Electromagnetic Scattering From Two-Scatterers Using the Extended Propagation-Inside-Layer Expansion Method

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Abstract

In this paper, the electromagnetic scattering from two scatterers is analyzed from a rigorous integral formulation solved by the method of moments (MoM). G. Kubické has recently developed the E-PILE (Extended Propagation-Inside-Layer Expansion) method to calculate the scattering from an object above a rough surface for a two-dimensional problem. This method allows us to calculate separately and exactly the interactions between the object and the rough surface. The purpose of this paper is to extend the E-PILE method to a three-dimensional problem.

1 Introduction

In recent years, composite electromagnetic scattering from an object near a randomly rough surface has attracted considerable interest in the fields of radar surveillance, target identification, object tracking, and so on. For a large 3-D problem, it is well-known that the method of moments (MoM) is limited by the memory requirement. Thus, hybrid methods are developed to overcome this limitation [1-3].

Recently, N. Déchamps [4] developed a fast numerical method, PILE (Propagation-Inside-Layer Expansion), devoted to the scattering by a stack of two one-dimensional interfaces separating homogeneous media. In that method only the upper surface was illuminated. More recently G. Kubické [5] extended the PILE method to the more general case of two illuminated surfaces, and applied this Extended PILE (E-PILE) method to an object above one-dimensional rough surface.

The purpose of this paper is to extend the E-PILE method to the more general case of a 3-D problem, in which two scatterers are illuminated. In addition, to accelerate the local interactions on each scatterer, the physical optics approximation is applied to the case of two parallel perfect electric conductor (PEC) plates. This will avoid us from calculating the inverse of the impedance matrix for each scatterer.

2 Mathematical Formulations

Let us consider an incident electromagnetic (EM) wave that illuminates the system composed of two PEC scatterers. The free-space Green function is written $G_0(\mathbf{R},\mathbf{R}') = \exp(-ik_0 r)/4\pi r$ with wavenumber k_0, where r denotes the distance between the two points $\mathbf{R}$ and $\mathbf{R}'$. Using the boundary conditions on the upper scatterer S_+, the first coupling integral equation between S_+ and S_- $\forall\, \mathbf{R}' \in S_+$ is obtained from

$$\begin{aligned}-\hat{\mathbf{n}}_+(\mathbf{R}') \times \mathbf{E}^i(\mathbf{R}') &= \hat{\mathbf{n}}_+ \times \int_{S_+} \left[jk_0\eta_0 \mathbf{J}_+(\mathbf{R}_+) G_0(\mathbf{R}_+,\mathbf{R}') + \frac{\eta_0}{jk_0} \left\{ \nabla \cdot \mathbf{J}_+(\mathbf{R}_+) \right\} \nabla G_0(\mathbf{R}_+,\mathbf{R}') \right] dS \\ &+ \hat{\mathbf{n}}_+ \times \int_{S_-} \left[jk_0\eta_0 \mathbf{J}_-(\mathbf{R}_-) G_0(\mathbf{R}_-,\mathbf{R}') + \frac{\eta_0}{jk_0} \left\{ \nabla \cdot \mathbf{J}_-(\mathbf{R}_-) \right\} \nabla G_0(\mathbf{R}_-,\mathbf{R}') \right] dS. \quad (1)\end{aligned}$$

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Using the boundary conditions on the lower scatterer S_-, the second coupling integral equation $\forall\, \mathbf{R}' \in S_-$ is obtained from

$$\begin{aligned} -\hat{\mathbf{n}}_-(\mathbf{R}') \times \mathbf{E}^i(\mathbf{R}') &= \hat{\mathbf{n}}_- \times \int_{S_-} \left[jk_0\eta_0 \mathbf{J}_-(\mathbf{R}_-) G_0(\mathbf{R}_-, \mathbf{R}') + \frac{\eta_0}{jk_0} \left\{ \nabla \cdot \mathbf{J}_-(\mathbf{R}_-) \right\} \nabla G_0(\mathbf{R}_-, \mathbf{R}') \right] dS \\ &+ \hat{\mathbf{n}}_- \times \int_{S_+} \left[jk_0\eta_0 \mathbf{J}_+(\mathbf{R}_+) G_0(\mathbf{R}_+, \mathbf{R}') + \frac{\eta_0}{jk_0} \left\{ \nabla \cdot \mathbf{J}_+(\mathbf{R}_+) \right\} \nabla G_0(\mathbf{R}_+, \mathbf{R}') \right] dS. \end{aligned} \quad (2)$$

