and mainly deals with characteristic equation (1.3). The speed and torque control (Rashid, 2004, Bose, 2002) can be done using one of the following control methods:

**a) Stator Voltage Control:**

This method stands on varying terminal voltages of the stator, so the motor torque is proportional to the square of stator voltages as indicated in (1.3) (Rashid, 2004, Bose, 2002).

Stator voltage can be varied by three phase AC voltage controller, three phase voltage-fed variable DC link inverter, and three phase pulse width modulation (PWM).

This method is mainly used in low power application, and may be used for starting high power induction motors to limit the inrush currents (Rashid, 2004).

**b) Rotor Voltage Control:**

This method used only in a wound rotor induction motor, because the rotor has acssesable terminals. An external three-phase resistance may be connected to the termials, and the developed torque is varied by varying this three phase resistor. To analyze this method, this resistor is added to the rotor and the developed torque are determined by applying characteristic equation (1.3) (Rashid, 2004, Bose, 2002).

This is an inefficient method, while there would be imbalance in voltages and current if the resistor is not uniform. In addition this method increase starting torque while limiting the starting current.

The three phase resistors may be replaced by three-phase diode rectifier and DC converter. This converter may be a DC converter with parallel resistor or AC inverter with step up transformer; and the secondary winding of the transformer are connected to the three phase supply. This type of drive is known as static Kramer drive.

Again, by replacing three phase resistor with three phase dual converter (or cycloconverter), a rotor voltage control occurs, this method is called static Scherbiuse drive.

The last two methods (static Kramer drive and static Scherbiuse drive) are used in large power pump and limited range of speed applications.

**c) Frequency Control:**

This method stands on changing the supply frequency. If the frequency above certain value is increased, the flux and torque would decreas. The synchronous angular frequency corresponding to rated frequency can be defined as $\omega_b$ ,then the synchronouse frequency at any other frequency becomes:

$$\omega_s = \beta\omega_b$$

and

$$s = \frac{\beta\omega_b - \omega_m}{\beta\omega_b} = 1 - \frac{\omega_m}{\beta\omega_b}$$

The characteristic equation then becomes:

$$T_d = \frac{3\grave{R}_r V^2}{s\beta\omega_b \left(R_s + \frac{\grave{R}_r}{s}\right)^2 + \left(\beta X_s + \beta\grave{X}_r\right)^2} \tag{1.4}$$

And figure (1.3) (Rashid, 2004) shows torque behavior by changing $\beta$.

**d) Voltage and Frequency Control:**

This method depends on keeping the voltage to frequency ratio constant, so the motor flux remains constant, and then maximum torque can be maintained approximately constant. In addition, at low frequencies, the air gap flux is reduced due to the drop in the stator resistance and the voltage has to be increased to maintain the torque level (Dubey, 1995, Rashid, 2004, Bose, 2002).

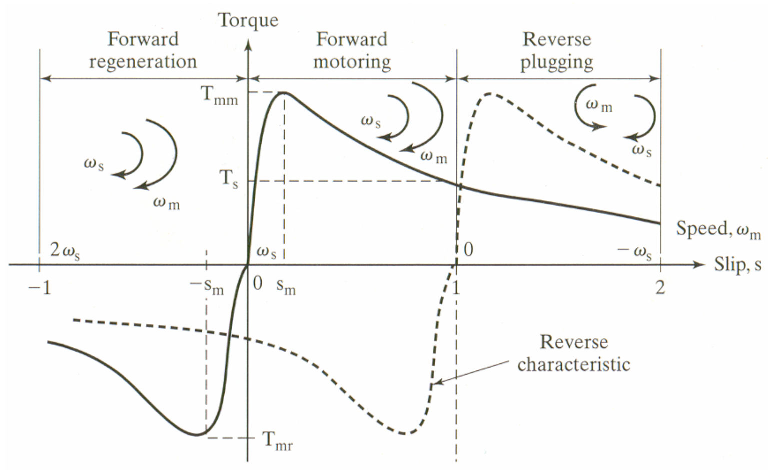



Figure 1.2: Torque-Speed Characteristic

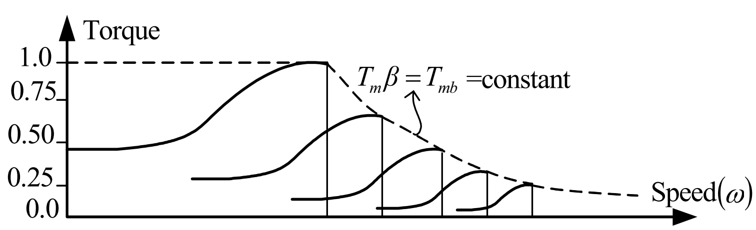



Figure 1.3: Torque-speed characteristic with frequency control.

This method is called volt/hertz control, and it is efficient and widely used in industry.

**e) Current Control:**

This method stands on varying the rotor current to control the induction motor torque. The input (stator) current is varied instead of rotor current using three phase current source inverter. Figure (1.4) (Dubey, 1995, Rashid, 2004) shows the torque speed characteristic by current controller.

**f) Voltage, Current, and Frequency Control**

The mechanical characteristic equation (1.3) depends on the type of control. It may be necessary to vary the voltage, frequency and current to obtain the mechanical requirements.

Figure (1.5) (Rashid, 2004) shows the control variables versus frequency, where there are three regions. In the first region we can vary speed with voltage (or current) control at a constant torque. In the second region, the motor is operated at a constant current and variable slip. In the third region, the speed is controled by frequency at reduced stator current (Rashid, 2004).

### 1.3.2 Vector Control:

The vector control refers not only to the magnitude but also to the phase of these variables. Matrices and vectors are used to represent the control quantities, and this method is also known as Field Oriented Control (FOC). Moreover, it allows a squirrel cage induction motor to be driven with high performance and

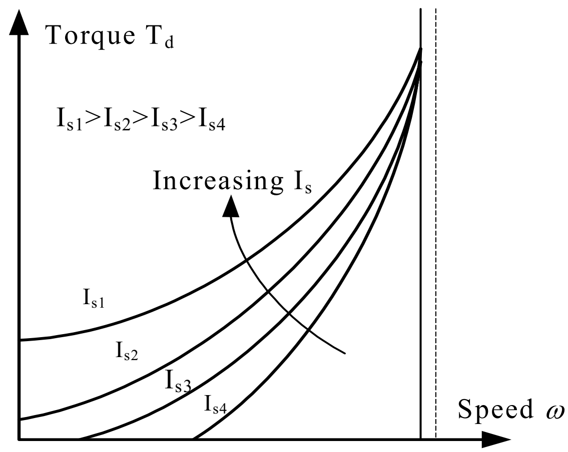



Figure 1.4: Torque-speed characteristic by current control.

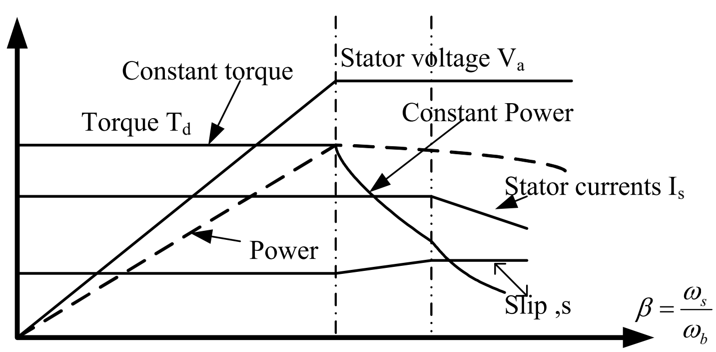



Figure 1.5: Control variables versus frequency

similar to the characteristic of a DC motor (Novotny *et al*, 1995, Leonard, 1985, Vas, 2000, Mohamad, 2000, Ho, Sen, 1988).

This method considers real mathematical equations that described the motor itself, and decouples the components of the stator current: one providing the air gap flux and another producing the torque that provides independent control of flux and torque. Although the induction motor have very simple structure; its mathematical model is complex due to the coupling factor between large number of variables and nonlinearities. To solve this problem, vector control uses the two primary transformations (NEC corporation, 2002). The first one is called Clark transformation and the other is called Park transformation. Clark transformation transforms the three stator currents ($i_a$, $i_b$ , and $i_c$) to two orthogonal current representations in stationary frame or frame attached to the stator ($i_\alpha$, $i_\beta$). To apply Clark transformation we have to measure only two of the phases currents (e.g. $i_a$, and $i_b$).

Park transformation transforms the stationary currents from stationary frame into rotating frame or frame attached to the rotor using the angle ($\theta$) between them, and to do so measuring the mechanical speed of the induction motor is needed.

In chapter two Clark, Park transformations and vector control implementation will be presented.

An induction machine model using vector control algorithm has been applied by several researchers in control, for example H. Zidan (Zidan *et al, 2000*) successfully applied the estimation method to control the induction motor drives without using speed sensors; they used simple speed estimation method for IM drive at low speed, this method uses the current and the input voltages in closed loop for rotor parameter estimation.

While, B. Hovingh *et al*, (Hovingh *et al*, In Press) presented an algorithm to estimate the rotor's speed and torque from the terminal voltage and input current to the motor. They showed that measurement of the stator voltage and currents are sufficient to determine the rotor position, speed and torque of an induction motor during any conditions, whether transient or steady state. Their work is being performed to analyze the response of a Field Orientated Control system when the estimated waveforms are used as an input into the control loop.

On the other hand, Ramirez and Canudas (Ramirez *et al,* In Press) presented experimental results of a nonlinear torque-flux optimal control for induction motor drives. This controller minimizes the stored magnetic energy and the coil losses, while satisfying torque tracking control objectives. They also presented an optimal design for current induction motor drives.

In addition, H. Rasmussen (Rasmussen, 2002) used an adaptive approach leading to a completely a new method called Field Angle Adaptation (FAA). The new contribution to the conventional current control system in rotor field oriented dq-coordinates is a signal added to the field angle in the transformation from rotor field coordinates to stator fixed coordinates. This signal adapts the field angle estimate to the correct rotor field angle.

Okoro (Okoro, 2003) used the vector control algorithm to simulate the dynamic performance of the induction motor by assuming the main flux inductance, stator, and rotor leakage inductances vary with the magnetization current.

Barambones (Barambones, In Press) presented indirect field oriented motor drive with sliding mode controller, including rotor speed estimation from measured stator terminal voltage and currents. The estimated speed is used as a feedback in an indirect vector control system achieving the speed control without the use of the shaft mounted transducer.

While Kim *et al* (Kim *et al*, 2001) used neural network technique to estimate rotor speed of the induction motor. They use backpropagation algorithm, and the training starts simultaneously with induction motor working. They realized speed sensorless drive.

On the other hand, Bose (Bose *et al*, 1997) showed the implementation of simple direct torque neuro-fuzzy control (DTNFC).

In this thesis, the optimal solutions of the induction motor (IM) fluxes and currents that minimize the total energy measure of the motor will be presented. The optimal control theory is used to solve a linear and nonlinear IM models based on vector control approach.

Following this introductory chapter, the vector control is disscussed in chapter two. In chapter three will discuss the optimal control theory and chapter four formulates the optimal control problem by applying optimal control theory on IM model. Chapter five shows the optimal control problem solution and simulation of the controller. Finally, conclusion and future work are discussed in chapter six.

# Chapter 2

# Vector Control

Vector control algorithm uses the dynamic equivalent circuit of the induction motor, and this equivalent circuit enables the induction motor to be controlled in a method similar to DC motor(Okoro, 2003).

Two primary famous transformation are used: Clark transformation which transforms the three stator current into two DC currents in stationary frame, and Park transformation which transforms the two DC currents into direct and quadrature axis or rotating frame as shown in figure (2.1).

**Vector Control Implementation:**

Figure (2.2) (NEC corporation, 2002) shows a block diagram of the vector control implementation with desired input ($i^*_{sq}$, $i^*_{sd}$) (Mathwork, 2002).

The purpose of this chapter is to describe the element of Clarck and Park transformations and flux estimation bolcks shown in figure (2.2) and to obtain the induction motor model using vector control technique.

