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Chapter One

Introduction and Motivations

1.1 Introduction

The earliest nuclear models were concerned mainly with the nuclear structure. The liquid drop and the shell models were developed to describe the nuclear structure, energy and interactions within the border of nuclear matter densities (Heyde, 1999). The nucleon-nucleon (NN) interacting systems have been investigated extensively and the general properties of nuclear force have been well established in the low-energy regime. Extension of such investigations requires going beyond the medium and the high energy regimes (regions of nuclear density beyond the nuclear matter density) which can be achieved by means of heavy-ion (HI) collisions. The density and temperature dependence of hadronic systems is an interesting topic in this branch of nuclear physics.

Experimentally, the nucleus-nucleus collisions at intermediate energies provide a unique opportunity to form a piece of hot nuclear matter in the laboratory with a density up to 2-3ρ_0, where ρ_0, is the normal nuclear matter density (~ 0.17 fm^{-3}) (Heyde, 1999). Thus, it is possible to study the properties of hadrons in hot and dense medium. Since this piece of dense nuclear matter exists only for a very short time (typically 10^{-23} *to* 10^{-22}s), it is necessary to use transport models to simulate the entire collision process in order to deduce the properties of the intermediate stage from the known initial conditions and the final state observables (Abu-Samreh, 1991). At intermediate energies, both the mean field and the two-body collisions play an equally important role in the dynamical evolution of the colliding system. In addition, they have to be taken into account in the transport models on an equal footing together with a proper treatment of the Pauli blocking for the in-medium two-body collisions. Relatively speaking, most collisions were accompanied by the emission of elementary particles in order to compensate conservation laws. Nowadays, more than 200 elementary particles have been produced and identified during nuclear reactions. They were classified and introduced according to their momenta and energies in order to fulfill the conservation of energy and momenta (Scadron, 1979).

Since particles are produced during HI collisions, a microscopic study of the dynamical processes of particle emission is of great importance in the collisions of heavy ions (Zhang and Wilets, 1992). Generally speaking, mesons, deltas as well as nucleons inside nuclides interact among themselves by means of strong interaction (Scadron, 1979). The strong interactions are referred to those processes involving baryons and pions (pi-meson). Such interactions give rise to nuclear forces between the nucleons in nuclides and to the formation processes such as decay of mesons and baryons in nuclear interactions with high energies. Processes in which strong interactions are manifested are known to be the fastest among other reactions and the characteristic lifetimes vary from 10^{-23} to 10^{-22} sec (Flügge, 1970).

1.2 The creation of Δ resonances in nuclear matter

HI collisions make it possible to go beyond the nuclear matter density and the medium energy range. This is resulted in producing other types of particles such as pions, mouns, leptons, deltas --etc. to maintain conservation laws. Accordingly, a great number of new short lived formations with a characteristic lifetime of strong interactions such as baryons and mesons have been observed in HI collisions reactions. These particles were called resonance particles, or resonant states or Fermi resonances (Yavorsky and Detalaf, 1972). Such particles have definite properties such as mass, specific momenta, and energies that enable resonances to be regarded as particles.

A beam of nucleons having energy above 300 MeV/nucleon may be resulted in producing an excited state of nucleon as a delta resonance. The Δ resonance, which was discovered by Fermi and Anderson in 1949, is considered as an excited state of the nucleon that can be excited via electromagnetic and strong interaction. However, the responses of these interactions are quite different. It was found that this state has an excitation energy of 300 MeV or more and transition width, Γ, of 116 MeV. The Δ resonance (particles) is generally characterized by spin and parity $J = \frac{3^+}{2}$ and its isospin $T = \frac{3}{2}$ (Perkins, 1987). The most general characteristics of Δ resonances are listed in Table 1.

Table 1. General properties of delta particle

Particle	**Charge**	**Spin**	**Isospin**	**Mass (MeV)**
Δ^{++}	++	$3/2$	$3/2$	**1232-1236**
Δ^{+}	+	$1/2$	$1/2$	**1232-1236**
Δ°	**Neutral**	$-1/2$	$-1/2$	**1232-1236**
Δ^{-}	-	$-3/2$	$-3/2$	**1232-1236**

Experimentally, there are three different types of Δ decays each of them is produced by a certain mechanism. For instance:

1) **Quasi-free decay of the Δ resonance**. In this case the Δ resonance is excited on one single nucleon in the target. The possible quasi-free decay modes are (Perkins, 1987):

$$\Delta^{++} \rightarrow \pi^{+} + \mathrm{P} \quad , \Delta^{+} \rightarrow \pi^{+} + n \ , and\ \Delta^{+} \rightarrow \pi^{0} + \mathrm{P} \tag{1.1}$$

2) **Absorption of the Δ resonance**. In this case, no pion can be detected following the decay of the Δ resonance. The pion is absorbed by two nucleons moving through the target and finally emitted from the target nucleons. The relevant absorption processes are (Perkins, 1987):

$$\Delta^{++} + n \rightarrow 2P \quad , \Delta^{+} + P \rightarrow 2P \quad \textit{and} \quad \Delta^{+} + n \rightarrow n + P \qquad (1.2)$$

3) **Coherent pion production**. In this case where one pion, (but no nucleon), is emitted from the target nucleus following the decay of the Δ resonance, (Perkins, 1987).

$$\Delta(A-1) \rightarrow \pi^{+} A \qquad (1.3)$$

These resonances are detected in states having densities as high as the nuclear matter ground state density or even more.

