

Impact of the Van Heil Model in Developing Abstract Thinking

Ibrahim Moh'D Arman¹, Sanaa Ali Abu Isbaa²

¹College of Education Science, Al-Quds University Abu, Palestine

²South Hebron Education Directorate, Hebron, Palestine

ABSTRACT

This study aimed to reveal the Impact of Van Heil's model in developing abstract thinking among 10th grade students in South Hebron Education Directorate.

In order to achieve the objectives of this study, the researcher used the experimental method, as the study was applied during the first semester of 2020/2021 academic year. The study population consisted of all the 10th-grade students who are registered in South Hebron Education Directorate, who are (2,967) students. The study was applied to an intentional sample that consisted of (44) female students from Dura Girls' Vocational Secondary School divided into two divisions, where one of the two divisions was considered an experimental group and the other was a control group, and each group reached (22) students. The experimental group was taught the Geometry construction unit in the 10th grade mathematics course using Van Heil's model, and the control group taught the same unit using the regular method.

The researchers prepared a tool of abstract thinking. The validity and reliability of the test was verified by appropriate methods.

The statistical analysis software (SPSS) was used to analyze the results of the study, by using analysis of covariance test (ANCOVA).

The results revealed the existence of statistically significant differences in the students' scores in the test of the level of abstract thinking according to teaching method variable, and in favor of the experimental group, the results revealed the presence of statistically significant differences in the test of the level of abstract thinking due to academic achievement variable, in favor of the group with high achievement, also the presence of statistically significant differences in the test of the level of abstract thinking due to interaction between method of teaching and level of academic attainment, in favor of the students with high achievement in the experimental group.

In light of the results, the researchers recommended the necessity of employing Van Heil's model in teaching Geometry units, the importance of training mathematics teachers to use it in teaching Geometry in mathematics courses is also recommended by the researcher, the researcher also recommended the need to conduct more studies on Van Heil's model using other dependent variables such as: generative thinking and trends towards Geometry or towards the model.

Keywords: Impact, Van Heil Model, Abstract Thinking

Article Received: 10 August 2020, Revised: 25 October 2020, Accepted: 18 November 2020

1. INTRODUCTION:

Geometry is one of the components of the mathematics curriculum and an important part of it, as NCTM (2000) and Al-Hanafi (2014) emphasized that it works to improve students' ways of thinking by linking facts and concepts, as the Geometry content has special advantages in the development of observation, experimentation, measurement and logical conclusion, writing proof and proving it, And that is through the student's perception of Geometry relations based on axioms and theories in light of what is given, in addition to that Geometry is related to the reality in which the student lives, so he finds the Geometry contents meaning, especially when he looks at the geometric shapes, models, models and constructions around him, and therefore Geometry is a fertile field for training On how to use thinking patterns to reach the required solutions, which helps in developing sound methods for different types of thinking.

On the subject of Geometry, Leung (2011) emphasized that Geometry construction activities help students to think deeply about Geometry

concepts and structures, as it requires a broad base of Geometry theories and properties to conduct any Geometry construction. The National Council of Mathematics Teachers in the United States of America has recommended organizing The process of teaching and teaching Geometry according to a model called Van Heil levels of Geometry thinking. Van Hail's model of mathematical Geometry thinking has received great attention by educators in the world, as interest in this model began in the former Soviet Union, then Europeans followed them, and then in the United States of America, and this interest indicates that understanding and knowing this model helps in teaching Geometry for students in different stages, and it shows to teachers the necessity for students to pass through levels of the model to help develop their levels of thinking.

Van Heil classified the thinking levels in Geometry into Five levels are: the level of visual recognition, the level of analysis, the level of non-formal inference, and the aforementioned three levels are considered appropriate for teaching students in basic grades, and the teacher should use them

gradually to enable students to comprehend concepts, generalizations and Geometry skills without the need for any logical proofs, while the fourth level inference The formal and the fifth level, the level of extreme accuracy, is available for students of the upper basic and secondary grades, because it is based on the perception of the Geometry structure and the use of mathematical proofs in theories and the solution of Geometry exercises (Rashid and Khashan, 2009).

These five levels are graduated from the simple level to the more complex level, so the student cannot reach the next level unless he has mastered the levels before him, and these levels depend largely on educational experiences, and not on chronological age or even the level of adulthood (Tannah, 2008).

