# Promoting Mathematical Understanding Using ICT in Teaching and Learning

Kamel Hashem, and Ibrahim Arman

***Abstract*—**Information and Communication Technologies (ICT) in mathematical education is a very active field of research and innovation, where learning is understood to be meaningful and grasping multiple linked representation rather than rote memorization, a great amount of literature offering a wide range of theories, learning approaches, methodologies and interpretations, are generally stressing the potentialities for teaching and learning using ICT. Despite the utilization of new learning approaches with ICT, students experience difficulties in learning concepts relevant to understanding mathematics, much remains unclear about the relationship between the computer environment, the activities it might support, and the knowledge that might emerge from such activities. Many questions that might arise in this regard: to what extent does the use of ICT help students in the process of understanding and solving tasks or problems? Is it possible to identify what aspects or features of students' mathematical learning can be enhanced by the use of technology? This paper will highlight the interest of the integration of information and communication technologies (ICT) into the teaching and learning of mathematics (quadratic functions), it aims to investigate the effect of four instructional methods on students' mathematical understanding and problem solving. Quantitative and qualitative methods are used to report about 43 students in middle school. Results showed that mathematical thinking and problem solving evolves as students engage with ICT activities and learn cooperatively.

***Keywords*—**Dynamic Geometry Software, Information and Communication Technologies, Visualization, Mathematical Education.

## I. Introduction

INFORMATION and Communication Technologies (ICT) in Mathematical Education is a very active field of research and innovation, where learning is understood to be meaningful and grasping multiple linked representation rather than rote memorization [1], there is a great amount of literature offering a wide range of theories, methodologies and interpretations, are generally stressing the potentialities for teaching and learning using ICT [2], [3].

Information and Communication Technologies can use powerful tools for learning mathematical reasoning and problem solving [4]. One powerful way to use ICT for learning mathematics is through the manipulation and construction of ICT-based mathematical models and simulations [5], [6], [7], [8]. In order to take greater advantage of the computer medium, learners should engage in technology-supported reasoning, including checking and inquiring assumptions arises. The acceptance of the partnership between ICT and humans in making mathematical reasoning breaks the "Fregean barrier". Frege said that what matters in mathematics is only the context of justification and reasoning not the context of discovery. The partnership between the mathematical learner and ICT has already transformed the culture of practicing mathematicians and will alter the mathematical learning culture [9], [10], [11].

The National Council of Teachers of Mathematics (NCTM 2000, 2006) have pointed out the relevance of enhancing the mathematical understating and problem solving as an integral part of learning in k-12. In addition, the NCTM reforms identify the use of technology as one of the key organizer principles in learning math, since it allows students to experiment and examine mathematical relationships from diverse angles or perspectives [12], [13], [14].

Despite the utilization of new learning approaches with ICT, students experience difficulties in learning concepts relevant to understanding mathematics, many remains unclear about the relationship between the computer environment, the activities it might support, and the knowledge that might emerge from such activities [15]. Many questions that arise in this regard: to what extent does the use of ICT help students in the process of understanding and solving tasks or problems? What is the role of teachers in an enhanced technology class? Is it possible to identify what aspects or features of students' mathematical learning can be enhanced by the use of technology?

The work presented in this paper is offered as a contribution to understanding the relationship between the dynamic geometry environment (GeoGebra), and the kind of mathematical thinking and problem solving that may develop as a result of interactions with the tool. This study focused on the effect of different modes of involvement in exploring mathematical activities, on students' mathematical understanding and problem solving. It is part of a more comprehensive study pursuing the goals: (1) to study the role of visualization in the learning process of mathematical equations and graphs; (2) to examine the contribution of different modes of involvement in the visualization process (e.g., intervention and manipulation, construction) to the students' understanding of mathematics; and (3) to examine the effect of the type of engagement (individualize vs. cooperative) on the student's learning.

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## II. Method

### A. Subjects

Participants were 43 students (ages ranging from 14 to 15 years old) composed of two 9th grade sections at Al-Quds preparatory school in Jerusalem old city. The two sections were divided according their level of involvement with tasks. One section is considered as a manipulation (MANI) group and divided into two groups based on the way of learning: individual (IND) and cooperative (COOP). The other section is considered as construction (CONST) group and divided into two groups based the way of learning: individual (IND) and cooperative (COOP). The resulting groups from the two sections are: (1) cooperative learning combined with ICT task-manipulation (COOP+MANI), (2) individualized learning combined with ICT task-manipulation (IND+MANI), (3) cooperative learning combined with ICT task-construction (COOP+CONST), and (4) individualized learning combined with ICT task-construction (IND+CONST). Students' distribution can be seen on Table I and Fig. 1.