Historically, the dynamics of the Δ-nucleus interaction has been one of the most challenging problems to nuclear physicists. The delta production in HI collisions should be included from which much more can be learned about the NN system. The delta, in general is assumed to propagate within the same mean field as the nucleons if not explicitly noted otherwise. Deviations from this average potential will not only influence the propagation of the resonances through the nucleus but also change the collision probabilities. That is, a difference between the nucleon and the resonance potentials will shift the energy in the

$RN \to NN$ reaction and the peak position in the $\pi N \to R$ process. Here R is used to designate resonance; while N is used for nucleon. The possibility of producing Δ matter (or in more general forms resonance matter) at beam energies about few hundred MeV/nucleon is increased when the nucleon is excited into higher states (Δ-resonance) (Gupta, 1988). If the density of these resonances is high as the nuclear matter ground state density, then a new state of matter, Δ matter, was created. One of the potential signals for the presence of Δ matter is the creation of pions as decay products of the Δ resonance (Bass *et al*., 1995). Delta resonances are initially produced through inelastic NN scattering. The produced resonances can either be reached either by inelastic scattering or by emitting a pion (Sehn and Wolter, 1996). The pion can then either freeze out or interact with nucleons to form Δ again, where the Δ can transfer energy via elastic scattering into another nucleon which then can scatter in-elastically and form a new delta. A nucleon interacts on the average about 3 times before it freezes out. Thus, systems such as: nucleon-delta (NΔ) and the delta-delta (ΔΔ) are of great importance as NN system. Accordingly, the delta production in HI collisions should be included in most investigations dealing with high energy nuclear reactions. This is because much more information could be extracted about the NN system.

1.3 Theoretical background

The strong interaction exists in two different forms: Firstly, as the force within the hadron, which holds it together. Secondly, as the force between different hadrons and it is mediated by the exchange of mesons between nucleons (Angelica, 2002). In HI collisions, various models of different natures depending on each specific process have been introduced and developed to investigate the various mechanisms involved and to understand the ΔN and ΔΔ interactions. Such models and investigation will be considered as a step for searching a general form of nuclear potential. However, most of them are restricted to specific energy regime or specific phenomenon and some of them have too many parameters to obtain physical conclusion from the analysis. Conventional models of the NN interactions are built on nonrealistic proton and neutron interaction via two-body potentials (Krane, 1987). Typical NN potential contains strong short range repulsion.

Nuclear systems are then described by the solution of many-body Schrödinger wave equation. Nucleons are treated as composite systems each with resonance structure which attributed to quarks consistent interacting by gluon exchange. Ideal model of NN interaction would start with a field described theoretically by means of quark-quark interaction. Therefore, no satisfactory theory has yet been developed (Feynman, 1969; Negele, 1982). The short range part of NN potential is generally treated in a phenomenological way which is reasonable until a better understanding of the role of quarks is reached. It was confirmed experimentally that most of attraction from two pion exchange process were nucleon resonances that might be exited intermediate state shifts (Wiringa *et al*., 1984; Hahn and Glendenning, 1987).

The interactions between two baryons such as $N\Delta$ or $\Delta\Delta$ were obtained from the chiral quark cluster model (Mota *et al*., 1999; Zhang and Wilets, 1992). The NΔ and the ΔΔ interactions have been derived in the past decades in the framework of meson-exchange models or phenomenological potential (Mota *et al*., 1999). These models have been used to fit NN data very accurately. However, in the NΔ and ΔΔ sectors, the experimental data are so limited so that it is not possible to obtain reliable values of the parameters involved in the interaction. The situation is different in the case of chiral quark cluster models (Zhang and Wilets, 1992). In these models the basic interaction is at a level of quarks involving only a quark-quark field (pion or gluon) vertex (Florkowski and Abu-Samreh, 1996). In such cases, the Pauli principle between particles determines the short-range behavior of different channels. In this way, even in the absence of experimental data (Mota *et* *al*., 1999), one has a complete scheme which starting from the NN sectors that allows us to make predictions in the NΔ and ΔΔ systems.

The NN long-range Paris potential and the quark-quark intermediate-ranged one were completely determined from pion-nucleon and pion-pion interactions (Negele, 1982; Hahn and Glendenning, 1987). The Δ (1232) plays an important role in processes which generate the intermediate-range attraction of NN interaction. Besides, the NΔ dispersion approach is important for investigating the $\pi\pi$ *and* πN interactions (Gavin, 1990; Bonutti, 2000). The $NN \rightarrow N\Delta$ transitions were found to be essential for the coupled channels approach model (Cassing *et al.*, 1990). This model can be used to construct realistic NN potentials that make sense if the ΔΔ as well as the NΔ intermediate states were included. A repulsive core should be presented in the NΔ and ΔΔ channels and a compression between strengths of such channels and that in NN channels can be easily made (Bjorken, 1983). A complete understanding of nuclear structure requires an understanding of both neutron and proton effects in the nucleus.