And even the transition from one level to a level higher than it also depends in large part on the levels of teaching appropriate to it, and therefore there are five levels of the teaching performance of the model, which are in order according to the five levels of Geometry thinking (inquiry, direct guidance, interpretation, free direction, integration. (Sadiq, 2001).

In order for the student to gain a full understanding of Geometry with its concepts and generalizations, he must pass through these levels and gradually master them. The model must be taken from the three main aspects: the existence of levels, the properties of levels, and the transition between the levels (Knight, 2006).

And since thinking is one of the most important basic goals of teaching school mathematics, mathematics teachers have adopted the topic of developing thinking as one of the main criteria for teaching mathematics, and many conferences have also included interest in the aspect of thinking of all kinds, and as a result many studies and research have been conducted in the current period on the topic of development Thinking, which called, recommended and emphasized all of the development of the field of thinking in its various types in light of its multiple skills, such as The study of (Al-Abdullah 2012).

From the interest in thinking emerged theories, and from the theories concerned with the development of the individual's mental and cognitive development and the development of his thinking, the Piaget theory, where he explained that there are differences in the mental and cognitive development of the individual at every stage of his thinking, starting from the perceptual thinking to abstract thinking (Al-Khasawneh, 1987).

As mathematics is an abstract subject, its study requires the individual to reach the stage of abstract thinking, which is the fourth stage of growth for Piaget, as this stage is distinguished from other stages by setting the individual to a series of assumptions and dealing with them as a whole to

reach the desired and appropriate conclusion, where the individual becomes able to relate The logical one between the issues, and his ability to deal with concepts in an abstract way away from the tangible (Al-Rajeh, 2009).

2. The Study Problem:

The great importance and impact of Van Heel model in mathematics, especially in Geometry units, and the new subject of Geometry construction in our Palestinian curriculum, and teachers have always been keen on developing different aspects of thinking among learners and reaching the stage of abstract thinking, the problem of the study came about the Impact of Van Heil model in developing the level of abstract thinking for 10th grade students.

2.1 Study Question: The study tried to answer the following question, which states:

What is the Impact of using Van Hail's model in developing abstract thinking among 10th grade students? Does this Impact differ according to the method and academic achievement, and the interaction between them?

2.1.1 The Null Hypothesis, which states:

"There are no statistically significant differences at the level of statistical significance ($\alpha \leq 0.05$) between the arithmetic averages of the scores of the 10th grade students in the abstract thinking test due to the method variable, academic achievement, and the interaction between them."

2.1.2 The Importance of the Study: The education of students must go beyond the limits of their memorization or even the teaching of science and knowledge, especially in mathematics, to the skillful and thinking aspects, especially in Geometry units. In response to the development and renewal of methods and methods of explaining Geometry units in mathematics in line with modern trends and achieving the educational goals that we all seek, this study may benefit mathematics teachers in explaining how to use the Van Heil model in teaching Geometry constructions in Geometry units..and this study may benefit Curriculum planners and those in charge of mathematics development projects, especially in Geometry, to include in our curricula many exercises and practical activities based on the Van Heil model, in order to achieve a gradual and smooth transition across levels of Geometry thinking of Van Heil.

2.1.3 Limitations of the Study: This study was limited to:

Human Frontiers: 10th grade students in the South Hebron District

Temporal limits: During the first semester of the 2020/2021 academic year

Spatial boundaries: The schools affiliated to the South Hebron District, which includes the 10th grade, and which amounts to (75) schools

2.1.4 Terminology of Study:

Qusay and Riyad (2014) defined Van Heil Model as a set of planned and organized steps that the teacher adopts to teach his students, and this model describes the various types of students' learning and teaching, from their transition from general perception of a geometric shape to the conclusion of proofs and is defined by five levels of instruction.

Al-Rafei (2001) defined Abstract thinking as a standard hypothetical thinking based on issues in the form of introductions whose truthfulness is recognized, then we try to conclude the logical consequences of these introductions.

3. Theoretical Framework:

Geometry is one of the important mathematical content standards, in addition to that it represents the bulk of physical mathematics, unlike other branches of mathematics that are completely abstract, such as the subject of algebra, as most Geometry subjects are easy to deal with, express, and are taught easily and easily if the teacher chooses Appropriate means, methods and strategies to simplify and learn (Abu-Lom, 2007).

