



Dual Rings and Their Modules

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Abstract



In this thesis, we introduce three definitions concerning the duality of rings namely dual ring, quasi – frobenius ring and quasi-dual ring.

Also, we introduce many other concepts in the rings, ideals, modules and the structure of rings.

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Introduction



In 1984, I. Kaplansky [5] has defined the duality between two partially ordered sets of left ideals in a ring R . And in 1972, C. Reis [1] has defined the dual ring by a left and right artinian ring with whose left and right ideals satisfy $l(r(L)) = L$ and $r(l(I)) = I$. In 1985, N.C. Norton [2] has defined the dual ring R by the ring with identity in which $I = r(l(I))$ for every right ideal I and $L = l(r(L))$ for every left ideal L of R . Moreover, in 2000, Y. Zhou [12] has defined the dual ring R by a ring that every right ideal of R is a direct summand of a right annihilator and every left ideal of R is a direct summand of a left annihilator.

In fact, these definitions are justified into three definitions, namely, dual ring, quasi frobenius ring, and quasi-dual ring.

This thesis consists of three chapters. The first chapter deals with rings, sub rings, ideals, homomorphism, modules and free modules.

The properties, definitions and examples of noetherian ring, artinian ring, simple ring, semi simple ring, radical ring, Jacobson radical, primitive ring and semi perfect ring are discussed in chapter 2 which is divided to 5 sections.