The use of the method of moments with point matching and pulse basis functions leads to the following linear system $\bar{\mathbf{Z}}\mathbf{X} = \mathbf{b}$, in which $\bar{\mathbf{Z}}$ is the impedance matrix of the total scene $(S_+ \cup S_-)$ of size $2(N_+ + N_-) \times 2(N_+ + N_-)$. The unknown vector $\mathbf{X}$ of length $2(N_+ + N_-)$ is equal to $\mathbf{X}^T = [\mathbf{X}_+^T\ \mathbf{X}_-^T]$, in which ^{T} stands for transpose operator. $\mathbf{X}_\pm$ of length $N_\pm$ contains the unknown currents $\mathbf{J}_\pm$ on the upper and lower scatterers

$$\mathbf{X}_\pm^T = \Big[\underbrace{J(R_+^1) \ldots J(R_+^{N_+})}_{\text{Upper Scatterer}} \underbrace{J(R_-^1) \ldots J(R_-^{N_-})}_{\text{Lower Scatterer}} \Big]. \quad (3)$$

The source term $\mathbf{b}$ is defined as $\mathbf{b}^T = [\mathbf{b}_+^T\ \mathbf{b}_-^T]$. To solve the linear system, the matrix $\bar{\mathbf{Z}}$ is expressed from sub matrices [5] as

$$\bar{\mathbf{Z}} = \begin{bmatrix} \bar{\mathbf{Z}}_+ & \bar{\mathbf{Z}}_\mp \\ \bar{\mathbf{Z}}_\pm & \bar{\mathbf{Z}}_- \end{bmatrix}. \quad (4)$$

$\bar{\mathbf{Z}}_+$ corresponds exactly to the impedance matrix of the first scatterer as if it is assumed to be alone (in free space), $\bar{\mathbf{Z}}_-$ corresponds to the impedance matrix of the second scatterer as if it is assumed to be alone (in free space). $\bar{\mathbf{Z}}_\mp$ and $\bar{\mathbf{Z}}_\pm$ can be interpreted as coupling matrices for the interaction between S_+ and S_-. The inverse of matrix $\bar{\mathbf{Z}}$ can be expressed as

$$\bar{\mathbf{Z}}^{-1} = \begin{bmatrix} \bar{\mathbf{T}} & \bar{\mathbf{U}} \\ \bar{\mathbf{V}} & \bar{\mathbf{W}} \end{bmatrix}, \quad (5)$$

with

$$\begin{cases} \bar{\mathbf{T}} = (\bar{\mathbf{Z}}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm)^{-1} \\ \bar{\mathbf{U}} = -(\bar{\mathbf{Z}}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm)^{-1} \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \\ \bar{\mathbf{V}} = -\bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm (\bar{\mathbf{Z}}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm)^{-1} \\ \bar{\mathbf{W}} = \bar{\mathbf{Z}}_-^{-1} + \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm (\bar{\mathbf{Z}}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm)^{-1} \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \end{cases}, \quad (6)$$

and the unknown vector $\mathbf{X}$ is obtained as :

$$\begin{bmatrix} \mathbf{X}_+ \\ \mathbf{X}_- \end{bmatrix} = \bar{\mathbf{Z}}^{-1} \begin{bmatrix} \mathbf{b}_+ \\ \mathbf{b}_- \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{T}}\mathbf{b}_+ + \bar{\mathbf{U}}\mathbf{b}_- \\ \bar{\mathbf{V}}\mathbf{b}_+ + \bar{\mathbf{W}}\mathbf{b}_- \end{bmatrix}. \quad (7)$$