Objectives of teaching Geometry: Al-Banna (1994) indicated that the goals of teaching the subject of Geometry at all academic levels are: The student's acquisition of appropriate information about geometric shapes, whether in level or in space, because of their importance in other studies such as trigonometry, calculus, and integration. And developing the student's understanding and appreciation of the inferential method, as a way of thinking and proof, while acquiring the skill of applying it in different and varied mathematical situations, as well as encouraging students' originality, initiative and fruitful thinking, and providing opportunities for them to practice innovative thinking through the study of Geometry. Where Geometry allows students to study different ways of thinking, especially when dealing with Geometry problems in the higher basic stages, which makes Geometry more fun in teaching and more effective during learning.

What is the Van Heil model:

Van Heil's model, after its translation into English in 1984, was well received in most countries, as I believed that one of the difficulties in learning Geometry is due in large part to the teacher, as he explains Geometry lessons or topics in a language that students may not understand when the teacher speaks at a certain level and the students They think on another level, meaning that the language used in teaching Geometry is the most important factor to it, and this is what Van Heil called the linguistic barrier, as each level of Van Heil has a special language that students understand (Sadiq, 2001).

Levels of the Van Heil Model:

Al-Balawi (2013) explained that learning a certain level of the Van Heil model requires learning for

the previous level, and that moving from one level to another requires time for it to mature before moving to the next level. Above it does not take place learning, but only memorization and revelation.

Malloy (2002) argues that one of the most important strengths of using the Van Heil model is that a student's progression from one level to the next depends on teaching more than the student's age or maturity.

Here is a description of each of the Van Heil levels:

The first level (visual level): is the level in which the learner judges the geometric shape and classifies it from its general appearance and distinguishes it as a whole, and does not know anything about its characteristics, and the student at this level cannot link between the characteristics and does not know the relationships between them (Bal, 2014). The student is expected here to be able to: Distinguish shapes according to their appearance, describe them in words, recognize their states, look at any geometric shape separately without generalizing, copy it with general names, and solve routine problems and some life Geometry problems that require dealing with them by measurement and counting or by cutting and repeating Installation

The second level (analysis): In this level, the learner begins to analyze geometric shapes, distinguishing the apparent characteristics through observation and experimentation, but he cannot connect between them and he can use many examples and be able to make generalizations but does not reach their interpretation and cannot emphasize the interrelationships between shapes and drawings Required Usiskin (1982), and the student at this level is expected to accomplish the following: Distinguish between shapes according to their general characteristics and components, and solve some exercises and activities on generalized properties, or exploratory approaches, and start using verbal and verbal expressions about the concepts they learned.

The third level (Informal Deduction): In this level, the learner's awareness of the relationships between the different geometric shapes includes the ability to formulate the definition of the geometric shape using words that have a logical character, and to find relationships between the properties of a single figure and link them with each other at the level of the shape or at the level of different shapes, The learner here can also complete a deductive proof of an Geometry problem, and he can understand the relationships between theories and axioms, but he cannot build a proof that starts from unfamiliar assumptions and has some concepts of necessary and sufficient conditions clear to him (Teppo, 1991).

The student at this level is expected to be able to: Perceive the properties that are sufficient to

distinguish one form from another, or construct an Geometry from another, deduce some properties of relationships, and arrange them without relying on complete evidence, and use basic Geometry properties in dealing with Geometry problems, and reach Results from data by informal methods.

The fourth level (the formal deduction level): where this level is determined by theoretical thinking as well as building proofs for Geometry theories, and the learner can also draw conclusions from specific properties and data, and he can also distinguish between undefined elements and between definitions and axioms as well as proofs, and state the reasons in logical terms and in dependence Theories and axioms (Al-Khasawneh, 2007).

It is expected that the student at this level will be able to: Identify defined and undefined terms, distinguish between what needs proof and what is taken for granted, the ability to justify the steps of proof, and the use of axioms And the relationships and steps that were explained in the previous level, and to prove the internal relationships between theories.

Fifth level (the complete abstract deductive level Rigor level): This level is considered the highest level of Van Heil's model, in understanding the origins of relationships to build theories and Geometry axioms, where the learner at this level can work in a variety of intuitive systems, and here he can also study non-Euclidean geometry This stage enables him to compare different geometries and he will look at geometry abstractly (Ebeid, 2010).