The relativistic Brüeckner model has been developed as a transport approach to HI collision frame a microscopic many-body approach based on NN scattering (Botermans and Malfliet, 1990). Relatively speaking, it would be tremendously difficult to solve the full Brüeckner problem. It has been only solved for idealized spherical nuclear matter configuration (Sehen and Wolter, 1996). The Dirac and Brüeckner model has been used to study the in medium effects of elastic and inelastic NN collisions (ter Haar and Malfliet, 1987). The Hamiltonian model has been used to calculate the pion interaction and its effects on HI collisions. In addition this model was used to derive the element $NN \rightarrow N\Delta$ cross sections, and used that in the Boltzmann`s equation for investigating HI collisions at 800 MeV/particle (Bertsch and Gupta, 1988). The Boltzmann equation includes the medium effects associated with the velocities and final phase- space of the colliding particles (Dalta *et al*., 1988).

The earliest nuclear dynamical models to describe the delta resonances is the Intra Nuclear Cascade model (INC) which was used to study the collisional properties of the nuclear many-body system in the relativistic HI collisions regimes with a beam of energy ranging from 250 MeV to 2 GeV per nucleon (Cugnon *et al*., 1981; Cugnon, 1982). This model assumes a multiple scattering but neglects the mean field effects and Pauli exclusion principle. For this approach to be valid, the inter particle distance must be larger than the size of the nucleon (Helgesson and Randrup, 1997). This model was used success to develop the relativistic HI collisions cross section.

A nonequilibruim dynamical theory of HI collision had been developed on the basis of quantum chromodynamics (QCD) (Zhang and

Wilets, 1992). Particle production in HI collision can be described by the collective excitations of the basic building blocks (quarks and gluons) of the system. This approach is beyond the scope of this study.

The Isospin Quantum Molecular Dynamics (IQMD), follows the same scheme as the INC but takes into account the nucleus potential which is calculated as the sum of all two-body potentials (Enagle *et al*., 1994.). Figure1.1 shows the delta-nucleon cycle in the IQMD model (Sorge *et al*., 1989; Bass *et al*., 1995). The scheme describes possible processes linked to the creation of Δ matter (for impact parameters b ≤ 5 fm and averaged over 60 fm/c) (Longo, 1973). The probabilities in the boxes were always referred to the vertices which are directly connected with Δ resonances that are initially produced via inelastic nucleon collision with Δ resonances and NN scattering. The produced resonances can be either reabsorbed via inelastic scattering or decay by emitting a pion (Blättel *et al*., 1993). The pion can then either freeze out or interact with a nucleon to form a Δ again (Bonutti, 2000). In this case, the Δ has been absorbed the corresponding high-energetic nucleon might have a second chance of becoming a Δ by inelastic scattering. In this case, energy could be transferred via elastic scattering into another nucleon that can be scattered elastically to form a new Δ.

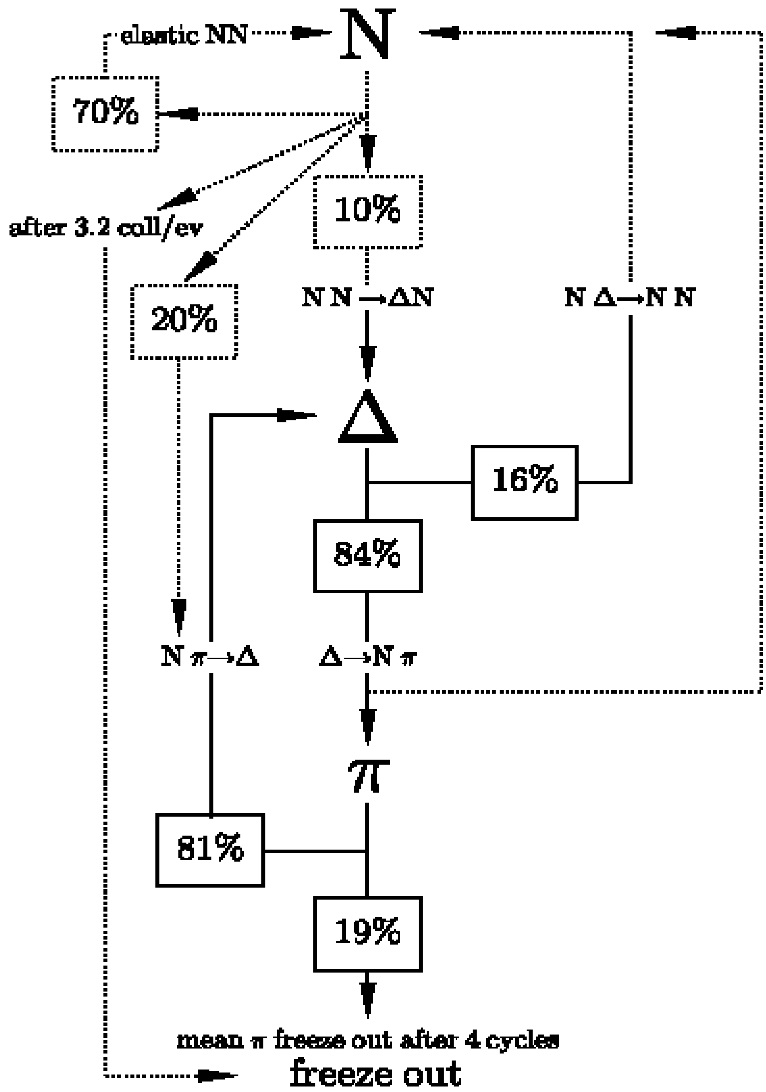


Figure 1.1 Pion-nucleon cycle in the IQMD model, the scheme describes (for b < 5 fm and time averaged) all possible processes in the linked to the creation of Δ matter. The main process for sustaining Δ matter is the $\Delta \rightarrow N\pi \rightarrow \Delta$ loop, which, however, first has to be fueled by the $NN \rightarrow \Delta N$ process (Bass *et al*., 1995).