TABLE I
STUDENTS' DISTRIBUTION ACCORDING TO THE LEVEL OF INVOLVEMENT AND THE WAY OF LEARNING

| Level of involvement | Way of Learning | | Total |
|---|---|---|---|
| | Individual | Cooperative | |
| construction | 12 | 11 | 23 |
| manipulation | 8 | 12 | 20 |
| Total | 20 | 23 | 43 |

### B. Research Instruments

(a) The learning environment comprising two components: (1) GeoGebra, free dynamic geometry software (DGS) and computer algebra system (CAS), created in 2002 by Markus Hohenwarter at University of Salzburg (see [16]), and (2) tasks and activities in which students run GeoGebra software and are requested to perform the tasks with.

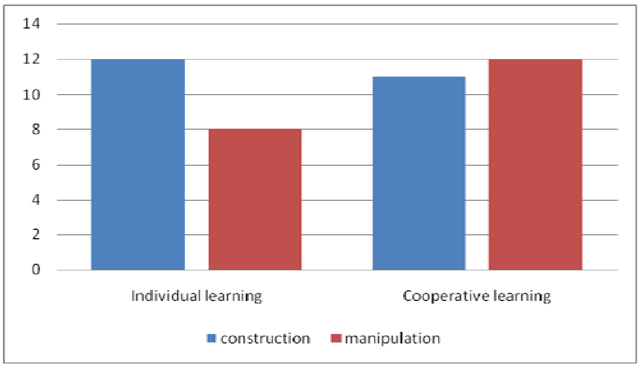


Fig. 1 Students' distribution to the different groups

(b) Data collection tools included: (1) pre-test comprising general background to evaluate students' prior knowledge of quadratic equations and graphs; (2) structured observation and data forms; (3) structured interview, focusing on students' attitudes toward the use of ICT in math learning; and (4) Post-test: (same as pre-test).

### C. Procedure

The study was carried in four stages: (a) Pre-test, (b) Treatment in four different groups. Generally speaking, all groups have attended a 50 minutes introduction to GeoGebra software environment (DGS), and were set to work in a 50 minutes session as follows:

(1) Cooperative learning combined with a manipulation activity (COOP+MANI) mode: the cooperative technique suggests that students learned in small groups (2-4 students) (see [13]: 287), and the manipulation engagement introduces students to a given initial set of conditions for a math activity and then requested to manipulate the variables according to the activity requirements, (2) Individualized learning combined with a manipulation activity (IND+MANI) mode: in which each student start to work on the activity using the manipulation technique, (3) cooperative learning combined with a construction activity (COOP+CONST) mode: each cooperative group start to construct the learning activity according to the given instructions, (4) individualized learning combined with a construction activity (IND+CONST) mode: in which each student start to work on the activity using the construction technique, (c) Interview after treatment: students were interviewed for their attitudes toward the use of technology in math learning, all responses were audio taped and (d) Post-test: (same as pre-test).

## III. Results

### A. Quantitative Analysis

In order to show how different modes of involvement affect learners' mathematical understanding, a paired-samples t test where done using SPSS software, the purpose of the analysis is to get a general sense of whether the students' understanding of the learning activities changed while using different modes of involvement. The results in Table II and Fig. 2 show that there was a significant increase ( $t(42) = -3.05$, $p < 0.01$) in students' scores on the pre-test and post-test indicating understanding of quadratic equations and graphs in all four groups (COOP+MANI, IND+MANI, COOP+CONST, and IND+CONST).

TABLE II
STUDENTS' MATHEMATICAL UNDERSTANDING

| Group | (N) | Pretest M | Pretest SD | Posttest M | Posttest SD |
|---|---|---|---|---|---|
| COOP+MANI | 12 | 7.92 | 7.18 | 12.67 | 8.33 |
| IND+MANI | 8 | 8.5 | 9.73 | 11 | 9.1 |
| COOP+CONST | 11 | 16.09 | 8.7 | 17.82 | 8.02 |
| IND+CONST | 12 | 18.17 | 9.77 | 18.33 | 9.98 |

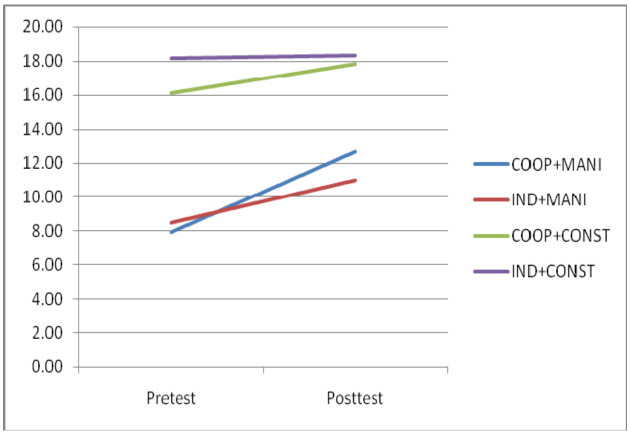


Fig. 2 Students' results on pre-test and post test for the different groups

The primary purpose of our study was to investigate the effect of four instructional methods on students' algebraic problem solving with regard to procedural tasks.