The student at this level is expected to be able to: develop methods for solving some Geometry problems, create general methods for problems, compare different Geometry systems based on axioms, and deduce and prove some theories in different Geometry systems.

3.1 Previous studies

Al-Rifai's study (2018): which aimed to investigate the Impact of activities based on Van Heil levels of Geometry thinking to develop Geometry understanding and to improve the direction towards Geometry, and the sample consisted of (92) second-grade middle school students from MaHeila al-Kubra governorate, and the sample was divided into two groups: (44) students were an experimental group, and (48) a control group, and the design for research was experimental, and the tools were to test criteria and foundations for the activities required to be achieved for Geometry understanding such as solving problems and drawing geometric shapes and others, and then testing the first four levels of Geometry thinking, and the achievement test in the two units Geometry, a scale for measuring the trend towards geometry, The results showed that the activities based on the levels of Van Heil model were

effective in developing Geometry understanding, and there were no statistically significant differences between the two groups to measure the trend towards Geometry, and the absence of a relationship between the levels of the Van Heil model and the Geometry understanding and the trend towards Geometry.

Mahmoud's study (2017): which aimed to reveal the effect of using an educational program based on Van Hail's theory supported by geogebra on achievement and Geometry thinking in the unit circle, and the sample consisted of (94) ninth grade students in Qalqilya, and the sample was divided into three groups: (33) a student in an experimental group using Van Heil with geogebra, (33) a student in an experimental group using Van Heil without geogebra, and (28) a student in a control group in the usual way, The results of the study showed that there were statistically significant differences between the scores of the groups in the achievement test in favor of the two experimental groups, and there were no statistically significant differences between the two experimental groups in the level of achievement, and the existence of statistically significant differences between the groups in the Geometry thinking test in favor of the two experimental groups, and the existence of related differences. Statistical significance between the scores of the two experimental groups in the Geometry thinking test for the benefit of the experimental group using the Van Heil model with geogebra.

Mustafa & etal (2017) study aimed to discover the effect of using the Van Hayel teaching model on the achievement goals of student teachers at Isfahan University in Iran, and the sample consisted of (176) student teachers in Iran, and a tool was prepared to search the phrase On a questionnaire divided into four dimensions that reflect four trends, and the experimental approach was used, the results showed that there are statistically significant differences between the averages of students 'performance on the first approach in favor of the experimental group that was studied using the Van Heil model, and there were no statistically significant differences between the average performance on the second trend. And the third and fourth.

The study sample: The study sample was chosen in an Stratified method, as it was represented by the Dura Girls Secondary Vocational School, in order to provide the possibility of applying the study in terms of the number of people needed, as well as the proximity of the school to the residence and the work of the two researchers to follow up on the implementation of the study. Professional the random way.

4. Conclusions

To answer the study's question, Means and standard deviations due to the teaching method

(Group) as it is in Table (1), and according to the level of academic achievement as it is in Table (2).
Table (1): The arithmetic means and standard deviations of the 10th grade students' scores in the pre and post abstract thinking level test, according to the teaching method.

Group	Number	Descriptive	Pre-test	Post-test
Control Group	22	Mean	1.36	3.68
		St. Deviation	0.66	1.52
Experiment. Group	22	Mean	1.68	6.77
		St. Deviation	1.43	3.09

It is evident from Table (1) that there are apparent differences in the arithmetic means of the grades of the 10th grade students in the post-test due to the teaching method, the experimental group mean is greater than the mean of the control group.

Table (2): The arithmetic means and standard deviations of the 10th due to the level of academic achievement.

Level	Number	Descriptive	Pre-test	Post-test
High	14	Mean	2.21	7.57
		St. Deviation	1.63	3.37
Low	30	Mean	1.20	4.13
		St. Deviation	0.55	1.80

It is evident from Table (2) that there are apparent differences in the arithmetic averages of the grades of the tenth grade students in the post-abstract thinking level test due to the variable of the academic achievement level, as the results showed that the arithmetic averages of the grades of the tenth grade students with low academic achievement are less than the arithmetic averages of the female students With a high level in the post-test, where the arithmetic mean of the marks of the tenth basic-grade female students with a high level reached (7.57) and a standard deviation of (3.37) in post-achievement.