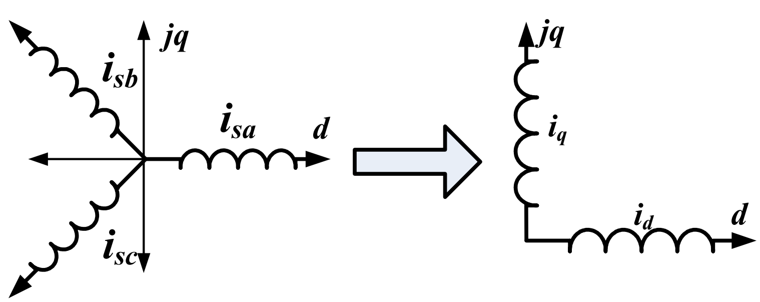



Figure 2.1: 3-phase to *d*-*q* equivalents

## 2.1 Direct and Quadrature Axis Transformation

Direct and quadrature axis transformation (Park transformation) was used to convert the three sinusoidal phase voltages and currents (with phase shift between any two phases: 120 degree) to two orthogonal voltages and currents respectively, this transformation can represent the machine parameters in a rotating d-q frame, and the following equation initialize d-q transformation: (Rashid, 2004, Okoro, 2003, Barambones, In Press, Marino *et al*, 1993):

$$\begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} = C \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = C \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

where

$$C = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}$$

and

$$\theta = (\omega_s - \omega_m)\, t$$

where $i_{as}, i_{bs}$,and $i_{cs}$ are stator currents in *abc* frame, $v_{as}, v_{bs}$ and $v_{cs}$ are stator voltages in *abc* frame, $i_{qs}$ and $i_{ds}$ are stator currents in *dq rotating* frame,$v_{qs}$ and $v_{ds}$ are stator voltages in *dq rotating* frame, $\theta$ : the phase angle between rotating frame and stationary frame, $\omega_s - \omega_m$ represent the relative speed between synchronously rotating reference frame (stationary) and frame attached to rotor (rotating frame). This difference is called slip speed($\omega_{sl}$) (Rashid, 2004).

## 2.2 Induction Machine Model

When the induction machine is modeled the following assumptions must be considered: (Okoro, 2003):

1. The machine is symmetrical with linear air-gap magnetic circuit.
2. Neglecting the saturation effect.
3. Neglecting skin, and temperature effects.
4. Neglecting harmonics content of MMF wave.
5. The stator voltages are balanced.

The suitable way to obtain the induction machine performance is through reducing the machine into two axis coils (d-q axis) model on both stator and rotor, as described by Krause and Tomas (Krause *et al*, 1965).

Figure (2.3) (Rashid, 2004, Okoro, 2003, Ozpineci *et al*, 2003) shows d-q equivalent circuit for three phase symmetrical induction reffered to arbitrary rotating frame.

From the dynamic equivalent circuit, the induction motor parameters can be expressed in matrix equation (2.2), regarding that the rotor bars in squirrel cage induction motor are shorted out and the rotor voltages equal zero (Rashid, 2004, Bose, 2002).

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + pL_s & \omega_s L_s & pL_m & \omega_s L_m \\ -\omega_s L_s & R_s + pL_s & -\omega_s L_s & pL_m \\ pL_m & (\omega_{s-}\omega_m) L_m & R_r + pL_r & (\omega_{s-}\omega_m) L_m \\ -(\omega_{s-}\omega_m) & pL_m & -(\omega_{s-}\omega_m) & R_r + pL_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \tag{2.2}$$

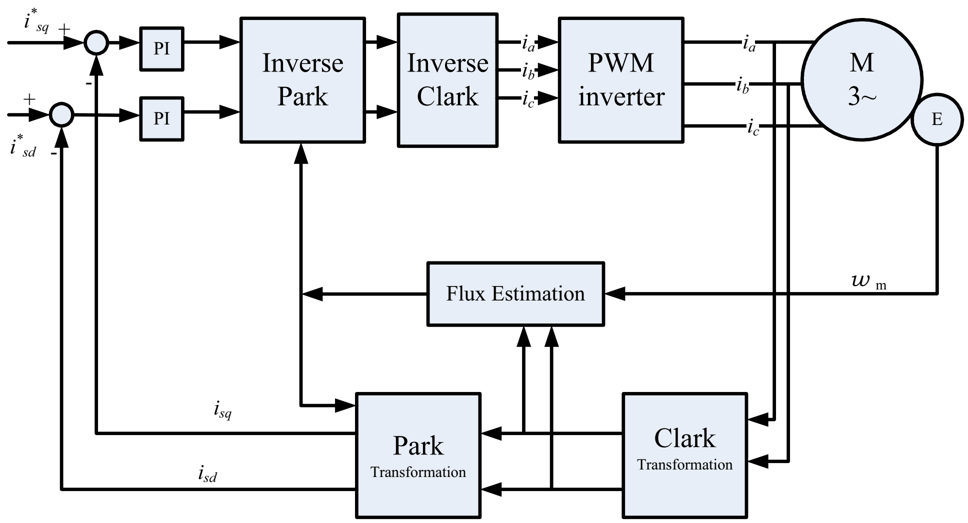



Figure 2.2: Vector control implementation

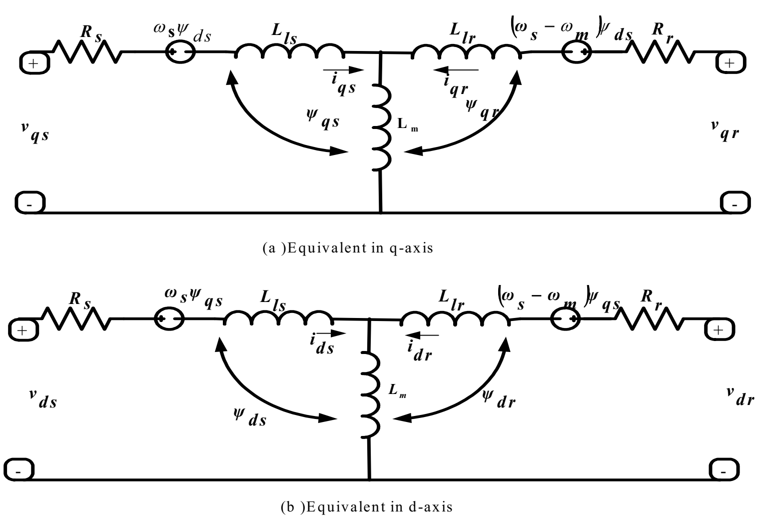



Figure 2.3: Dynamic equivalent circuit for induction motor.

and

$$\begin{aligned} L_s &= L_{ls} + L_m \\ L_r &= L_{lr} + L_m \end{aligned}$$

Where $R_s, R_r$ are stator, rotor resistance per phase respectively, $L_s, L_r$ are stator, rotor inductance per phase respectively, $p = \dfrac{d}{dt}$ operator, $\omega_s, \omega_m$ are synchronous and rotor speeds respectively, and scripts $l$ and $m$ represent leakage and magnetizing inductances, respectively. The stator flux linkages are given by matrix equation (2.3):

$$\begin{bmatrix} \psi_{qs} \\ \psi_{ds} \end{bmatrix} = \begin{bmatrix} i_{qr} & i_{qs} \\ i_{dr} & i_{ds} \end{bmatrix} \begin{bmatrix} L_m \\ L_s \end{bmatrix} \tag{2.3}$$

While rotor flux linkages are given by matrix equation (2.4):

$$\begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} = \begin{bmatrix} i_{qs} & i_{qr} \\ i_{ds} & i_{dr} \end{bmatrix} \begin{bmatrix} L_r \\ L_m \end{bmatrix} \tag{2.4}$$

Solving equation (2.4) for $i_{qr}, i_{dr}$ we obtain equation (2.5):

$$\begin{bmatrix} i_{qr} \\ i_{dr} \end{bmatrix} = \begin{bmatrix} \psi_{qr} & i_{qs} \\ \psi_{dr} & i_{ds} \end{bmatrix} \begin{bmatrix} \frac{1}{L_r} \\ \frac{-L_m}{L_r} \end{bmatrix} \tag{2.5}$$

The air gap flux linkages are given by matrix equation (2.6):

$$\begin{bmatrix} \psi_{qm} \\ \psi_{dm} \end{bmatrix} = \begin{bmatrix} i_{qr} & i_{qs} \\ i_{dr} & i_{ds} \end{bmatrix} \begin{bmatrix} L_m \\ L_m \end{bmatrix} \tag{2.6}$$

From equation(2.2) we obtain equation (2.7):

$$\frac{d}{dt} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} = \begin{bmatrix} i_{qr} & \psi_{qr} \\ i_{dr} & \psi_{dr} \end{bmatrix} \begin{bmatrix} -R \\ -(\omega_s - \omega_m) \end{bmatrix} \tag{2.7}$$

Substituting equation (2.5) in equation (2.7), we get the second order model of the three phase induction machine as follows:

$$\begin{bmatrix} \dot{\psi}_{qr} \\ \dot{\psi}_{dr} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & -(\omega_s - \omega_m) \\ (\omega_s - \omega_m) & -\frac{L_r}{R_r} \end{bmatrix} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} + \frac{L_m}{L_r} R_r \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} \tag{2.8}$$

This model (2.8) is a second order nonlinear model because $(\omega_m)$ is function of the states and the controls as shown in equation 2.13. It will be used to determine the optimal fluxes and input currents of the induction motor using optimal control theory. In addition, the torque developed by induction machine is given by (Rashid, 2004):

$$T_d = z\left[\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right] \tag{2.9}$$

Substituting equation (2.5) in equation (2.3), the following equation is yielded:

$$\begin{bmatrix} \psi_{qs} \\ \psi_{ds} \end{bmatrix} = \begin{bmatrix} i_{qr} & i_{qs} \\ i_{dr} & i_{ds} \end{bmatrix} \begin{bmatrix} L_s - \frac{L_m^2}{L_r} \\ \frac{L_m}{L_r} \end{bmatrix} \tag{2.10}$$

and by substituting equation (2.10) in equation (2.9), the following equation is yielded:

$$T_d = z\frac{L_m}{L_r}\left[\psi_{dr} i_{qs} - \psi_{qr} i_{ds}\right] \tag{2.11}$$

The mechanical model of the induction motor is described by:

$$\frac{I}{z}\left(\frac{d\omega_m}{dt}\right) = T_d - T_l - \frac{F}{z}\omega_m \tag{2.12}$$

Where $I$ is the moment of inertia, $F$ is viscose friction coefficient, $T_l$ is the load torque. By substituting equation (2.11) in equation (2.12), the following differential equation is obtained (Ouhrouche *et al*, 2000):

$$\dot{\omega} = \frac{z^2 L_m}{I L_r}\left(i_{sq}\psi_{rd} - i_{sd}\psi_{rq}\right) - \frac{F}{I}\omega_m - \frac{z}{I}T_l \tag{2.13}$$

Rewriting equations (2.8) and (2.13) in matrix form, we get the following differential equation:

$$\begin{bmatrix} \dot{\psi}_{qr} \\ \dot{\psi}_{dr} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & -\omega_s & \psi_{dr} \\ \omega_s & -\frac{L_r}{R_r} & -\psi_{qr} \\ -\dfrac{z^2 L_m}{I L_r} i_{sd} & \dfrac{z^2 L_m}{I L_r} i_{sq} & -\dfrac{F}{I} \end{bmatrix} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{L_m}{L_r} R_r i_{qs} \\ \frac{L_m}{L_r} R_r i_{ds} \\ -\dfrac{z}{I} T_l \end{bmatrix} \tag{2.14}$$

Equation (2.14) represents the nonlinear third-order model for the induction machine which will be used to determine the optimal fluxes, input currents, speed and load torque of IM using nonlinear optimal control theory.

# Chapter 3

# Optimal Control

There are several methods to control different systems.To determine the control signals that will cause a process to satisfy a physical constrains and at the same time minimize (or maximize) some performance index, the optimal control theory should be used. In addition, optimal control approach helps to deal with modern and complex systems.

## 3.1 General Optimal Control Problem

Assuming that the plant is described by nonlinear time varying dynamical differential equation:

$$\dot{x}(t) = f(x(t), u(t), t) \tag{3.1}$$

Where $x(t) \in R^n$ is the states of the plant, $u(t) \in R^m$ is the control input.

With the system associate performance measure:

$$J(t_0, T) = \phi(x(T), T) + \int_{t_0}^{T} L(x(t), u(t), t)dt \tag{3.2}$$

While $[t_0, T]$ is the interval of interest. The final weighting function $\phi(x(T), T)$ depends on the final state and final time, and the weighting function $L(x(t), u(t), t)$ depends on the state and input at intermediate times in $[t_0, T]$ ,(Lewis *et al*, 1995).

Now the optimal control problem is to find the input $u^*(t)$ on the time interval to control the plant equation (3.1) along trajectories $x^*(t)$ that minimize the cost function (3.2) as shown in figure (3.1).

After the mathematical model of the system (equation 3.1) is determined, and the physical boundary values $t_0$, $T$, $x(t_0)$, $x(T)$ are also determined, the next step is to define the physical performance measure. Section 3.2 shows some useful performance measures (or performance indices) and their meanings.