By using equations (6) and (7), the current on the upper scatterer $\mathbf{X}_+$ can be expressed as

$$\mathbf{X}_+ = (\bar{\mathbf{I}} - \bar{\mathbf{Z}}_+^{-1} \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm)^{-1} \bar{\mathbf{Z}}_+^{-1} (\mathbf{b}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \mathbf{b}_-), \quad (8)$$

where $\bar{\mathbf{I}}$ is the identity matrix. Let us introduce the characteristic matrix $\bar{\mathbf{M}}_{c,+}$ as

$$\bar{\mathbf{M}}_{c,+} = \bar{\mathbf{Z}}_+^{-1} \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm. \quad (9)$$

The first term in equation (8) can be expanded as an infinite series over p

$$(\bar{\mathbf{I}} - \bar{\mathbf{Z}}_+^{-1} \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \bar{\mathbf{Z}}_\pm)^{-1} = \sum_{p=0}^{p=\infty} \bar{\mathbf{M}}_{c,+}^p. \quad (10)$$

For the numerical computation, the sum must be truncated at the order $P_{\text{E-PILE}}$. From equations (8) and (10), the unknown current on the upper scatterer $\mathbf{X}_+$ is then expressed as

$$\mathbf{X}_+ = \left[\sum_{p=0}^{p=P_{\text{E-PILE}}} \bar{\mathbf{M}}_{c,+}^p \right] \bar{\mathbf{Z}}_+^{-1} (\mathbf{b}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1} \mathbf{b}_-) = \sum_{p=0}^{p=P_{\text{E-PILE}}} \bar{\mathbf{Y}}_+^{(p)}, \quad (11)$$

in which

$$\begin{cases} \mathbf{Y}_+^{(0)} = \bar{\mathbf{Z}}_+^{-1}(\mathbf{b}_+ - \bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1}\mathbf{b}_-) & \text{for } p = 0 \\ \mathbf{Y}_+^{(p)} = \bar{\mathbf{M}}_{c,+}\bar{\mathbf{Y}}_+^{(p-1)} & \text{for } p > 0 \end{cases}. \tag{12}$$

The unknown vector $\mathbf{X}_-$ is obtained by substituting in equations (10), (11), and (12), subscripts $+, -, \pm, \mp$

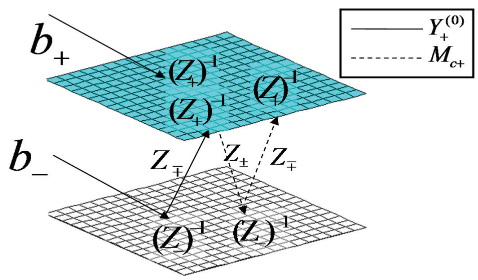
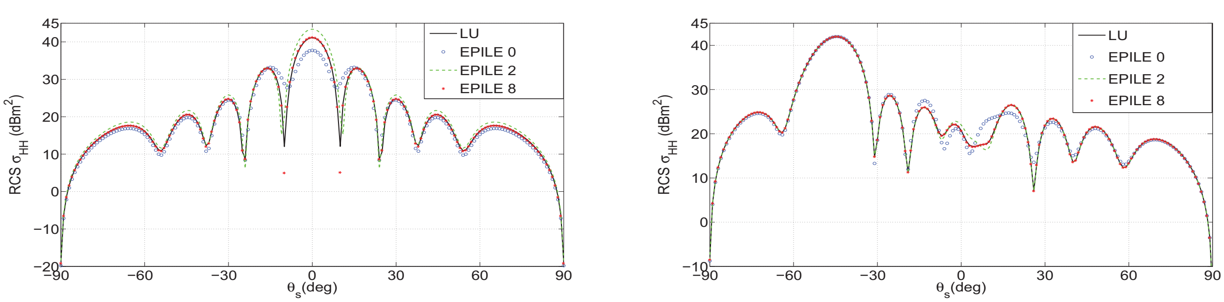


Figure 1: Physical interpretation of the E-PILE method for the case two parallel plates.