A nucleon on the average interacts about 3 times before it freezes out (Emelyanov and Panti, 1995). This value fluctuates considerably, depending on whether the nucleon was in the participant zone (geometrical overlap of the colliding heavy ions) or in the spectator zone of the collision. The presence of Δ matter is accompanied by the creation of pions as decay products of the Δ resonance (Bertsch, 1976). Unfortunately, the probability for a nucleon to undergo inelastic scattering and to form a Δ during the heavy-ion collision is as low as 10%. The main process for sustaining Δ matter is the $\Delta \to N\pi \to \Delta$ loop, which, however, first has to be followed by the $\mathrm{NN} \to \Delta\mathrm{N}$ process. On the average delta passes approximately 3 times through this loop (it has been created by the decay of a hard Δ). However, 30% of delta passes more than 6 times through the loop. For nucleons, the probability of forming a soft Δ, i.e., via, $\pi N \to \Delta$, is almost twice as high (Δ-matter pump) than the probability of forming a hard Δ via $\mathrm{NN} \to N\Delta$ (Bass *et al.*,1995).

The most elaborate theoretical approach for the describing the delta production is based on the microscopic kinetic theories. Transport models like the Boltzmann-Uehling-Uhlenbeck (BUU) models (Uehling and Uhlenbeck, 1933) are widely applied to investigate HI collisions (Bertsch and Gupta, 1988). The quantum statistical Kinetic model or the BUU (Danieleicz, 1984; Aichelin, 1985) provides a semi-classical approximation to HI collision. During the past decade, such model has been widely used in studying the HI collisions in high energy regimes (Aichelin and Bertsch, 1985). The NN reactions in the low and intermediate energy regimes had been analyzed using Uehling-Uhlenbech-Nordheim modification in nuclear matter and the relaxation-times for the equilibration as a function of density and final temperature of the equilibrated system (Abu-Samreh and Kohler, 1993). These models are successful used in describing the average properties of the one-body observations associated with nuclear collisions as nucleon spectra, collective flows and particle production (Heisenberg and Wong, 1996). This type of approach includes the propagation of pions and nucleon resonances as well as their mutual interactions (Enagle *et al*., 1994). Each ΔN resonance forms a structure as an ordinary proton or neutron and the fact that these resonances are extremely short-lived should not affect including them in a list of particles.

The BUU is quite successful in describing the experimental data on pion production in proton-nucleus as well as nucleus–nucleus collisions (Engel *et al*., 1994), the pion-nucleus reactions were analyzed using a microscopic transport model of the BUU type which propagates nucleons, pions, deltas and N (1440) resonances explicitly in space and time. Some dynamical simulation of nucleus-nucleus collisions based on the transport theories indicated the most likely excite resonance matter, Δ matter in hot and dense nuclear matter (Florkowski and Abu-Samreh, 1996). Experimental observations had indicated that of an incident energy of (2 GeV/nucleon) more than 30% of the nucleons in the collision core are excited to states where deltas are most likely be created there (Bertsch *et al*., 1988). It seems to be that deltas play an important role at intermediate and high energy HI collisions. It would, therefore, be interesting and worthwhile to develop BUU transport equation type for the delta distribution function, which should be solved simultaneously with that of the nucleons in HI collision energy. It is worth mentioning here that experimental determination of NΔ and the ΔΔ scattering cross section is inaccessible even in free space. In almost, all transport models they were assumed to be equal to free NN scattering cross section (Li and Gross, 1999).

The Boltzmann-Uehling-Uhlenbeck (BUU) equation (Cassing and Bratkovskaya, 1999) and quantum molecular dynamics (QMD) (Weber *et al*., 2003) as well as their relativistic extensions (RBUU and RQMD) (Wong, 1996) are widely used describing the intermediate energy of HI reactions (Zhang and Wilets, 1992). In general, transport model have been very successful in describing particle production in HI collisions (Bertsch and Gupta, 1988). One of the remaining open problem in the transport-theoretical description of HI collisions at energies (~ 1 GeV) or above is an over prediction of the pion multiplicity (Helgessaon and

Randrup, 1997). At the beam energies of a few GeV the dominating mechanism of the pion production is the excitation of the Δ(1232) resonance in a NN collision followed by several decay processes as: $\mathrm{NN} \rightarrow \mathrm{N}\Delta$, $\Delta \rightarrow \mathrm{N}\pi$. Accordingly, the pion multiplicity depends crucially on the value of the in-medium $\mathrm{NN} \rightarrow \mathrm{N}\Delta$ cross section which – by detailed balance – also determines the pion reabsorption (Blättel *et al.*, 1993).