## 3.2 Useful Performance Indices

In this section, some common performance indices will be discussed, so one of them could be selected for system equation (3.1) (Lewis *et al*, 1995, Pryson *et al,* 1975, Kirk, 1970):

### 3.2.1 Minimum Time Problem:

Assuming that the control input $u^*(t)$ should be found to drive the system from the given initial states ($x(t_0) = x_0$) to a desired final states $x_f$ in minimum time, this performance index could be selected:

$$J = T - t_0 = \int_{t_0}^{T} dt$$

In this case, the final weighting function $\phi(x(T), T) = T - t_0$ and the

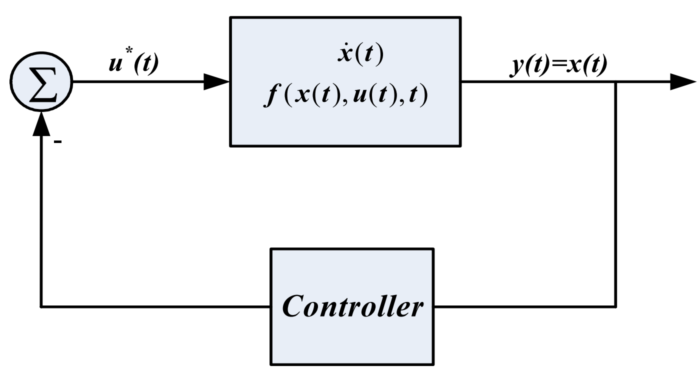



Figure 3.1: Optimal control problem representation

weighting function $L(x(t), u(t), t) = 0$ or equivalently $\phi(x(T), T) = 0$ and $L(x(t), u(t), t) = 1$ (Kirk, 1970).

### 3.2.2 Minimum Fuel Problem:

Assuming that the control input $u^*(t)$ should be found to drive the system from the given initial states ($x(t_0) = x_0$) to a desired final states $x_f$ at fixed time $T$ using minimum fuel, this performance index could be selected:

$$J = \int_{t_0}^{T} |u(t)| \, dt$$

In this case, the final weighting function $\phi(x(T), T) = 0$ and the weighting function $L(x(t), u(t), t) = |u(t)|$.

### 3.2.3 Tracking Problem:

Assuming that the control input $u^*(t)$ should be found to drive the system with a closed state vector ($x(t)$) to a desired state vector ($r(t)$) as possible at fixed time $T$. this performance index could be used:

$$J = \int_{0}^{T} (x(t) - r(t))^T Q \, (x(t) - r(t)) \, dt$$

Where $Q$ is a real $(n \times n)$ positive definite and symmetric state weighting matrix $(Q \geq 0)$.

In this case, the final weighting function $\phi(x(T), T) = 0$, and the weighting function $L(x(t), u(t), t) = (x(t) - r(t))^T Q \, (x(t) - r(t))$ (Kirk, 1970).

### 3.2.4 Minimum Energy Problem:

Assuming that the control input $u^*(t)$ should be found to minimize the energy of the final state, and intermediate states, and also controls at fixed time $T$, this performance index could be used (Kirk, 1970):

$$J = \frac{1}{2}x^T S_f x + \frac{1}{2}\int_0^T \left(x^T Q x + u^T R u\right) dt$$

where $S$ and $Q$ are real $(n \times n)$ a positive semidefinite and symmetric state weighting matrix $(S, Q \geq 0)$, and $R$ is real $(m \times m)$ positive definite symmetric control weighting matrix $(R > 0)$.

In this case, the final weighting function $\phi(x(T), T) = \frac{1}{2}x^T S_f x$, and the weighting function $L(x(t), u(t), t) = \frac{1}{2}x^T Q x + \frac{1}{2}u^T R u$.

Using this performance index corresponds keeping the state vector ($x$) and control vector ($u$) close to zero (Kirk, 1970).

In this thesis, minimum- energy problem to minimize the total energy in the induction motor is used, which is the sum of the stored magnetic energy in the inductance, the dissipated energy in the rotor and stator resistances, the dissipated energy due to core losses (eddy currents and magnetic hysteresis), and mechanical energy (Ramirez *et al,* In Press, Georges *et al*, In Press-a).

### 3.3 Linear Quadratic Optimal Control Problem

Section 3.1 showed the optimal control problem statement for general nonlinear system. Now we will consider the linear time invariant system:

$$\dot{x}\left(t\right) = Ax\left(t\right) + Bu\left(t\right)$$

Where $x \in R^n$, $u \in R^m$ with associate quadratic performance index:

$$J = \frac{1}{2}x^T(T)\,S_f x(T) + \frac{1}{2}\int_0^T \left(x^T(t)\,Qx(t) + u^T(t)\,Ru(t)\right)dt \qquad (3.3)$$

The time interval over which were interested in the behavior of the plant is $[0, T]$, we have to determine the control vector $u^*(t)$ on $[0, T]$ that minimizes the performance index (3.3) for feedback control.

The initial system state $x(0)$ is given, state weighting matrices $S$ and $Q$ are symmetric positive semi-definite, and control weighting matrix $R$ is symmetric positive definite for all $t \in [0, T]$.

The solution of this optimal control problem is shown in (Lewis *et al*, 1995, Pryson *et al,* 1975, Kirk, 1970) in detail, and the result of this solution is:

$$u^*(t) = -Kx(t) \qquad (3.4)$$

Where K is called the Kallman gain and is defined by:

$$\begin{aligned} K(t) &= R^{-1}B^T S \\ t &\leq T \end{aligned}$$

Where $S$ is the solution of the following algebraic differential equation:

$$-\dot{S} = A^T S + SA - SBR^{-1}B^T S + Q$$

The control system shown in equation (3.4) is a time varying state feedback. The closed loop plant can be written as:

$$\dot{x}(t) = (A - BK)\,x(t)$$

Since $A$, $B$, $Q$, and $R$ are time invariant, this can be used to find the optimal state trajectory $x^*(t)$ given any $u^*(t)$.

For the case of time invariant and $\lim_{T\to\infty} S_f(T) = 0$, the optimal control problem is given by:

$$\underset{u}{\text{Min}}\ J = \frac{1}{2}\int_0^\infty \left(x^T(t)\,Qx(t) + u^T(t)\,Ru(t)\right)dt$$

subject to the system equation

$$\dot{x}(t) = Ax(t) + Bu(t)$$

The optimal state feedback control that solves this problem is given by:

$$u^*(t) = -Kx(t)$$

And the Kallman matrix is given by:

$$K = R^{-1}B^T S$$

where $S$ is the solution of the following Algebraic Ricatti Equation (ARE) (Lewis *et al*, 1995, Pryson *et al,* 1975, Kirk, 1970):

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{3.5}$$

## 3.4 Linear System with Known Disturbance

For any linear time invariant systems with known disturbance $h(t)$:

$$\dot{x}(t) = Ax(t) + Bu(t) + h(t)$$

Where $x \in R^n$, $u \in R^m$, $h \in R^n$ with associate performance index:

$$J = \frac{1}{2}x^T(T)\,S_f x(T) + \frac{1}{2}\int_{t_0}^T \left(x^T(t)\,Qx(t) + u^T(t)\,Ru(t)\right)dt$$

The initial system state $x(t_0)$ is given, state weighting matrices $S$ and $Q$ are symmetric positive semi-definit, and control weighting matrix $R$ is symmetric positive definite for all $t \in [t_0, T]$.

The optimal state feedback control that solves this problem is:

$$u^*(t) = -Kx\left(t\right) + R^{-1}Bv\left(t\right)$$

And the Kallman matrix is given by:

$$K = R^{-1}B^T S$$

Where $S$ is the solution of the following algebraic differential equation:

$$\begin{aligned} -\dot{S} &= A^T S + SA - SBR^{-1}B^T S + Q \\ t &\leq T \end{aligned}$$

And $v$ is the solution of the differential equation:

$$\dot{v}\left(t\right) = \left(A - BK\right)^T \; v\left(t\right) + Sh(t)$$

For the case of time invariant and $\lim\limits_{T \to \infty} S(T) = 0$, the optimal contorl problem will become:

$$\underset{u}{\text{Min}} \; J = \frac{1}{2} \int\limits_0^\infty \left(x^T\left(t\right) Qx\left(t\right) + u^T\left(t\right) Ru\left(t\right)\right) dt$$

subject to the system equation

$$\dot{x}\left(t\right) = Ax\left(t\right) + Bu\left(t\right) + h(t)$$

Then, the optimal state feedback control that solves this problem can be given by:

$$u^*(t) = -Kx\left(t\right) + R^{-1}Bv\left(t\right)$$

where the Kallman matrix is given by:

$$K(t) = R^{-1} B^T S$$

and $S$ is the solution of the following Algebraic Riccati Equation (ARE):

$$\begin{aligned} A^T S + SA - SBR^{-1} B^T S + Q &= 0 \\ t &\leq T \end{aligned}$$

And $v$ is given by:

$$v(t) = \left( (A - BK)^T \right)^{-1} Sh(t)$$

### 3.5 Nonlinear Optimal Control Problem

Since many problems are described by a strongly nonlinear differential equation, or consist of nonlinear or complex performance index, we must apply a numerical method to solve the optimal programming and control problem (Pryson *et al,* 1975).

Moreover, several methods have been proposed to solve a nonlinear optimal control problem. For example, discretization method, parameterization method (Goh, Teo, 1988, Frick *et al*, 1995), steepest decent method, and quazilinearization method (Vlassenbroeck *et al*, 1988, Jaddu, Shimemura, 1999).

Converting an optimal control problem into mathematical programming problem using discretization technique or parameterization technique is classified as direct technique (Jaddu, 2002). In addition, steepest decent method converts a nonlinear optimal control problem into a mathematical programming problem using gradient of Hamiltonian function and considering an analog calculus problem.

On the other hand, quazilineariztion method can be used in nonlinear optimal control problem in two different ways (Bellman *et al*, 1965):

**1. Linearizing two point boundary value problem (TPBVP)**.

The widely used method is to linearize the Euler-Lagrange system of differentialequations around nominal trajectories, and the optimal control problem can be solved by solving successively a sequence of two linear point boundary value problem.

**2. By solving a sequence of linear quadratic optimal control problems.**

In this method, the performance index were expanded up to the second order, and we linearize the system differential equation around nominal trajectories. Therefore, the original optimal control problem can be solved be solving sequence of linear quadratic (LQ) problems

In this thesis a quasilineariztion method will be used by solving a sequence of LQ problem to optimize the induction machine performances.

### 3.6 Quazilineariztion Method

First, let us define the problem statement:

Find the optimal control $u^*(t)$ that minimizes the performance index:

$$J = \frac{1}{2}\int_0^T \left(x^T Q x + u^T R u\right) dt \tag{3.6}$$

subject to the nonlinear system equation:

$$\dot{x}(t) = f(x(t), u(t), t) \tag{3.7}$$

and initial states

$$x(0) = x_0 \tag{3.8}$$

Where $x(t) \in R^n$ is the states of the plant, $u(t) \in R^m$ is the control input, $t \in [0, T]$, $Q$ is real $(n \times n)$ a positive semidefinite and symmetric state weighting matrix $(Q \geq 0)$, and $R$ is real $(m \times m)$ positive definite symmetric control weighting matrix $(R > 0)$.

Applying the quazilineariztion method, the optimal control problem equations (3.6) to (3.8) can be replaced with following linear quadratic optimal control problem (Jaddu, 2002):

Find the optimal control $u^{*^{(k+1)}}(t)$that minimizes the performance index:

$$J = \frac{1}{2}\int_0^T \left( x^{(k+1)^T} Q x^{(k+1)} + u^{(k+1)^T} R u^{(k+1)} \right) dt$$

subject to the linearized system state equation (3.9):

$$\dot{x}^{(k+1)} = A^{(k)} x^{(k+1)}(t) + B^{(k)} u^{(k+1)}(t) + h^{(k)}(t) \tag{3.9}$$

with initial condition:

$$x^{(k+1)}(0) = x_0$$

where script $k$ represents the iteration number, $A^{(k)}$, $B^k$,and $h^{(k)}$ can be written:

$$A^{(k)} = \frac{\partial \dot{x}(x, u, t)}{\partial x} \mid_{x^{(k)}, u^{(k)}}$$

$$B^{(k)} = \frac{\partial \dot{x}\left(x,u,t\right)}{\partial u}\mid_{x^{(k)},u^{(k)}}$$

$$h^{(k)}\left(t\right) = \dot{x}^{(k)} - Ax^{(k)}\left(t\right) + B^{k}u^{(k)}\left(t\right)$$

By considering the term $h^{(k)}\left(t\right)$ is disturbance input for the system and $T = \infty$, an optimal control problem is solved by the theory presented section 3.4.