We analyzed the data through the examination between the different instructional methods of learning on the activities scores (see Table III and Fig. 3) using a one-way ANOVA showing a significant differences between groups ($F$ (3, 39) = 6.05, $p < 0.01$).

Post Hoc (LSD) analysis was done showing that: (a) significant differences between the (IND+CONST) group and the (COOP+CONST) group ($p < 0.01$), in examining Table III, it shows that the (COOP+CONST) group got the highest scores on doing the activities, on the other hand there were no significant differences between the (IND+CONST) group and both of the (IND+MANI) group and the (COOP+MANI) group, (b) significant differences between the (COOP+CONST) group and the other three groups ($p < 0.05$), in examining Table III, it shows that the (COOP+CONST) group got the highest scores on doing the activities followed by the (COOP+MANI) group followed by the (IND+CONST) group followed by the (IND+CONST) group, (c) significant differences between the (IND+MANI) group and the (COOP+CONST) group ($p < 0.01$), in examining Table III, it shows that the (COOP+CONST) group got the highest scores on doing the activities, on the other hand there were no significant differences between the (IND+CONST) group and both of the (IND+MANI) group and the (COOP+MANI) group, and (d) significant differences between the (COOP+MANI) group and the (COOP+CONST) group ($p < 0.05$), in examining Table III, it shows that the (COOP+CONST) group got the highest scores on doing the activities, on the other hand there were no significant differences between the (COOP+MANI) group and both of the (IND+MANI) group and the (IND+COST) group.

TABLE III
SCORES ON ACTIVITIES FOR THE DIFFERENT GROUPS

| Group type | N | Mean | Std. Deviation |
|---|---|---|---|
| IND+CONST | 12 | 50.75 | 24.488 |
| COOP+CONST | 11 | 86.45 | 16.348 |
| IND+MANI | 8 | 44.25 | 23.759 |
| COOP+MANI | 12 | 59.58 | 29.657 |
| Total | 43 | 61.14 | 28.264 |

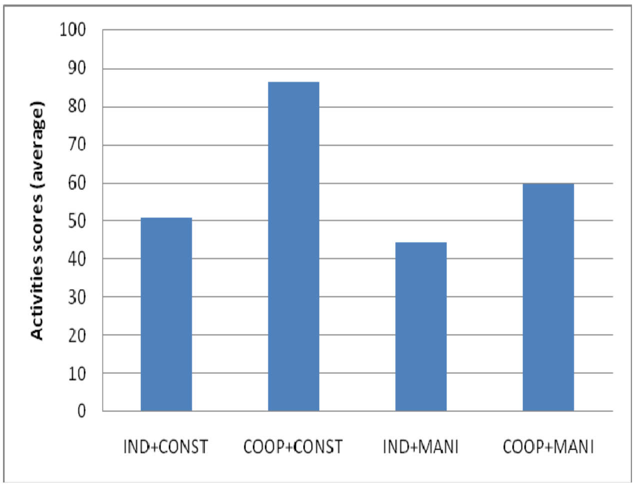


Fig. 3 Students' achievements when solving quadratic equations

### *B. Qualitative analysis*

An examination of students' responses indicated additional qualitative differences between the different groups. Students' answers show that information and communication technology has made connections "more evident and clearer" as stated by Aya and Mais. These students stated that technology made things easier because the dynamic construction of the equation and manipulating graphs made them understand the process better and relate the points to the graph.

## IV. CONCLUSION

GeoGebra as a dynamic geometry software (DGS) offers support for teaching much more than geometry. In this work we have given activities that demonstrate changeable diagrams that can show a generalization and support the teaching of algebra. Diagrams produced by students themselves were useful for many teaching and learning situations; manipulating the diagram and working in groups improve their mathematical thinking and give them the ability to take up the challenge of making use of algebra to solve it.

This paper reports on a study about the interaction between modes of learning with ICT tool and mathematical problem solving, there are many concepts that we never directly experience or that violate our intuitions and challenges of our cognitive and meta-cognitive resources. The implementation of such an instructional approach in the curriculum would have many benefits for learners, such as new ways of thinking, exploration of tools to think with, and construction of diagrams linking between theory and practice. In addition, GeoGebra has produced changes not only in the type of tasks and questions that students examine during their activity processes; but also in the role played by both teachers and students throughout the development of the class.

By introducing this new perspective in learning using computer DGS for learning mathematics, mathematics learning will be more motivational and truthful, more inclusive and accessible to the great majority of students, the use of the DGS allows effective reasoning about the mathematical problem solving, in addition, this study's results have clear implications for the design of learning environments that can support learning about mathematics.

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