The finite-temperature Greens-function, formalism was introduced and developed to study the equation of state of a hot interacting delta at zero chemical potential and calculated the in-medium single-delta self-energy and the $\pi\pi$ scattering amplitude in the quasi particle approximation ((Danieleicz, 1984; Rapp and Wambach, 1995). This model was used to study thermalization processes in HI collision at beam energy $E_{lab} \approx 400$ MeV/nucleon. This Green function model has been used for developing microscopic dynamical theory of relativistic HI collision on the bases walecka model (Serot and Walecka, 1986). A hadronic transport theory which includes $\mathrm{N}, \Delta, \sigma, \pi, \omega$ elementary particles has been developed somewhere else (Wong, 1996) approach is too difficult and if needs a lengthy computer calculations. The time-dependent Hartree-Fock (TDHF) approximation emerges as natural dynamical extension to Greens function model. This approximation assumes along mean for path and neglects the imaginary part of the effective interaction between particles (Negle, 1982).

The isospin-dependent transport model has been introduced to study the isospin and momentum relaxation-times in the heavy residues formed in HI collisions at intermediate energies (Li and Ko, 1998.). It was found that only at incident energies below the Fermi energy, chemical and thermal equilibrium can be reached before dynamical instability is developed in heavy residues (Song and Koch, 1993). Also the isospin relaxation-time is shorter (longer) than that for momentum at beam energies (higher) than the Fermi energy (Li and Ko., 1998).

In order to obtain the collision integrals which describe particle production, are go beyond the mean-field approximation (MFA) (Zhang and Wilets, 1992). A transport model with chrial symmetry was developed from the quark level to describe the high-energy HI collisions beyond the MFA.

The main purpose of our work is to develop a model, which can be used to describe the relaxation-time of some nuclear reactions with a minimum number of parameters and a wide range of applicability. We shall propose the BUU incorporated with the relaxation-time approximation model (RTAM) in order to achieve our goals (Köhler, 1985). The modified BUU model requires the calculation of the in-medium $NN \leftrightarrow N\Delta$ cross sections. The calculated cross sections are then used in the BUU model to study the delta production in HI collisions at intermediate and high energies. The choice of such models is motivated by several reasons. Firstly, the (BUU) model, describes reactions of the dynamics of the interacting particles, and is commonly used to extract the information of the nuclear collision particles. Secondly, it can be used in describing the types of collisions (elastic and inelastic) by including the conservation of energy and so momentum before and after collisions. Thirdly, transition rate can be estimated using this model by investigating the development of the system from non-equilibrium states to equilibrium ones. Fourth, the RTA model gives a good result with least parameters and less computer work.

1.4 Statement of the problem

We shall assume a nuclear matter consists of protons and neutrons. These particles can interact with each others in the ground state of nuclear matter to form a new state, or an excited state which is usually called a resonance state. In such state, a new particle such as delta particles may be created at certain threshold energy. In this study, such particles were treated as free particles, the scattering transition rate or resonance width and cross sections were needed when modeling theoretically this phenomenon. The BUU kinetic model can be used to accomplish this goal.

The equation of motion for the time evolution of the one-body phase-space distribution $f(\vec{r},\vec{p}_1;t)$ is (Balescu, 1975):

$$\frac{\partial f(\vec{r},\vec{p}_1;t)}{\partial t}+\frac{\vec{p}_1}{m}.\vec{\nabla}_r f(\vec{r},\vec{p}_1;t)-\vec{\nabla}_r U(\vec{r}).\vec{\nabla}_{p_1} f(\vec{r},\vec{p}_1;t)=I[f(\vec{r},\vec{p}_1;t)] \quad (1.4)$$

The first term on the left-hand-side (l.h.s) is the scattering term (particles are scattered into and out of the volume element $d\vec{r}d\vec{p}$). The second term represents the streaming term (particles pass in and out of the volume element around $\vec{r}$ because of their motion) and the third term represents the change of the distribution function because of the applied external forces (Engel *et al*., 1994). The $U(r)$ is the particle mean-field potential. The term on the right-hand-side (r.h.s) is the collision term.

This term plays a vital role in this study and it needs to be discussed in more details in chapter two.

In this work, the calculations can be simplified by combining the BUU model with the relaxation-time approximation (RTA) model (Köhler, 1985). This approximation will be used in calculating the width (transition rate) of the reaction and so that the transition rates cross sections can be estimated. The RTA model will be used in this study as a simple approximation to estimate the relaxation rates or the resonance width. In the RTA each collision term is inversely proportional to the relaxation-time which characterizes that scattering mechanism. Thus (Kolomietz *et al*., 1998),

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = -\nu\left[f\left(\vec{p},\vec{r},t\right) - f_{eq}\left(\vec{p},\vec{r},t\right)\right] \qquad (1.5)$$

Where $f_{eq} = \left\{ e^{\left[\left(\frac{E_p-\mu}{T}\right)\right]} + 1 \right\}^{-1}$ is the local equilibrium distribution with local temperature T, chemical potential μ and ν^{-1} is the relaxation-time. The relaxation to the equilibrium state is connected with elastic scattering collision frequency which is density dependent (Emelyanov and Panti, 1995).

This approach uses the utilities of kinetic models to estimate the transition rates and their dependence on energy and momentum. The study is motivated by the fact this approximation contains the transition rate as well as the scattering cross sections which makes it different from the classical and quantum approaches (Green's function model, intra-cascade model, ---). In transport simulations, one deals with the partial lifetime τ_i of the resonances with respect to decay into different channels, including re-scattering processes. If one uses $\frac{1}{\Gamma_{N\Delta}}$ as the lifetime for the Δ with respect to NΔ decay channel, this corresponds to the use of the "standard" cross section $\frac{d\sigma_{\Delta N \to NN}}{d\Omega}$. If the particle lifetime is changed then the cross-section has to be changed accordingly.