So that the optimal state feedback control is:

$$u^{*(k+1)} = -K^{(k+1)}x^{(k+1)} + R^{-1}B^{(k)}v^{(k+1)}$$

$$K^{(k+1)} = R^{-1}B^{(k)^{T}}S^{(k+1)}$$

while $S^{(k+1)}$ solve the Algebraic Racciti Equation:

$$A^{(k)^{T}}S^{(k+1)} + S^{(k+1)}A^{(k)} - S^{(k+1)}B^{(k)}R^{-1}B^{(k)^{T}}S^{(k+1)} + Q = 0$$

and

$$v^{(k+1)} = -\left(A^{(k)} - B^{(k)}K^{(k+1)^{T}}\right)^{-1}S^{(k+1)}h^{(k)}$$

We guess the values $x^{(0)}(t)$, $u^{(0)}(t)$ at the beginning to find the matrices $A^{(0)}, B^{(0)}, h^{(0)}$, then we solve the problem as a linear quadratic optimal problem with disturbance input to find $v^{(1)}, S^{(1)}, K^{(1)}, x^{(1)}$, and $u^{(1)}$. Then by using $x^{(1)}$ and $u^{(1)}$ we find $A^{(1)}, B^{(1)}$, and $h^{(1)}$ for next iteration, and so on, until an acceptable convergence is reached.

# Chapter 4

# Induction Machine Optimal Control

Chapter two showed the induction machine model using attractive method "Field Oriented Algorithm", and chapter three presented directed approaches that were used to analyze and control certain systems, satisfying physical constrains and maintaining certain performance index at the same time.

In this chapter, the optimal control problem of the induction motor model, which is a very famous physical system, is formulated.

Because the induction motors are widely used in industry, especially in high power ranges of motors, reducing (or increasing efficiency) the total energy consumed by induction motor is a very important aspect for engineers. Optimal control method help us to approach these aspects.

To apply the optimal control theory on the induction machine system, first we will study the energy measure of the induction motor.

## 4.1 Energy Measure of the Induction Motor

The total energy consumed by induction motor is equal to the total energy in the electro mechanical system, which are:

1. Stored magnetic energy inductance.
2. Dissipated energy in the rotor and stator resistances (Copper losses).
3. Dissipated energy causes by fluctuate currents and hystersis (Core

Loses).

4. Mechanical energy measure.

The previous four energy measures were discussed to obtain the total energy measure in the induction motor.

### 4.1.1 Magnetic Energy Measure:

The total stored magnetic energy in the induction motor is (Ramirez *et al,* In Press):

$$W_{mag.} = \frac{1}{2}L_s\left[1-\frac{L_m^2}{L_sL_r}\right]\left(i_{sq}^2+i_{sd}^2\right)+\frac{1}{2L_r}\left(\psi_{rq}^2+\psi_{rd}^2\right)$$

$$= \frac{1}{2}L_s\left[1-\frac{L_m^2}{L_sL_r}\right]\left[\begin{array}{cc} i_{sq} & i_{sd}\end{array}\right].\left[\begin{array}{c} i_{sq} \\ i_{sd}\end{array}\right]+\frac{1}{2L_r}\left[\begin{array}{cc} \psi_{rq} & \psi_{rd}\end{array}\right].\left[\begin{array}{c} \psi_{rq} \\ \psi_{rd}\end{array}\right]$$

$$W_{mag.} = \frac{1}{2}\left[L_s.\sigma.i_{sqd}^T.i_{sqd}+\frac{1}{L_r}.\psi_{rqd}^T.\psi_{rqd}\right]$$

while $i_{sqd} = \left[\begin{array}{cc} i_{sq} & i_{sd}\end{array}\right]^T$, $\psi_{rqd} = \left[\begin{array}{cc} \psi_{rq} & \psi_{rd}\end{array}\right]^T$,and $\sigma = \left[1-\frac{L_m^2}{L_sL_r}\right]$.

### 4.1.2 Copper Losses Measure:

The copper losses measure equal the total energy dissipated by stator and rotor resistances ($R_s$, and $R_r$), and it is given by the integral of the coil losses:

$$P_{cu} = \frac{1}{2}R_si_{sq}^2+\frac{1}{2}R_si_{sd}^2+\frac{1}{2}R_ri_{rq}^2+\frac{1}{2}R_ri_{rd}^2$$

$$P_{cu} = \frac{1}{2}R_s\left[\begin{array}{cc} i_{sq} & i_{sd}\end{array}\right].\left[\begin{array}{c} i_{sq} \\ i_{sd}\end{array}\right]+\frac{1}{2}R_r\left[\begin{array}{cc} i_{rq} & i_{rd}\end{array}\right].\left[\begin{array}{c} i_{rq} \\ i_{rd}\end{array}\right] \qquad (4.1)$$

Substituting equation (2.5) in equation (4.1), the total coil losses will be:

$$P_{cu} = \frac{1}{2}\left(\left[R_s+\frac{R_rL_m^2}{L_r}\right]\left[\begin{array}{cc} i_{sq} & i_{sd}\end{array}\right].\left[\begin{array}{c} i_{sq} \\ i_{sd}\end{array}\right]+\frac{R_r}{L_r}\left[\begin{array}{cc} \psi_{rq} & \psi_{rd}\end{array}\right].\left[\begin{array}{c} \psi_{rq} \\ \psi_{rd}\end{array}\right]\right)-\Delta P$$

$$= \frac{1}{2}\left(\left(R_s+\frac{R_r}{L_r}L_m^2\right).i_{sqd}^T.i_{sqd}+\frac{R_r}{L_r}\psi_{rqd}^T.\psi_{rqd}\right)-\Delta P$$

while $\Delta P = L_m \left(\psi_{rq}.i_{sq} + \psi_{rd}.i_{sd}\right)$ .

### 4.1.3 Core Losses Measure:

Core losses are the losses due to fluctuate currents and hysteresis, and they are very difficult to model . In addition , they do not make matter compared with copper losses, especially in well design induction machines. Therefore, we will not consider them here (Ramirez *et al,* In Press, Georges *et al*, In Press-a, Georges *et al*, In Press-b).

### 4.1.4 Mechanical Energy Measure:

Mechanical energy usually is defined by the desired acceleration and velocity time profile, so that no mechanical energy minimization is required. However, the desired or load torque profile is computed from the mechanical equation (2.12), and the mechanical energy is the integral of the equation (4.2):

$$P_{mech} = \omega_m.T_L \tag{4.2}$$

### 4.1.5 Total Energy Measure:

From the second order model of equation (2.8), we can easily write the flux norm variation as:

$$\frac{d}{dt}\left[\frac{\psi_{rqd}^T.\psi_{rqd}}{2}\right] = -\frac{R_r}{L_r}\psi_{rq}^2 - \left(\omega_{s-}\omega_m\right)\psi_{rd} - \frac{R_r}{L_r}\psi_{rd}^2 + \left(\omega_{s-}\omega_m\right)\psi_{rq}$$

$$-\frac{R_r L_m}{L_r}\left[\begin{array}{cc}\psi_{rq} & \psi_{rd}\end{array}\right]\left[\begin{array}{c}i_{sq}\\ i_{sd}\end{array}\right] \tag{4.3}$$

Noting that the value $\left(\omega_{s-}\omega_m\right)$ is very small, and term $\left(\left(\omega_{s-}\omega_m\right)\psi_{rd}\right)$ is also a very small value with respect to $\left(\frac{R_r}{L_r}\psi_{rq}^2\right)$, equation (4.3) could be rewritten as:

$$\frac{d}{dt}\left[\frac{\psi_{rqd}^T.\psi_{rqd}}{2}\right] = -\frac{R_r}{L_r}\psi_{rqd}^T.\psi_{rqd} - \frac{R_r L_m}{L_r}\psi_{rqd}^T i_{sqd}$$

And the total energy measure using the flux norm variation equal the integral of the value:

$$P = \frac{1}{2}\left\{\left(R_s + \frac{R_r}{L_r^2}L_m^2\right).i_{sqd}^T.i_{sqd} - \frac{R_r}{L_r^2}\psi_{rqd}^T.\psi_{rqd} - \frac{1}{L_r}\frac{d}{dt}\left(\psi_{rqd}^T.\psi_{rqd}\right)\right\} \tag{4.4}$$

Since the integral of the last term in equation (4.4) depends on the boundary values $[0, T]$, this means that:

$$W = \frac{1}{2L_r}\int_0^T \frac{d}{dt}\left(\psi_{rqd}^T.\psi_{rqd}\right)dt = \frac{1}{2L_r}\left[\psi_{rqd}^T(T).\psi_{rqd}(T) - \psi_{rqd}^T(0).\psi_{rqd}(0)\right]$$

This value of losses does not change the energy dissipated by coil losses with the interval [0,T], so that we can omit this term.

To summarize , the suitable energy , on the form of cost function to be minimized could be (Ramirez *et al,* In Press):

$$L\left(\psi_{rqd}, i_{sqd}\right) = \alpha_1 W_{mag} + \alpha_2 \bar{P}$$

$$= \frac{1}{2}\left(\alpha_1 L_s\left(1 - \frac{L_m^2}{L_s L_r}\right) + \left(\alpha_2\left(R_s + R_r\frac{L_m^2}{L_r^2}\right)i_{sqd}^T.i_{sqd}\right)\right.$$
$$\left. + \left(\frac{\alpha_1}{L_r} - \frac{\alpha_2}{L_r^2}\right)R_r\psi_{rqd}^T.\psi_{rqd}\right)$$

$$= \frac{1}{2}\left(i_{sqd}^T.R.i_{sqd} + \psi_{rqd}^T.Q.\psi_{rqd}\right)$$

Where $\bar{P} = P + \frac{1}{2L_r}\frac{d}{dt}\left(\psi_{rqd}^T.\psi_{rqd}\right)$, $\alpha_1 > 0, \alpha_2 > 0$ satisfy $(\alpha_1 L_r - \alpha_2 R_r > 0)$ used to scale quantities in defined power energy combined convex criteria. $Q$

at least is semi-definite positive matrix, and $R$ is positive definite matrix.

Minimizing the stored energy and coil losses causes maximizing the machine efficiency, and maximum efficiency is usually obtained at rated operating points (rotor and stator currents, rotor and stator fluxes, and torque) (Ramirez *et al,* In Press, Leonard, 1985, Seleme *et al,* 1992). By choosing weighting matrix R and Q different optimal solutions can be obtained.

## 4.2 Problem Formulation

The optimal control problem as presented in section 3.1 was applied on the induction machine plant to minimize the total stored energy and coil losses or maximize machine efficiency. This is stated as:

Find the state feedback control $u^*$ that minimize the performance index

$$J = \phi(x(T), T) + \int_0^T L(x(t), u(t), t)dt$$

subject to machine state equation

$$\dot{x}(t) = f(x(t), u(t), t), \ \ (x(0) = x_0)$$

And the induction machine run on the interval $[0, \infty]$, i.e. it runs in continuous duty or $S1$ duty(Dubey, 1995), and weighting function $\phi(x(T), T) = 0$.

So that the optimal control problem becomes:

Find the state feedback control $u^*$ that minimizes the performance index

$$J = \int_0^\infty L(x(t), u(t), t)dt$$

subject to machine state equation

$$\dot{x}(t) = f(x(t), u(t), t), \ \ (x\,(0) = x_0)$$

Here $L(x(t), u(t), t) = \frac{1}{2}\left(x^T Q x + u^T R u\right)$, and state vector $x = \psi_{rqd} = \left[\begin{array}{cc} \psi_{rq} & \psi_{rd} \end{array}\right]^T$ ,and control vector $u = i_{sqd} = \left[\begin{array}{cc} i_{sq} & i_{sd} \end{array}\right]^T$.

In this thesis, we have two problems to be treated :

**Problem I**: Based on the first model (2.8), two linear optimal control problems are solved, which are obtained by linearizing the nonlinear model (2.8) using some properties of IM.

The optimal control problem then states: find the control input $u^*\,(t)$ to minimize the performance index

$$J = \frac{1}{2}\int\limits_0^\infty \left(\psi_{rqd}^T.Q.\psi_{rqd} + i_{sqd}^T.R.i_{sqd}\right) dt \tag{4.5}$$

subject to the IM model equation (2.8).

**Problem II**: Based on the second model (2.14), two cases of nonlinear optimal control problem are solved. In the first case the torque is considered as input and in the second case the torque is considered as a fixed load.

Then optimal control problem: find the control input $u^*\,(t)$ to minimize the performance index equation (4.5) subject to the IM model equation (2.14).

In the next chapter, these problem solutions and the simulation results are shown.

# Chapter 5

# Induction Machine Optimal Control Problem Solution

Chapter 4 showed the design theory for optimal state feedback control to the induction motor, this design should be stable and make the system run with minimum energy or maximum efficiency.