To find out whether these apparent differences in the arithmetic means of the grades of the tenth grade students are statistically significant, the accompanying test of variance (ANCOVA) was used as shown in Table (3).

Table (3): ANCOVA due to the Group, level, and the interaction between them.

Source of variance	Sum of squares	Degree of freedom	Mean Squares	f-value	Sig.	Effect Size
Pre	60.40	1	60.40	39.67	0.001	0.504
Group	61.79	1	61.79	40.58	*0.001	0.510
Level	32.22	1	32.22	21.16	*0.001	0.352
Group*level	0.04	1	0.04	0.03	*0.001	0.001
Error	59.39	39	1.52			
Total	1556	44				

* Statistical function at the level of statistical significance ($\alpha \leq 0.05$)

4.1 Results related to the group: It is evident from Table (3) that the value of the significance level calculated between the mean scores of the tenth grade students in the control and experimental groups according to the teaching method is (0.001), which is less than the level of statistical significance ($\alpha \leq 0.05$), and accordingly The null

hypothesis is rejected, that is, there are statistically significant differences at the level of statistical significance ($\alpha \leq 0.05$) between the mean scores of the tenth grade students in the test of the level of abstract thinking due to the teaching method, and to know the direction of the differences, the adjusted arithmetic averages and standard errors of the teaching method variable were calculated, As shown in Table (4).

Table (4): Marginal Estimated means and standard Deviation errors due to group

Group	Marginal Estimated Means	Standard Deviation Errors
Control	4.26	0.284
Experiment	6.95	0.302

It is evident from Table (4) that the marginal mean of the experimental group is (6.95), which is higher than the control group.

The results of Table (3) indicate that the effect size of the teaching method reached (0.510), which is a value greater than (0.14) the reference criterion for the size of the effect, which indicates that the teaching method according to the Van Hail model

has a great impact on the development of the level of abstract thinking among the 10th grade students.

4.2 Results related to the level of academic achievement:

The results of Table (3) indicate that the level of significance due to the level was (0.001), which is less than the level of statistical significance ($\alpha \leq 0.005$), and therefore the null hypothesis is rejected, and to know the benefit of those

differences, the marginal estimated means and standard errors calculated as shown in Table (5).

Table (5): Marginal Estimated means and standard Deviation errors due to level

Level	Marginal Estimated Means	Standard Deviation Errors
High	6.65	0.361
Low	4.56	0.235

It is evident from Table (5) that the arithmetic average adjusted for the level of high academic achievement in the Abstract Thinking Test is (6.65), which is higher than the modified arithmetic average for the students of the low achievement level, and this is evidence that the differences between the two levels of academic achievement in the test of developing abstract thinking were in favor of High achievement students.

4.3 Results related to the interaction between Group and Level:

The results of Table (3) indicate that the level of significance for the interaction between the group and the level was (0.001), which is less than the level of statistical significance ($\alpha \leq 0.05$), and therefore the null hypothesis is And to know the benefit of these differences, the marginal means and standard errors were calculated due to the interaction between the Group and the level, as shown in Table (6).

Table (6): The Marginal Estimated Means and Standard Errors due to the interaction between the Group and the Level

Control	Level	Marginal Estimated means	Standard Errors
Control	High	5.27	0.467
	Low	3.25	0.321
Experiment	High	8.03	0.562
	Low	5.88	0.333

It is evident from Table (6) that the adjusted arithmetic average for tenth grade students with high academic achievement in the experimental group is (8.03), which is higher than the adjusted arithmetic average for those with a low level in the same group, reaching (5.88), which indicates that the differences were For the benefit of high-level students in the experimental group, which is evident that the high-level students benefited from the teaching of the Geometry construction unit according to the Van Heil model more than the low-level students in the same group in developing their abstract thinking.