The probabilities of the $NN \to N\Delta$ with the Δ resonance in the final state are not modified. Thus we have assumed that the overall lifetime is given by the total width according to $\tilde{\Gamma} = \frac{1}{\tau}$. Generally speaking, $\tilde{\Gamma}$ can be decomposed into modified partial widths as:

$$\Gamma_i = \sum_i \tilde{\Gamma}_i \tag{1.6}$$

Another important aspect that can be introduced here is the well known modified branching ratios $\frac{\tilde{\Gamma}_i}{\Gamma}$ as: $\tilde{\Gamma}_i = \Gamma_i \tilde{\Gamma} / \Gamma = \Gamma_i (\Gamma \tau)^{-1}$.

This type of ratios can be used to ensure that the measurable cross sections for multistep processes are correct and produce the experimental observations. Therefore, the probabilities of processes where the Δ resonance is present in the initial state are all multiplied by the same factor $\tilde{\Gamma}/\Gamma = (\Gamma\tau)^{-1}$ to keep the branching ratios constant (Leupold, 2001).

$$\frac{d\tilde{\sigma}_{\Delta N\to NN}}{d\Omega} = \frac{d\sigma_{\Delta N\to NN}}{d\Omega}\left(\Gamma\tau\right)^{-1} \qquad (1.7)$$

Where $\frac{d\sigma_{\Delta N\to NN}}{d\Omega}$ is the differential cross section obtained by using the following expression (Leupold, 2001):

$$\frac{d\sigma_{\Delta N\to NN}}{d\Omega} = \frac{1}{64\pi^2}\overline{|\mathrm{X}|^2}\,\frac{\mathrm{P}_{NN}}{\mathrm{P}_{N\Delta}s}\times\frac{4}{C_{NN}} \qquad (1.8)$$

where P_{NN} and $\mathrm{P}_{N\Delta}$ are the center-of-mass (c.m) momenta of incoming and outgoing particles respectively, s is the c.m. energy squared, $\overline{|\mathrm{X}|^2}$ is the spin-averaged matrix element squared and C_{NN} =2 (1) if the final particles are identical (different).

Chapter Two

The model and the corresponding kinetic equations

2.1 Introduction

Nuclear reactions reveal various aspects of hadronic many-body problem as a function of the incident energy and the momentum of the target and projectile combination involved. In the excited nuclear systems, it is important to study the equilibration and thermalization in statistical mechanics by using BUU equation. The corner stone in studying thermalization of non-equilibrium is the calculations of the total time rate of change of the distribution function. This requires the evaluation of the collision term. Because of the short range of nuclear forces, we can evaluate the collision term in a local density approximation by using local density approximation to define the UU collision term which includes the conservation of energy and momentum. In this chapter we shall develop adequate theoretical model based on the BUU kinetic equation to calculate the nucleon relaxation-times as well as the delta particle resonance width. Besides, the concept of the delta cross sections will be discussed in a proper way to fit the experimental data.

2.2 The model

In order to investigate the width of delta resonance in HI collisions theoretically, we shall introduce and develop a certain model adequate to interpret the experimental data. In this study, a theoretical model has been introduced on the basis on following assumptions:

1- We shall consider a nuclear system composed of nucleons and delta out of equilibrium and only two-body binary collision processes between particles, such as nucleon-nucleon (NN), nucleon-delta (NΔ), delta-delta (ΔΔ) -----etc, will bring the system into equilibrium. Besides we are concerned mainly with only elastic collision processes between particles.

2- For simplicity, we restrict ourselves to the semi-classical evolution of spin-isospin averaged phase-space densities and consider only elastic binary collisions. The non-equilibrium state of the nuclear system under study is a result of a collision between delta and nucleons.

3- Particles momenta and energies were considered to be conserved quantities during the collision process.

4- The microscopic state of the system can be described in terms of a single-particle distribution function $f(\vec{r}, \vec{p}, t)$. The distribution function of the system in the overlapping region of nucleons and delta will correspond to a state of non-equilibrium. In order to investigate the effect of two-body collisions in HI collisions, each small volume can be studied separately in a local density approximation because of the short range of the nuclear forces.

5- The collision between nucleons and delta will change the distribution function due to the transition of nucleons and delta into and out of states. The collision process between NN and NΔ in a microscopic picture and the distribution function of the system in the overlapping region will correspond to a state of non-equilibrium. The collision process between NN and NΔ with nucleon will destroy the equilibration state of the system at a certain time known as the relaxation-time. In this region the momentum distribution at each point in coordinate space is locally deformed. The distribution function of the system in the overlapping region of nucleons and deltas will correspond to a state of equilibrium.

6- For the calculation of the $\mathrm{NN} \rightarrow \mathrm{N}\Delta$ cross section in nuclear matter we apply the non-relativistic one-boson-exchange model (OPEM) with readjusted parameters (Bertsch *et al.*, 1988). We have chosen the non-relativistic version since it incorporates the contact nuclear interactions in a natural way, which is necessary in the in-medium calculations (Druce and Moszkowski, 1986).

7- The relaxation-times can be calculated by equating the collision term with the relaxation-time collision term (Abu-Samreh and Kohler, 1993).