In this chapter, the optimal control technique shown in chapter 4 is applied to induction motor models equations (2.8) and (2.14) to find the IM states and controls. Moreover, the behavior and response of these controls will be shown for different cases.

To demonstrate the presented solutions for different cases "either in linear or nonlinear cases" digital simulation programs were used, and they are MATLAB 7.0.1 and SIMULINK 6.1 (see Appendices A1, and A2).

Built in functions by MATLAB 7.0.1 like (lqr, and care) were used. These functions help us to solve the continuous Algebraic Ricatti Equation.

Moreover, to demonstrate the simulations we used the following motor parameters: Power= 1hp, Rated speed =1440 rpm, z= 2 pole pairs, $R_s$= 1.15 $\Omega$, $R_r$ =1.44 $\Omega$, $L_m$=0.144 H, $L_s$=$L_r$ $=$ 0.156 H , $I$ $=$ 0.013 kg.m$^2$, $F$ $=$ 0.002 Nm.s/rad.

## 5.1 Linear Optimal Control Cases

To over come the problem of nonlinearity of the induction machine equation

(2.8), we will consider two cases based on the induction machine properties:

**Case 1** *Frequency difference between the synchronous speed and mechanical speed is zero, i.e.* (Mathwork, 2002, Ismail *et al,* In Press)*:*

$$\omega_{s-}\omega_m = 0$$

Therefore the optimal control problem becomes: Find the state feedback control vector $\left(u^* = i_{sqd}^* = \begin{bmatrix} i_{sq}^* & i_{sd}^* \end{bmatrix}^T\right)$ that minimizes the performance index

$$J = \frac{1}{2}\int_0^\infty \left(\psi_{rqd}^T.Q.\psi_{rqd} + i_{sqd}^T.R.i_{sqd}\right) dt$$

subject to the induction motor model:

$$\frac{d}{dt}\begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & 0 \\ 0 & -\frac{L_r}{R_r} \end{bmatrix}\begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} + \frac{L_m}{L_r}R_r\begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix}$$

For simplicity, this system can be rewritten in a compact form "$x = AX + Bu$" with $A = \begin{bmatrix} -\frac{L_r}{R_r} & 0 \\ 0 & -\frac{L_r}{R_r} \end{bmatrix}$, and $B = \begin{bmatrix} \frac{L_m}{L_r}R_r & 0 \\ 0 & \frac{L_m}{L_r}R_r \end{bmatrix}$.

This problem can be solved using the optimal control technique presented in section 3.3, i.e.

$$u^* = i_{sqd}^* = -K\psi_{rqd}^*$$

$$\textit{Kallman Gain:} \quad K = -R^{-1}B^TS$$

$$A^TS + SA - SBR^{-1}B^TS + Q = 0$$

**Case 2** *Frequency difference between the synchronous speed and mechanical speed equals slip speed* (Mcpherson *et al,* 1990, Novotny *et al*, 1995, Rashid, 2004)*, i.e:*

$$\text{slip speed} = s\omega_s = \omega_{s-}\omega_m$$

Therefore the optimal control problem becomes:

Find the state feedback control vector $\left(u^* = i_{sqd}^* = \begin{bmatrix} i_{sq}^* & i_{sd}^* \end{bmatrix}^T\right)$ that minimizes the performance index

$$J = \frac{1}{2} \int_{0}^{\infty} \left( \psi_{rqd}^{T}.Q.\psi_{rqd} + i_{sqd}^{T}.R.i_{sqd} \right) dt$$

subject to the induction motor model: $\dot{\psi}_{rqd} = A\psi_{rqd} + Bi_{sqd}$, while $A = \begin{bmatrix} -\frac{L_r}{R_r} & -s\omega_s \\ s\omega_s & -\frac{L_r}{R_r} \end{bmatrix}, B = \begin{bmatrix} \frac{L_m}{L_r}R_r & 0 \\ 0 & \frac{L_m}{L_r}R_r \end{bmatrix}$.

Also, this problem can be solved using optimal control theory presented in section 3.3 similar to the previous case.

## 5.2 Simulation of Linear OCP

To simulate the linear optimal control problem (OCP) solved in section 4.2, different weighting matrices $Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$,and $R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$ have been considered, with their eigenvalues $q_1, q_2 > 0$ , and $r_1, r_2 > 0$.

The simulation has been performed using five sets of $(Q$ and $R)$; $q_1, q_2 = 0.01, 0.1, 1, 10, 100$ , and $r_1, r_2 = 0.01, 0.1, 1, 10, 100$ respectively and with initial state $x_0 = [-5 - 5]$ weber .

Figures (5.1), (5.2), and (5.3) show the simulation result of case 1. Figure (5.1) the state feedback control vector (stator currents).

Figure (5.2) shows the phases currents $(I_a,\ I_b,\ and\ I_c)$ of IM.

While figure (5.3) shows the optimal system state trajectories (rotor fluxs).

Moreover, to show the simulation result of the rotor speed, we used the speed differential equation (2.13), and figure (5.4) shows the mechanical speed under constant torque 1.5Nm and zero initial speed .

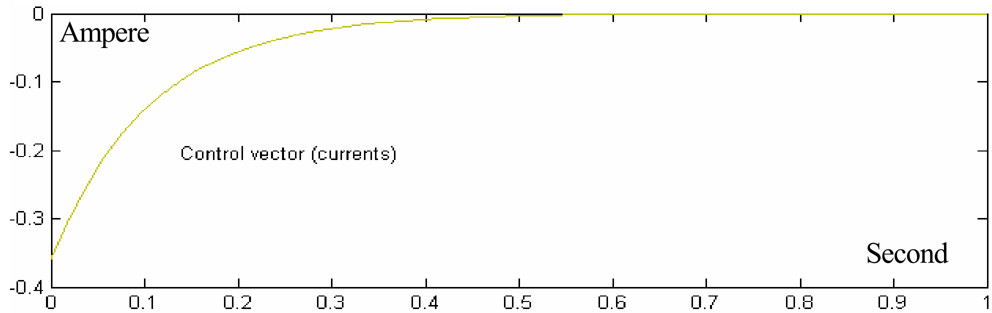



Figure 5.1: State feedback control $(i_d, i_q)$ using LQR method for 1hp, 4 poles IM and load torque 1.5 Nm (case1)

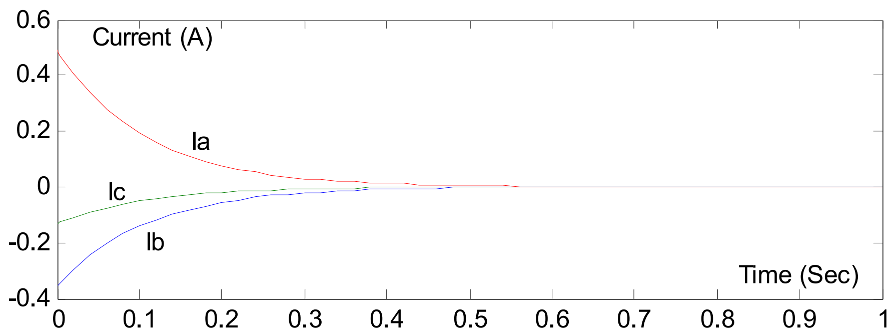



Figure 5.2: Phases currents simulation usnig LQR OCP $(I_a, I_b, I_c)$ for 1hp, 4 poles IM and load torque 1.5 Nm (case1)

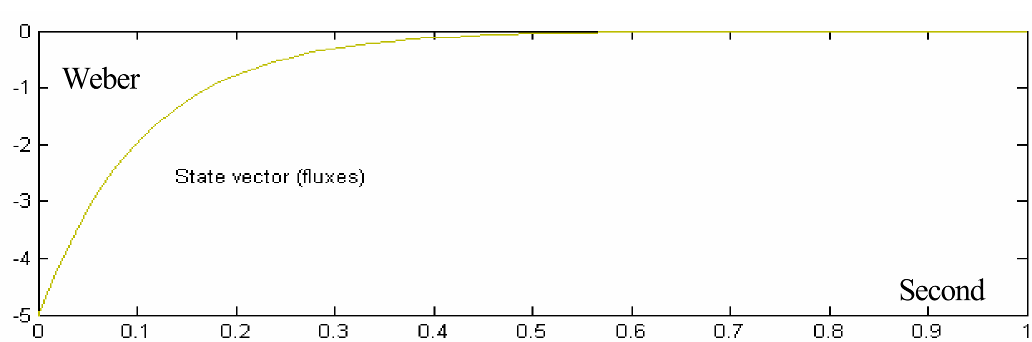



Figure 5.3: States $(\psi_d, \psi_q)$ using LQR method for 1hp, 4 poles IM and load torque 1.5 Nm (case1)

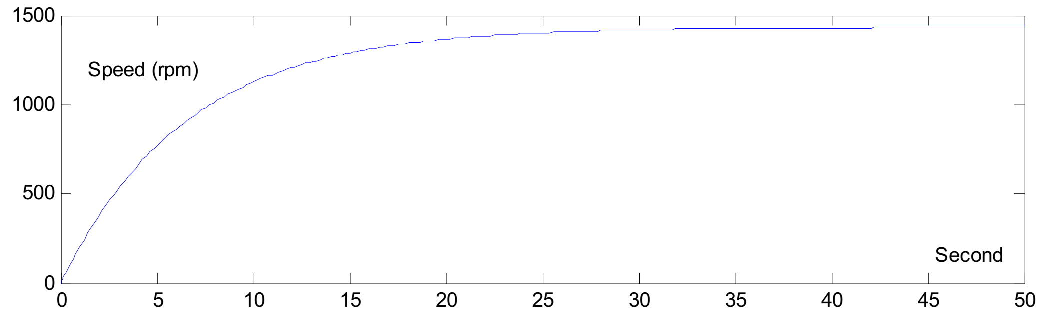
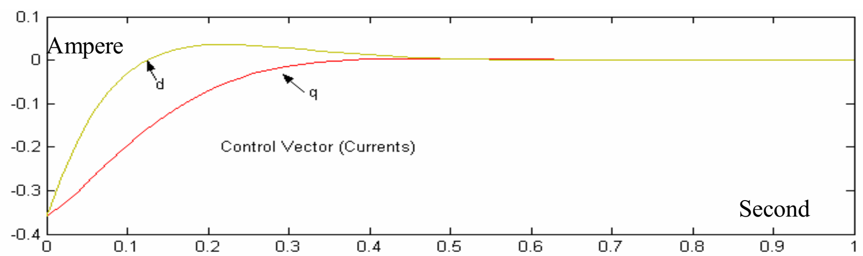



Figure 5.4: Speed simulation $(n)$ result using LQR method for 1hp, 4 poles IM and load torque 1.5 Nm (case1)




(b)

Figure 5.5: State feedback control $(i_d, i_q)$ using LQR method for 1hp, 4 poles IM and load torque 1.5 Nm (case2)

Figures (5.7), (5.5), and (5.8) show the simulation results of the case 2 of section 4.2. Where figure (5.7) shows the state feedback control trajectories (fluxes), figure (5.5) shows the state trajectories (currents), and figure (5.8) shows the motor speed trajectories under constant torque 1.5 Nm.

Figure (5.6) shows the phases currents $(I_a,\ I_b,\ and\ I_c)$ of IM.

From previous simulations, we notice that changs in weighting matrices do not affect the optimal controls or optimal states.

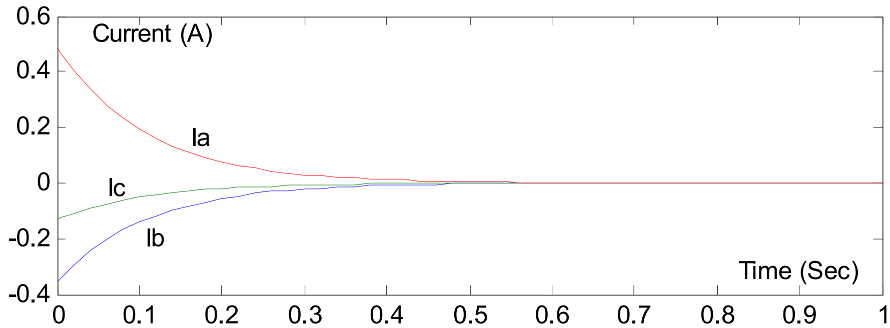



Figure 5.6: Phases currents simulation usnig LQR OCP ($I_a, I_b, I_c$) for 1hp, 4 poles IM and load torque 1.5 Nm (case2)

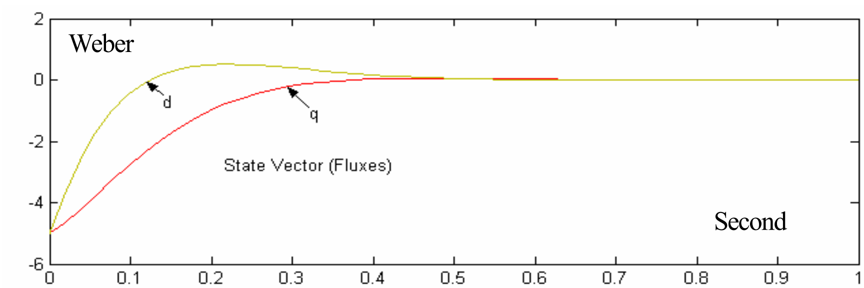



Figure 5.7: States ($\psi_d, \psi_q$) using LQR method for 1hp, 4 poles IM and load torque 1.5 Nm (case2)

## 5.3 Nonlinear Optimal Control Cases

The second approach to solve the optimal performance measure for the induction motor is solving a nonlinear optimal control problem. Therefore, we used the third order model of the induction motor equation (2.14). In addition, we did not make any modification on the induction motor model.