5. Discussions of the Conclusions:

Teaching according to the Van Heil model at its five levels enables students to develop their level of abstract thinking in the Geometry Construction Unit from the tenth grade mathematics course, and the researchers attribute this result to the Impact of teaching using the Van Heil model in developing the level of abstract thinking among the tenth grade students, where it contributed The use of the Van Heil model in teaching the Geometry construction unit for the tenth grade in helping the students to think deeply about Geometry concepts and structures, and the use of thinking patterns to reach the required solutions gradually from the concrete to reaching the level of abstract thinking required in this study, As this model worked on the transition between levels of Geometry thinking from visual to analytical to non-formal conclusion to formal conclusion to an abstract level, especially with the expansion of the amount of Geometry knowledge in this unit and the discoveries and continuous additions and rapid changes in all aspects of Geometry, which helped students in The

abstraction of ideas and concepts and the application of multiple activities on them, and the processes of linking abstract Geometry concepts with applied ideas helped students to create a balance between the cognitive side and the applied side of their skills.

The results indicated that there are significant differences in favor of female students with a high level of achievement, and this can be explained by the interest of this group of students in the interest in learning and the acquisition of new experiences and their desire to develop their mental abilities and their mathematical skills, where it is apparent that the high-level students are keen to show the ability to conclude Theories in various postulates systems, and the ability of this class to draw an abstract conclusion so that it enables them to understand geometry and analyze conclusions from axioms And from Geometry definitions, in order to be able to learn by developing new axioms and methods of solving different from the above, and to conduct complex mental processes such as analysis and inference to be distinguished by them in her class, so that high-level students are the highest in the value of the arithmetic mean in the scores of the test of developing the level of abstract thinking for this study.

The results also showed that there are statistically significant differences according to the interaction between the teaching method variable and the academic achievement variable, and this is evidence that teaching using the Van Heil model was more appropriate for high-level students in the experimental group than for students with low achievement in the experimental group in abstract thinking. This is also mentioned previously in

terms of the interest of this group of high achievers in adhering to the educational tasks and homework imposed on them, And keenness to help themselves to always reach the levels of higher and abstract thinking, as Van Heil's model helps them not to accumulate information in their minds, but rather to help them raise the level of their thinking and direct it to discover more and understand the relationships that bind their ideas, which helps them to be creative and productive in various fields, unlike the female students. Low achievement.

6. Recommendations

In light of the findings of the study, the study recommended inviting those in charge of developing and updating Palestinian mathematics books to reconsider the formulation of the content of Geometry units in mathematics courses for all stages in line with Van Hail's model. And the inclusion of the teacher's guide for the new mathematics curriculum by preparing some procedural lessons for the Geometry units in it according to the Van Heil model, so that mathematics teachers can review it. And holding training courses for teachers of mathematics through which they are introduced to Van Heil model on levels of Geometry thinking, its use and application in planning, preparing and implementing Geometry lessons in textbooks. And directing the attention of mathematics teachers to enriching the mathematics curriculum with activities that help students develop the level of thinking.

References:

- Abu-Lom, Kh. (2007). **Geometry Teaching Methods and Strategies**, 2nd edition. Dar Al-maseerah, Amman.
- Al-Balawi, A. (2013). **A training program based on interactive programs in teaching and learning mathematics**. PhD thesis, College of Education, Umm Al-Qura University, Saudi Court.
- AL-Hanafi, A. (2014). **The Impact of a program based on mobile mixed learning in developing levels of Geometry thinking among students teachers in the mathematics Sections**. Unpublished PhD thesis, Menoufia University, Egypt.
- Al-Rajeh, N. (2009). Abstract thinking according to the theory of Piaget among students of the Faculty of Education and its relationship to the trend towards mathematics and teaching skill, **Journal of Educational Sciences**. Issue 4, pp. 117-195, Egypt.
- Rashid, M., and Khashan, Kh. (2009). **Mathematics Curricula and Teaching Methods for Primary Stages**, First Edition. Janadriyah House for Publishing and Distribution, Amman, Jordan.
- Al-Rafei, A. (2018): Employing activities based on the Van Heil model to develop Geometry understanding and the trend towards geometry among middle school students, **The Education Journal**. Issue 51, College of Education, Sohag University, Tanta, pp. 141-198.
- Sadiq, I. (2001). **Methods of Teaching Mathematics**, Dar Al Fikr Al-Arabi, Cairo.
- Tannah, R. (2008): **An analysis of the content of the mathematics curriculum for the eighth grade in light of the levels of geometric reasoning by Van Heil**. Unpublished message, College of Education, Islamic University, Gaza, Palestine.
- Bal, A. (2014): Predictor variables for primary school students related to Van Hiele geometric thinking , **Jornal of theory and practice in education**. 10(1), pp259-278.
- knigh, K. (2006):**An Investigation into the change in the Van Hiele levels of understanding geometry of pre – service Elementary and Secondary Mathematics Teachers**. A thesis Submitted in partial Fulfillment of the Requirements for the Degree of master of Science in Teaching the Graduate School The university of Maine.
- Leung, H. (2011):**Enhancing students' ability and interest in geometry learning through geometric constructions**. HKU theses Online (HKUTO).
- Malloy. C. (2002): **The Van Hiele framework, Navigating Through Geometry in Grades 6-8 , USE , Reston va : (NCTM) , INC , WWW.nctm.org .**
- Mustafa, M., Javad , L & Reza , O .(2017): the effect of Van Hiele theory-based teaching educational package on Achievement goal orientation of student teachers, **Canadian center of science and education**, 9(1), pp93-105.
- NCTM. (2000): **Principles and standards for school Mathematics**, Reston, VA, Author.
- Teppo, A. (1991): Van Hiele levels of Geometric thought revisited, **Mathematics Teacher**, 84(3), pp210-221.
- Usiskin, Z. (1982): **Van Hiele levels and achievement in Secondary School Geometry**, Department of Education the University of Chicago, USA.
- Van Hiele P.M (1986):**Structure and insight: A theory of Mathematics Education**, Orlando, Florida, academic press, Inc.
- Mahmoud, M. (2017): The effect of using an educational program based on Van Heil's theory on achievement and engineering thinking in mathematics among ninth grade students in Qalqilya Governorate. Unpublished MA Thesis, College of Graduate Studies, An-Najah University, Nablus, Palestine.
- Khasawneh, A. (2007). Levels of Thinking in Space Engineering for Tenth Grade Students, **The Jordanian Journal of Educational Sciences**. Issue 3, Part 1, pp. 11-32, Jordan.
- Khasawneh, F. (1987). Test the Impact of both the inductive method and the deductive method for teaching

concepts in the acquisition of chemical concepts by first-grade secondary school students, Yarmouk University, Jordan, **Journal of Mathematics Pedagogy**. Volume 21, Issue 10, Part 1, Egypt, pp. 1-73.

Qusay, T. & Riyadh, J. (2014). The effect of the Van Heel model on developing self-confidence among fifth-grade scientific students in mathematics, **Journal of Educational and Psychological Sciences**. Iraq, Issue 101, pp. 78-129.

Al-Banna, M. (1994): **A proposed program for the development of thinking in engineering in light of the Van Heil model**. Unpublished PhD thesis, Girls' College, Ain Shams University, Egypt.

Ebaid, W. (2010). **Teaching Mathematics to All Children**, 2nd edition. House of the March for Publishing and Distribution, Amman, Jordan.

DARWINS, AK, M. SATHEESH, and G. RAMANAN. "EXPERIMENTAL ANALYSIS OF FRICTION STIR WELDING OF DISSIMILAR ALLOYS AA7075 AND MG ZE42 USING BUTT JOINT GEOMETRY." *International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)* 8.1 (2018) 505-512

Jadhav, SUBHASH V., PRASHANT M. Pawar, and BABRUVAHAN P. Ronge. "Analysis of pin-fin geometry effect on microchannel heat sink performance." *Int J Mech Prod Eng Res Dev* 8.4 (2018) 653-666.

JOSEPH, G. BRITTO, and TN VALARMATHI. "EFFECT OF GMAW PROCESS PARAMETERS ON IS 2062 STEEL CLAD GEOMETRY." 8.4 (2018) 855-862

Widakdo, Widi Aulia. "ANALYSIS OF PRIOR KNOWLEDGE IN LEARNING GEOMETRY." *IMPACT: International Journal of Research in Applied, Natural and Social Sciences (IMPACT: IJRANSS)* 6.5 (2018) 141-148

Irfan, Osama Mohamed. "INFLUENCE OF SPECIMEN GEOMETRY AND LUBRICATION CONDITIONS ON THE COMPRESSION BEHAVIOR OF AA6066 ALUMINUM ALLOY." *International Journal of Mechanical Engineering (IJME)* 5.1: 14-24.

Singh, S. S., et al. "Experimental Investigation of Thermal Efficiency of Boiling Pot with Modified Surface Geometry." *International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)* 8.1 (2018) 479-486.