The system which we shall study consists of nucleons that at $t = 0$ occupy states described by two Fermi spheres and separated by the relative momentum of the two nucleons. When the two-body collisions start, the occupation numbers will change and the Fermi spheres will be distorted and delta particles production may take place. That means the equilibration process will start. In this study, the elastic collisions between NN, NΔ, and ΔN were taken into account and this restricts the possible number of collisions. The most well known interactions between particles are: proton-proton (pp), proton-neutron (pn), neutron-proton (np), delta-proton (Δp), delta-neutron (Δn), and delta-delta ($\Delta\Delta$), which can be chosen at the range of energy (300-1500) MeV.

Thus, the following reactions might be playing an important role in the present study.

$$
\begin{aligned}
&p + p \longrightarrow p + \Delta \\
&p + n \longrightarrow p + \Delta \\
&p + \Delta \longrightarrow p + \Delta \\
&n + n \longrightarrow n + \Delta \\
&n + p \longrightarrow p + \Delta \\
&n + \Delta \longrightarrow n + \Delta \\
&\Delta + \Delta \longrightarrow \Delta + \Delta \\
&\Delta + p \longrightarrow \Delta + p \\
&\Delta + n \longrightarrow \Delta + n
\end{aligned} \tag{2.1}
$$

The Feynman diagrams of the above reactions are displayed in Figures 2.1 to 2.4.

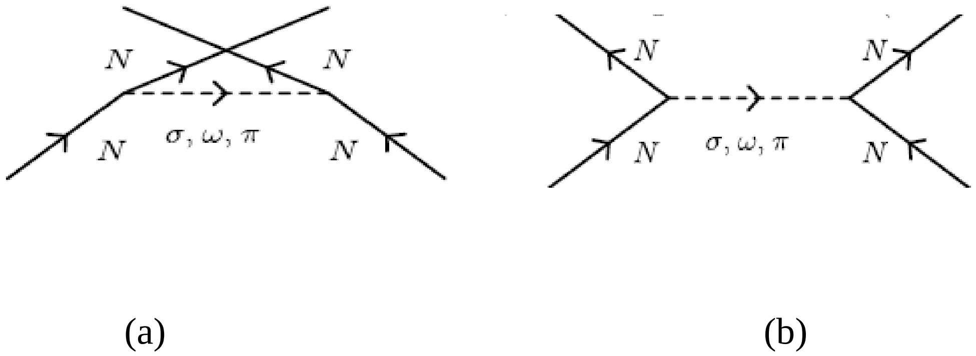


Figure 2.1 Feynman diagrams for the scattering amplitudes contributing to the $NN \to NN$ reaction (Mao, 2002).

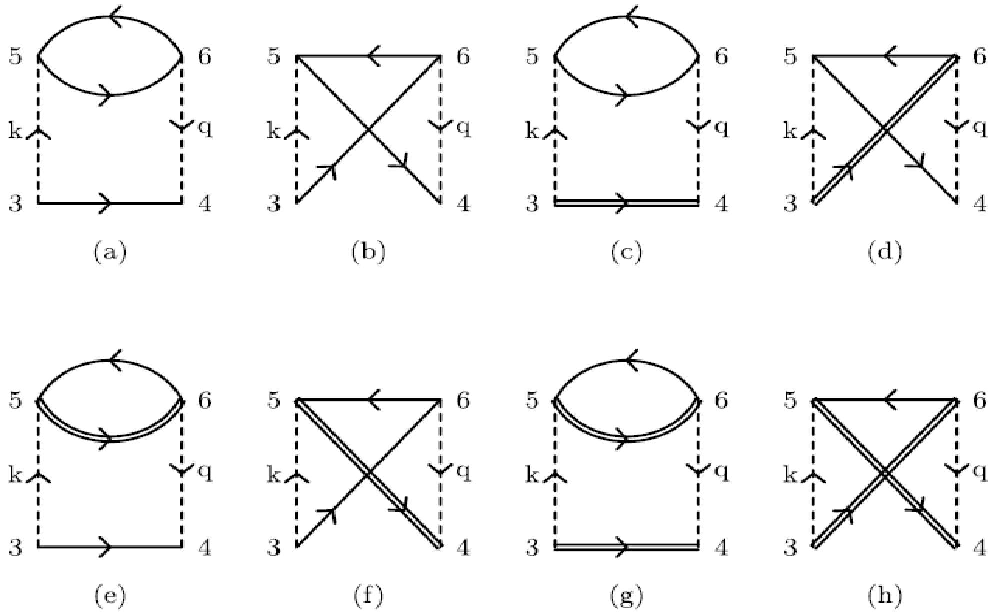


Figure 2.2 Feynman diagrams contribute to the Born term of the nucleon self-energy. The dashed line denotes the exchanged virtual mesons, the solid line and the double line represent the nucleon and delta. The imaginary part of (a) and (b) contributes to the $NN \to NN$ elastic cross section, and (c)-(f) to the $NN \to N\Delta$, (g) ,(h) to the $NN \to \Delta\Delta$ inelastic cross section, respectively (Mao, 2002).