Thus, the nonlinear optimal control problem given by:

Find the input control vector $(u^*)$ that minimizes the performance index

$$J = \frac{1}{2}\int_0^\infty \left(x^T.Q.x + u^T.R.u\right) dt$$

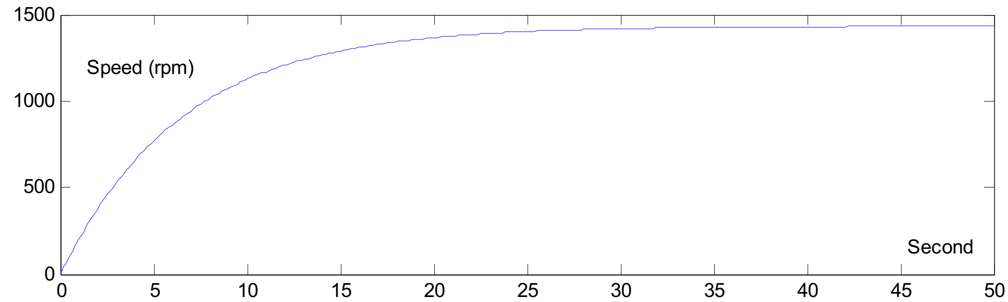



Figure 5.8: Speed simulation $(n)$ result using LQR method for 1hp, 4 poles IM and load torque 1.5 Nm (case2)

subject to the induction motor model equation (2.14)

And this nonlinear optimal control problem can be solved using optimal control technique presented in section 3.6.

Moreover, we will consider two cases while we are solving this problem.

**Case 3** *Load torque is control input for the induction motor system (e.g. brake).*

Thus, the third order model is $(\dot{x} = Ax + Bu)$, the control vector $u = \begin{bmatrix} i_{sq} & i_{sd} & T_l \end{bmatrix}^T$ and the state vector is $x = \begin{bmatrix} \psi_{rq} & \psi_{rd} & \omega_m \end{bmatrix}^T$.

To convert our nonlinear optimal control problem to a sequence of linear quadratic optimal control problem as presented in section 3.6, we must linearize the model around known trajectories, so that:

$$\begin{bmatrix} \dot{\psi}_{qr}^{(k+1)} \\ \dot{\psi}_{dr}^{(k+1)} \\ \dot{\omega}^{(k+1)} \end{bmatrix} = A^{(k)} \begin{bmatrix} \psi_{qr}^{(k+1)}(t) \\ \psi_{dr}^{(k+1)}(t) \\ \omega_m^{(k+1)}(t) \end{bmatrix} + B^{(k)} \begin{bmatrix} i_{qs}^{(k+1)}(t) \\ i_{ds}^{(k+1)}(t) \\ T_l^{(k+1)}(t) \end{bmatrix} + h^{(k)}(t) \qquad (5.1)$$

where

$$A^{(k)} = \begin{bmatrix} -\frac{L_r}{R_r} & -\omega_s + \omega_m^{(k)} & \psi_{dr}^{(k)} \\ \omega_s - \omega_m^{(k)} & -\frac{L_r}{R_r} & -\psi_{qr}^{(k)} \\ -\dfrac{z^2 L_m}{I L_r} i_{sd}^{(k)} & \dfrac{z^2 L_m}{I L_r} i_{sq}^{(k)} & -\frac{F}{I} \end{bmatrix}$$

$$B^{(k)} = \begin{bmatrix} -\frac{L_m}{L_r} R_r & 0 & 0 \\ 0 & -\frac{L_m}{L_r} R_r & 0 \\ \dfrac{z^2 L_m}{I L_r} \psi_{dr}^{(k)} & -\dfrac{z^2 L_m}{I L_r} \psi_{qr}^{(k)} & -\frac{z}{I} \end{bmatrix}$$

$$h^{(k)}(t) = \begin{bmatrix} -\omega_m^{(k)} \psi_{dr}^{(k)} \\ \omega_m^{(k)} \psi_{qr}^{(k)} \\ -\frac{z^2}{I} \frac{L_m}{L_r} i_{qs}^{(k)} \psi_{dr}^{(k)} + \frac{z^2}{I} \frac{L_m}{L_r} i_{ds}^{(k)} \psi_{qr}^{(k)} \end{bmatrix}$$

Then the optimal control problem becomes:

Find the state feedback control vector $\left( u^* = \begin{bmatrix} i_{sq}^* & i_{sd}^* & T_l^* \end{bmatrix}^T \right)$ that minimizes the performance index:

$$J = \frac{1}{2} \int_0^\infty \left( \left( x^{(k+1)} \right)^T Q x^{(k+1)} + \left( u^{(k+1)} \right)^T R u^{(k+1)} \right) dt \qquad (5.2)$$

Subject to the state equation (5.1).

We guess the values $x^{(0)}(t)$, $u^{(0)}(t)$ at the beginning to find the matrices $A^{(0)}, B^{(0)}, h^{(0)}$, then we solve the problem as linear quadratic problem to find $x^{(1)}$, $u^{(1)}$, then finding $A^{(1)}, B^{(1)}$, and $h^{(1)} \ldots$.etc. (Sequence of LQ Problems).

Anyway, the solution of this optimal control problem at any iteration $(k)$ is:

$$u^{*(k+1)} = -K^{(k+1)} x^{(k+1)} + R^{-1} B^{(k)} v^{(k+1)}$$

$$K^{(k+1)} = R^{-1} {B^{(k)}}^T S^{(k+1)}, \text{ \textit{Kallman Gain}}$$

while $S^{(k+1)}$ solves the algebraic racciti equation:

$$ {A^{(k)}}^T S^{(k+1)} + S^{(k+1)} A^{(k)} - S^{(k+1)} B^{(k)} R^{-1} {B^{(k)}}^T S^{(k+1)} + Q = 0$$

and

$$v^{(k+1)} = -\left(A^{(k)} - B^{(k)}K^{(k+1)^T}\right)^{-1} S^{(k+1)}h^{(k)}$$

$$x^{(k+1)}(0) = x_0^{(k+1)}$$

With initial guess $x^{(0)}(t)$, $u^{(0)}(t)$ .

**Case 4** *Load torque is constant, or it is an input disturbance to the induction motor model.*

Thus the control vector $u = \begin{bmatrix} i_{sq} & i_{sd} \end{bmatrix}^T$, state vector $x = \begin{bmatrix} \psi_{rq} & \psi_{rd} & \omega_m \end{bmatrix}^T$

and the induction motor model :

$$\begin{bmatrix} \dot{\psi}_{qr} \\ \dot{\psi}_{dr} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & -\omega_s & \psi_{dr} \\ \omega_s & -\frac{L_r}{R_r} & -\psi_{qr} \\ -\dfrac{z^2L_m}{IL_r}i_{sd} & \dfrac{z^2L_m}{IL_r}i_{sq} & -\dfrac{F}{I} \end{bmatrix} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{L_m}{L_r}R_r i_{qs} \\ \frac{L_m}{L_r}R_r i_{ds} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \dfrac{z}{I}T_l \end{bmatrix}$$

The statment of the optimal control problem for a new case will be:

Find the state feedback control vector $\left(u^* = \begin{bmatrix} i_{sq}^* & i_{sd}^* \end{bmatrix}^T\right)$ that minimize the performance index equation (5.2) subject to the state equation:

$$\dot{x}^{(k+1)} = A^{(k)}x^{(k+1)}(t) + B^k u^{(k+1)}(t) + h^{(k)}(t)$$

while

$$A^{(k)} = \begin{bmatrix} -\frac{L_r}{R_r} & -\omega_s + \omega_m^{(k)} & \psi_{dr}^{(k)} \\ \omega_s - \omega_m^{(k)} & -\frac{L_r}{R_r} & -\psi_{qr}^{(k)} \\ -\dfrac{z^2L_m}{IL_r}i_{sd}^{(k)} & \dfrac{z^2L_m}{IL_r}i_{sq}^{(k)} & -\frac{F}{I} \end{bmatrix}$$

$$B^{(k)} = \begin{bmatrix} -\frac{L_m}{L_r}R_r & 0 \\ 0 & -\frac{L_m}{L_r}R_r \\ \dfrac{z^2L_m}{IL_r}\psi_{dr}^{(k)} & -\dfrac{z^2L_m}{IL_r}\psi_{qr}^{(k)} \end{bmatrix}$$

$$h^{(k)}(t) = \begin{bmatrix} -\omega_m^{(k)}\psi_{dr}^{(k)} \\ \omega_m^{(k)}\psi_{qr}^{(k)} \\ -\frac{z^2}{I}\frac{L_m}{L_r}i_{qs}^{(k)}\psi_{dr}^{(k)} + \frac{z^2}{I}\frac{L_m}{L_r}i_{ds}^{(k)}\psi_{qr}^{(k)} - \frac{z}{I}T_L \end{bmatrix}$$

Again, to solve this optimal control problem, we may use the same technique that was used in the pervious case.

## 5.4 Simulation of Nonlinear OCP

To simulate the nonlinear optimal control problem (OCP) solved in section 4.2, different weighting matrices have been considered:

$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}, and\ R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$ for case 3 and $R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$ for case 4, with their eigenvalues $q_1, q_2, q_3 > 0$ , and $r_1, r_2, r_3 > 0$.

The simulation has been performed using five sets of $(Q$ and $R)$; $q_1, q_2 = 0.01, 0.1, 1, 10, 100$ , and $r_1, r_2 = 0.01, 0.1, 1, 10, 100$ respectively.

Figures (5.9), (5.10), and (5.11) show the simulation result to nonlinear optimal control problem (case3), the simulation performed with initial states

$x_0 = \begin{bmatrix} \psi_{rq_0} & \psi_{rd_0} & \omega_{m_0} \end{bmatrix}^T = \begin{bmatrix} -5 & -5 & -100 \end{bmatrix}^T$, and initial guess and $x^{(0)} = \begin{bmatrix} \psi_{rq}^{(0)} & \psi_{rd}^{(0)} & \omega_m^{(0)} \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and

$u^{(0)} = \begin{bmatrix} i_{sq}^{(0)} & i_{sd}^{(0)} & T_l^{(0)} \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$.

Figure (5.9) shows the state trajectories (motor fluxes and speed).

Figure (5.6) shows the phases currents $(I_a,\ I_b,\ and\ I_c)$ of IM in case 3 .

While figure (5.11) shows control trajectories (input currents and load torque)

On the other hand, figures (5.12), (5.13) and (5.14) show the simulation result

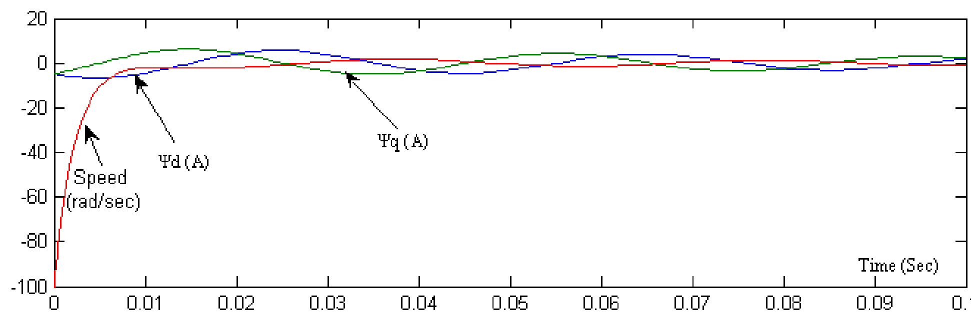



Figure 5.9: States $(\psi_d, \psi_q, \omega_m)$ using Quasilinearization method for 1hp, 4 poles IM and brake torque (case 3).