for subscripts $-, +, \mp, \pm$, respectively. $\bar{\mathbf{M}}_{c,+}$ has a clear physical interpretation as shown in Figure 1 : in the zeroth order terms, $\bar{\mathbf{Z}}_+^{-1}$ accounts for the local interactions on the upper scatterer, so $\mathbf{Y}_+^{(0)}$ corresponds to the contribution of the scattering on the upper scatterer, when it is illuminated by the direct incident field ($\mathbf{b}_+$) and the direct scattered field by the lower scatterer ($-\bar{\mathbf{Z}}_\mp \bar{\mathbf{Z}}_-^{-1}\mathbf{b}_-$). In the first order term, $\mathbf{Y}_+^{(1)} = \bar{\mathbf{M}}_{c,+}\mathbf{Y}_+^{(0)}, \bar{\mathbf{Z}}_\pm$ propagates the resulting upper field information, $\mathbf{Y}_+^{(0)}$, toward the lower scatterer, $\bar{\mathbf{Z}}_-^{-1}$ accounts for the local interactions on this scatterer, and $\bar{\mathbf{Z}}_\mp$ re-propagates the resulting contribution toward the upper scatterer; finally, $\bar{\mathbf{Z}}_+^{-1}$ updates the field values on the lower scatterer. So the characteristic matrix $\bar{\mathbf{M}}_{c,+}$ realizes a back and forth between the upper scatterer and the lower one. The order $P_{\text{E-PILE}}$ of PILE, corresponds to the $P_{\text{E-PILE}}$ back and forth between the upper and the lower scatterer. In the same manner, $\bar{\mathbf{M}}_{c,-}$ realizes a back and forth between the lower scatterer and the upper one. Once the equation $\bar{\mathbf{Z}}\mathbf{X} = \mathbf{b}$, is solved for $\mathbf{X}$ the scattered fields is computed from using Huygen's principle on the electric current densities.

3 Numerical Results and Discussion

In this section, a first numerical example is presented for the case of two parallel horizontal PEC plates of dimensions $5\lambda \times 5\lambda$, and the distance between the two plates is 7λ. The plates are discretized into



(a) Convergence of E-PILE versus its order, $\theta_i = 0^0$.

(b) Convergence of E-PILE versus its order, $\theta_i = 45^0$.

Figure 2: Comparison of the bistatic scattering from 2-D parallel plates, $S = 5\lambda \times 5\lambda$ for each plate, $\Delta x = \Delta y = \lambda/10$.

$\Delta x = \Delta y = \lambda/10$. The incident EM plane wave is horizontally polarized, with angles $\theta_i = 0°$ and $\phi_i = 0°$ (normal incidence) for Figure 2a, and $\theta_i = 45°$ for Figure 2b. Comparisons, not shown here, with results computed from the commercial software FEKO, showed good agreements. This validates our code. To quantify the convergence of the E-PILE versus its order, we compare the results obtained by E-PILE with those computed from a direct LU inversion. Figures 2a and 2b show that as the order increases E-PILE converges to LU. Other simulations, not depicted here, showed that the E-PILE method converges more quickly as the distance between the two plates increases, since the electromagnetic coupling is less important than the previous case. In fact, the order $P_{\text{E-PILE}}$ is directly related to the coupling between the two scatterers.

4 Conclusion

In this paper, a new efficient method to study the electromagnetic scattering from a 3-D problem with two scatterers is presented. The method is based on the rigorous E-PILE method, originally developed to predict the field scattered from an object above a one-dimensional rough surface. This E-PILE method was applied in this work to the 3D problem composed of two 2D parallel smooth surfaces. The numerical results showed that the E-PILE method converges rapidly. Moreover, the E-PILE order is linked to the number of reflections between the two smooth surfaces.

One of the advantages of this method is to apply algorithms valid for a single scatterer (in free space). Thus, to accelerate the E-PILE method a hybridization, based on the physical optics approximation, will be presented during the conference. By this way there is no need to calculate the inverse of the impedance matrix for one of the scatterers (or both), which will allow us to reduce the complexity of the E-PILE method to $\mathcal{O}(N^2)$ instead of $\mathcal{O}(N^3)$ from a direct LU inversion, with N the number of unknowns on the two scatterers.

5 References

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