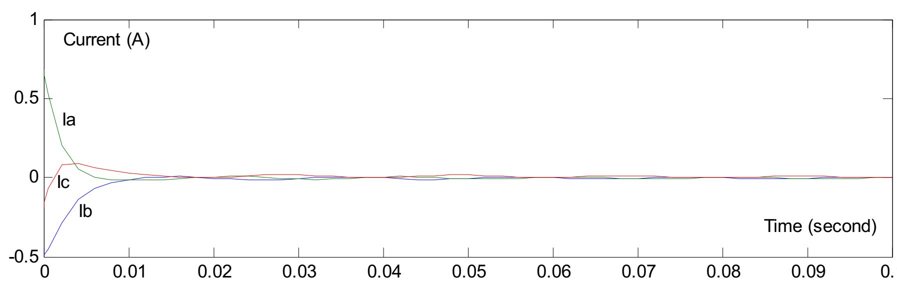



Figure 5.10: Phases currents simulation usnig Quasilinearization method $(I_a, I_b, I_c)$ for 1hp, 4 poles IM and load brake torque (case3)

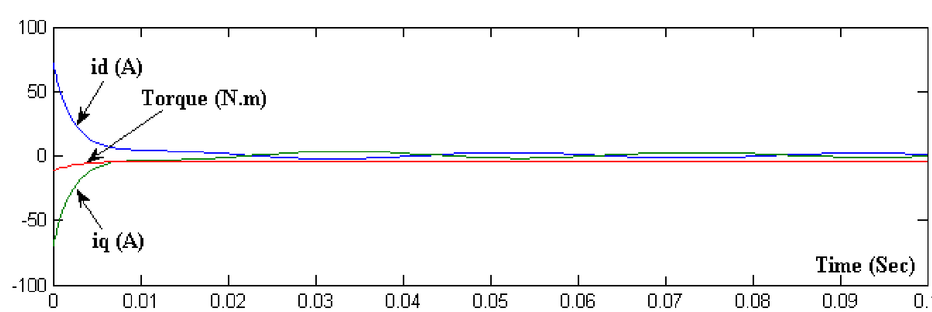



Figure 5.11: States $(\psi_d, \psi_q,\ T_L)$ using Quasilinearization method for 1hp, 4 poles IM and brake torque (case3)

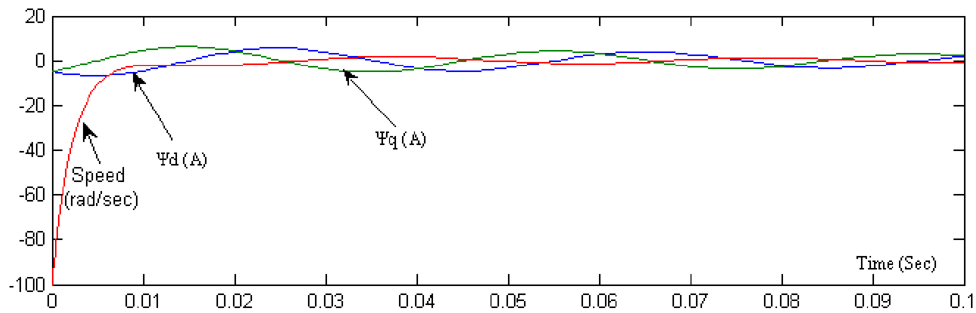



Figure 5.12: States ($\psi_d, \psi_q, \omega_m$) using Quasilinearization method for 1hp, 4 poles IM and load torque 1.5 Nm (case 4)

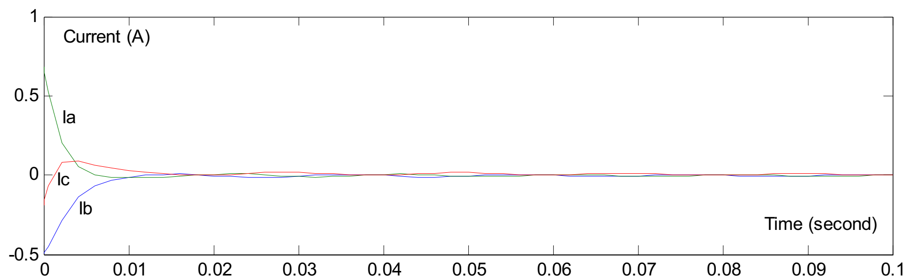



Figure 5.13: Phases currents simulation usnig Quasilinearization ($I_a, I_b, I_c$) for 1hp, 4 poles IM and load torque 1.5 Nm (case 4)

to nonlinear optimal control problem (case3), the simulation performed with initial states:

$x_0 = \begin{bmatrix} \psi_{rq_0} & \psi_{rd_0} & \omega_{m_0} \end{bmatrix}^T = \begin{bmatrix} -5 & -5 & -100 \end{bmatrix}^T$, and initial guess and $x^{(o)} = \begin{bmatrix} \psi_{rq}^{(0)} & \psi_{rd}^{(0)} & \omega_m^{(0)} \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and $u^{(0)} = \begin{bmatrix} i_{sq}^{(0)} & i_{sd}^{(0)} \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$.

Where figure (5.12) shows the state trajectories (motor fluxes and speed).

Figure (5.6) shows the phases currents $(I_a,\ I_b,\ and\ I_c)$ of IM in case 4

And figure (5.14) shows control trajectories (input currents) at fixed load torque.

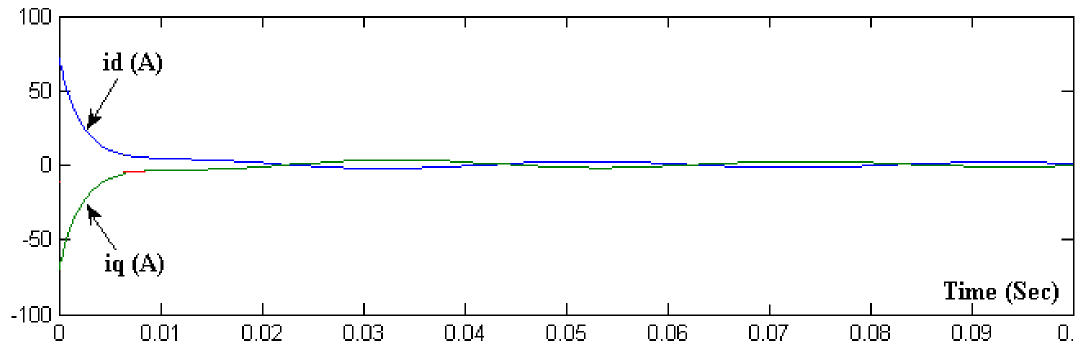



Figure 5.14: State feedback control $(i_d, i_q)$ using Quasilinearization method for 1hp, 4 poles IM and load torque 1.5 Nm (case 4)

# Chapter 6

# Conclusion and Future Work

## 6.1 Conclusion

This thesis obtained optimal trajectories of induction motor states (rotor fluxes and mechanical speed) and controls (stator currents and load torque).

These trajectories are obtained by minimizing the quadrature performance measure or performance index that represents a measure of total energy caused by magnetizing energy, core and copper losses, and mechanical energy of the induction motor.

On the other hand, these trajectories are composed using vector control algorithm, and rotating direct quadrature axis, and they give final compact, attractive, simple, and controllable model of the induction machine.

Because of nonlinearity of the obtained model, an optimization was done using to different approaches: first, by simplifying the motor model using some of induction machine properties, and the second approaches is dealing with obtained nonlinear model then solve nonlinear optimal control problem.

After simplifying the obtained model using some useful properties, we got a linear model of the induction motor, so that we made the optimization to the linear quadratic optimal control problem to minimize the total energy of the motor. We did the optimization by solving an algebraic Riccati equation (ARE) for our problem and using different weighting matrices Q and R.

In addition, we solve the nonlinear optimal control problem using quasilineariztion technique by converting the problem to sequence of linear quadratic optimal control problems. These problems were easily solved by solving algebraic Riccati equation (ARE) and ordinary differential equation.

Finally, the simulation of the optimizations are carefully done in two approaches to obtain controls (stator currents and/or load torque) and states (rotor fluxes and/or mechanical speed) using digital computer programs which are MATLAB and SIMULINK programs.

Using different state and control weighting matrices (Q and R) didn't affect the response of fluxes, currents, speed, and torque as shown in simulation figures, and this shows the stability and robustness for the IM state feedback controllers.

The responses of induction machine using linear optimal control theory are similar to those obtained by using nonlinear optimal control theory.

## 6.2 Recommendation and Future Work

This thesis presented the mathematical design and simulation of optimal trajectories of the induction motor system (rotor fluxes, stator currents, rotor or mechanical speed, and load torque) with field oriented algorithm. We recommend for future work, an implementation of these controllers by building, and tuning.

## References


Barambones, O., Garrido A.J., Maseda, F.J., (In Press), A Sensorless Robust Vector Control of Induction Motor Drives, Universidad del Pa´ıs Vasco. Plaza de la Casilla, 48012 Bilbao (Spain) pp1 -6.

Beguenane, R., Ouhrouche, M., (2003), MRAC-IFO Induction Motor Control with Simultaneous Velocity and Rotor-Inverse Time Constant Estimation, IASTED International Conference PES'2003.

Bellman,R., Kalaba, R., (1965), Quasilinearization and Nonlinear Boundary Value Problem, Elsevier, New York.

Bose, B. K., (2002), Modern Power Electronics and AC Drive, Prentice Hall PTR.

Bose, B. K., Patel, N., Rajashekara, K., (1997), A Neuro-Fuzzy based On-Line Efficiency Optimization Control of a Stator Flux-Oriented Direct-Vector Controlled Induction Motor Drive, IEEE Trans. on Ind. Electronics, Vol. 44, pp 270 – 273.

Dubey, G. K., (1995), Fundamentals of Electrical Drives, Narosa Publishing House.

Frick, P., Stech, D., (1995), Solution of the optimal control problems on parallel machine using epsilon method, Optimal Control Appl. Methods 16, (1-17).

Georges, D., Ramirez, J., Canudas C., (In Press), Nonlinear H2 and H$\infty$ Optimal Controllers for Current-feeded Induction Motors , Submitted to the IEEE Trans. on control systems technology.

Georges, D., Canudas C., Ramirez, J., (In Press), Performance Evaluation of Induction Motors Under Optimal-Energy Control, submitted to IEEE Trans.

Goh, C., Teo, K., (1988), Control parametrization: a unified approach to optimal control problem with genral constaints, Automatic

Ho, E. Y., Sen, P. C., (1988), Decoupling Control of Induction Motors, IEEE Tran. on industial electronics, Vol. 35 No. 2, pp 253-262.

Hovingh, B., Keerthipala, W., Yan, (In Press), Sensorless Speed Estimation of an Induction Motor in a Field Orientated Control System, School of Electrical and Computer Engineering Curtin University of Technology, Australia.a 24, 3-18.

Ismail, Z., Luc, L., Christophe F., (In press), An extended filter and appropriate model for the real time estimation of the induction motor variables and parameters, LEC, UTC.

Jaddu, H., (2002), Direct of nonlinear optimal control problems using quasi-linearization and chebyshev polynomials, Journal of Franklin Institute, 399, pp 479-498.

Jaddu, H., Shimemura, E., (1999), Compination of optimal control trajectories using Chebyshev polynomials: Parametrizationa and quadratic programing, Optimal Control Appl. Methods 20 21-42.

Kim, S., Park, T.,Yoo, J., Park, G., (2001), Speed-Sensorless Vector Control of an Induction Motor Using Neural Network Speed Estimation, IEEE Tran. On Industrial Electronics, Vol. 48, No., pp 609-615.

Kirk, O., (1970), Optimal Control, Prentice Hall Inc.

Krause, P., Thoms, C., (1965), Simulation of symetrical induction machinary, IEEE Trans. PAS-84 Vol.11 pp1038-1056.

Leonard, W., (1985), Control of Electrical Drive, New York, Springer-Velag .

Lewis, F., Syrmos, V., (1995), A Wiley Intersciece Publication, Second edition.

Mathwork, (2002), MATLAB (6.5) and SIMULINK (5.5) tutorials.

Marino, R., Peresada, S., Valigi, P. (1993), Adaptive input output linearizing control of induction motor, IEEE transaction automatic control, vol. 38, no. pp 208-221.

Mcpherson, G., and Laremore, R., (1990), Electrical Machines and transformer, John Wiley & Sons, $2^{nd}$ edition.

Mohamad, N., (2000), Electrical Drive: An Integrated Approach, Minneapolis,

MN:MNPERE.

NEC corporation, Application Note, (2002), An Introduction To Vector Control of AC Motor Using V850.

Novotny, D.W., and Lipo, T.A., (1995), Vector Control and Dynamics of AC Drives, Oxford Science Publications.

Okoro, O. I., (2003), MATLAB simulation of induction machine saturable leakage and magnetizing inductance, Pracific Journal of science and technology, vol. 5 no. 1.

Ouhrouche, M., Volat, C., (2000), Simulation of a Direct Field-Oriented Controller for an Induction Motor Using MATLAB/SIMULINK Software Package, Proceeding of the IASTED International Conference Modelling and Simulation (MS'2000) - Pittsburgh, Pennsylvania, USA, pp 308-082-308-087, May 15-17.

Ozpineci, B., Tolbert, M., (2003), simulink implmetation of induction motor machine model- A modular approche, IEEE pp 728-734.

Rryson, A., Ho, Y., (1975), Applied Optimal control, Hemisphere publication corporation.

Ramirez, J. and Canudas,W., (In Press), Optimal Torque Control for Current-Fed Induction Motors, Submitted to the IEEE Trans. on control systems technology.

Rashid, M., (2004), Power Electronics circuit, devices and applications, Pearson Prentice Hall.

Rasmussen, H., (2002), Adaptive Field Oriented Control of Induction Motors,Aalborg Univesity.

Seleme S., Canudas, C., (1992), Minimum Energy Operation Conditions of Induction Motors Under Torque Regulation, Workshop on Motion Control for Intelligent Automation, Vol. 1, pp. 127-133. Pergamon Press.

Texas Instrument, (1996), Application Note (BPRA043), Digital Signal Processing solution for AC Induction Motor.

Vas, D. , (1992), Electrical Machines and Drive: A Space Vector Theory Approach, London, UK, Clarendon Press.

Vlassenbroeck, J., Van Doreen, R., (1988), A. Chebyshev technique for solving nonlinear optimal control problem, IEEE Trans. Automat. Control 33, 333-340.

Wasynczuk, O., Sudhoff, S., Hansen, I., Taylor, L., (In Press), a maximum torque per ampere control strategy for induction motor drives, pp 1-8.

Zidan, H., Fujii, S., Hanamoto, T., Tsuji, T., (2000), Simple Sensorless Vector Control For Variable Speed Induction Motor Drives, T IEE, Japan, Vol. 120-D, No. 10, pp 1165-1170.

# Appendix A .

Induction motor ratings: Power= 1hp, Rated speed =1440 rpm, z= 2 pole pairs, $R_s$= 1.15 Ω, $R_r$ =1.44 Ω, $L_m$=0.144 H, $L_s$=$L_r$ = 0.156 H , $I$ = 0.013 kg.m$^2$, $F$ = 0.002 Nm.s/rad.

## A.1 Simulink Charts for Linear Optimal Control Problem Cases 1 and 2

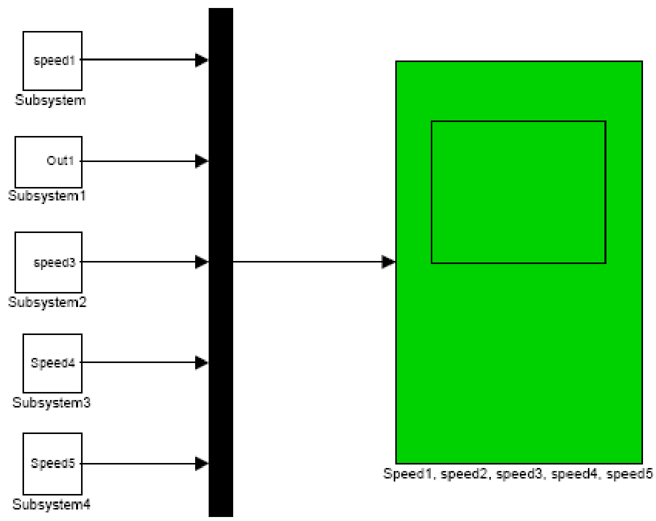



Figure A.1: Simulink chart for case 1 and

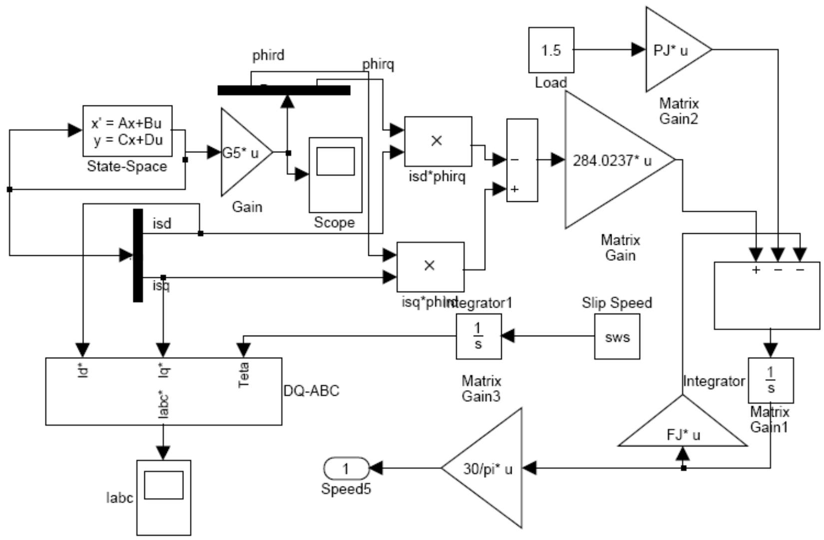



Figure A.2: Block contents of subsystems speed1 to speed 5 for cases 1 and 2

## A.2 Simulink Charts for Nonlinear Optimal Control Problem Cases 3and 4

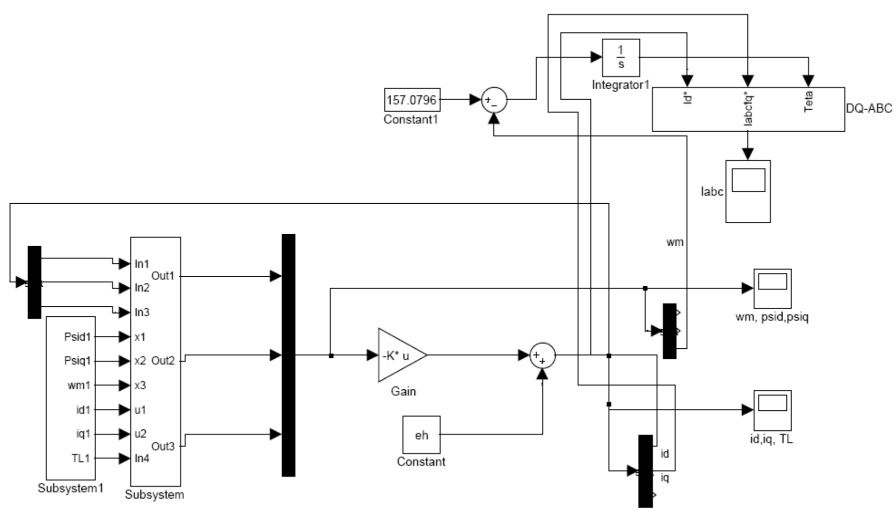



Figure A.3: Simulink chart for case 3 and case 4

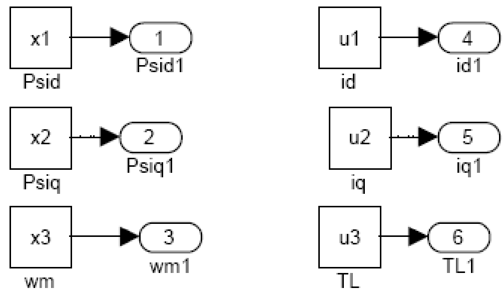



Figure A.4: Subsystem input for cases 3 and 4

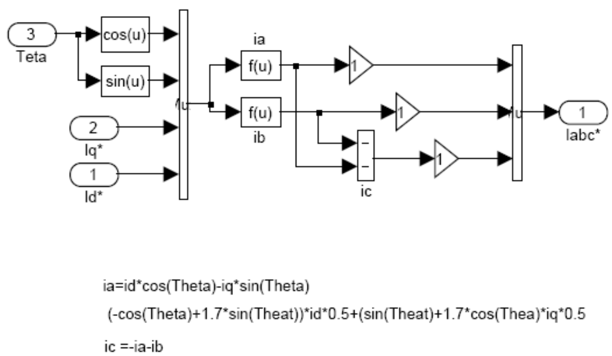



Figure A.5: Phase currents calculations

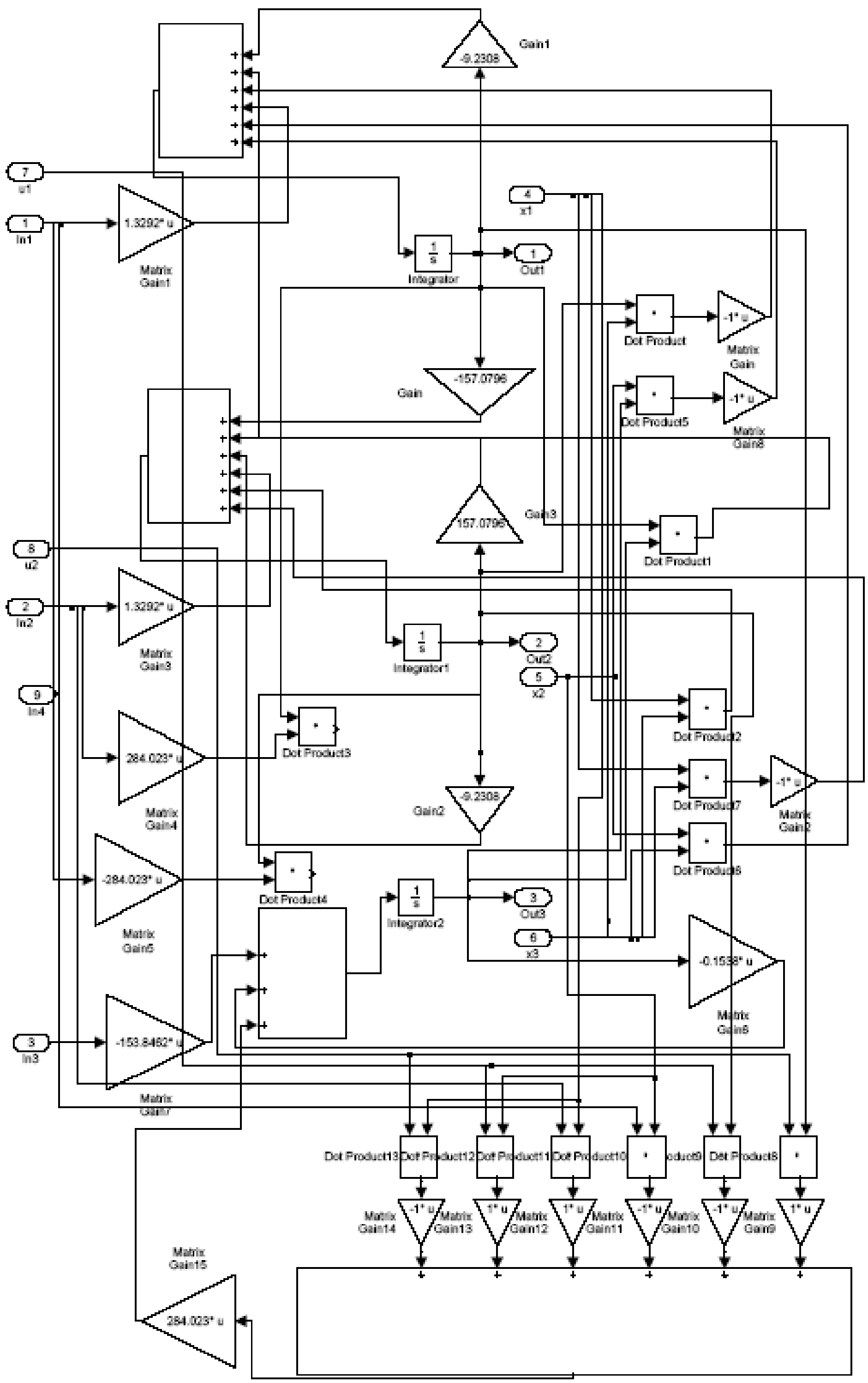



Figure A.6: Block contents of subsystem for cases 3 and 4

## Publications

Tamimi J. M. Kh., Jaddu H. M., "Optimal Vector Control of Three-Phase Induction Motor". Accepted for publication in the 25th IASTED International Conference on MODELLING, IDENTIFICATION, and CONTROL, Lanzarote, Canary Islands, Spain, 2006.

Tamimi J. M. Kh., Jaddu H. M., "Nonlinear Optimal Controller of Three-Phase Induction Motor Using Quasilinearization". Accepted for publication in the second *IEEE-EURASIP* International Symposium on Control, Communications, and Signal Processing Confrance, Marrakech, Morocco, 2006.