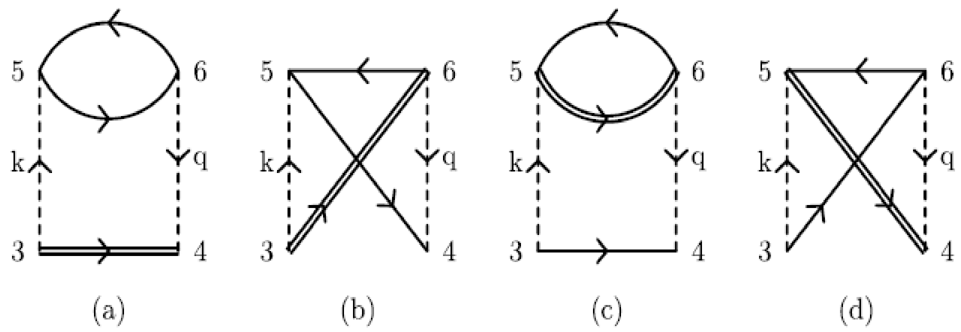


Figure 2.3 Feynman diagrams contribute to the Born term of the nucleon self-energy. The dashed line denotes the exchanged virtual mesons, the solid line and the double line represent the nucleon and delta. The imaginary part of (a)-(d) contributes to the $\Delta N \rightarrow N\Delta$ (Mao, 2002).

Hence, the differential cross sections entering the collision term are chosen to reproduce the main features of the NN, $N\Delta$ and ΔN for reactions { pp, nn, np, pn , $p\Delta$, Δp , $n\Delta$ and Δn } scattering in the energy range 300-1500 MeV per particle.

The collision between $\Delta\Delta$ was neglected, because $\Delta\Delta$ scattering occurs in ultra-relativistic HI collision**.** Accordingly, this type of scattering is not considered in the present study because we shall take the collisions at intermediate energy range whereas the NN collision is occurred.

8- The quantal effects are included through the Pauli blocking factors $(1 - f_i)$ for delta and nucleons (fermions particles) where $i = 1,2,3,4$

and different spin and isospin between nucleons and delta were considered.

9- The dynamical description of hadronic-nucleus or nucleus-nucleus reactions based on the equation of motion for the time evolution of the nucleon one-body phase-space distribution $f(\vec{r},\vec{p};t)$ is given by (Wannier, 1966):

$$\frac{\partial f(\vec{r},\vec{p};t)}{\partial t} = I\left[f(\vec{r},\vec{p};t)\right] \tag{2.2}$$

where I the collision term.

10- The delta density **n** and energy density ε will be decreased due to the expansion of nuclear matter and hence the conservation of the energy- momentum during the expansion requires that:

$$\left.\begin{aligned} n(t) &= \frac{n_0 t_0}{t} \\ \varepsilon(t) &= \varepsilon_0 F(t) \end{aligned}\right\} \tag{2.3}$$

Where F (t) can be obtained from the solution of the following integral equation:

$$\varepsilon(t) = \frac{g}{(2\pi)^3}\int \frac{\sqrt{p^2 + m_\Delta{}^2}\, d^3p}{\exp\left\{\left[\sqrt{p^2 + m_\Delta{}^2} - \mu(t)\right]/T(t)\right\} + 1} \tag{2.4}$$

Where g = 4 for 4–deltas state.

For each time t we have two self-consistent equations, for the local chemical potential $\mu(t)$ and local temperature T(t). Supposing that we have at some initial distribution function f_0 and the initial values n_0, we can find $\mu(t)$ for any $t > t_0$ and define the function $f(\vec{r},\vec{p},t)$.

2.3 The basic kinetic equations

The time evolution of one-body phase-space distribution function, $f(\vec{r}_1,\vec{p}_1;t)$, according to the BUU model can be written as (Uehling and Uhlenbeck, 1933):

$$\frac{\partial f(\vec{r}_1,\vec{p}_1;t)}{\partial t}+\frac{\vec{p}_1}{m}.\vec{\nabla}_{\vec{r}} f(\vec{r}_1,\vec{p}_1;t)-\vec{\nabla}_{\vec{r}_1}U(\vec{r}_1).\vec{\nabla}_{\vec{p}_1} f(\vec{r}_1,\vec{p}_1;t)=I\left[f(\vec{r}_1,\vec{p}_1;t)\right] \quad (2.5)$$

The first term on l.h.s is the scattering term (particles are scattered into and out of the volume element $d\vec{r}d\vec{p}$), the second term represents the streaming term (particles pass in and out of the volume element around $\vec{r}$ because of their motion), the third term represents the change of the change of the distribution function because of the applied external forces (Engel *et al.*, 1994). The term on the r.h.s is known as the collision term. The general form of this term can be rewritten as (Uehling and Uhlenbeck, 1933):

$$\mathrm{I}[f(\vec{r},\vec{p};t)]=\frac{g}{(2\pi)^9}\int d^3\vec{p}_2 d^3\vec{p}_3 d^3\vec{p}_4\,\omega_{12,34}(f_3 f_4 \overline{f_1}\,\overline{f_2}-f_1 f_2 \overline{f_3}\,\overline{f_4}) \quad (2.6)$$

Where f_1, f_2, f_3 *and* f_4 are the Fermi-Dirac distributions of particles in state 1, 2, 3, and 4 and $\bar{f}_1 = 1 - f_1$, $\bar{f}_2 = 1 - f_2$, $\bar{f}_3 = 1 - f_3$ *and* $\bar{f}_4 = 1 - f_4$ are the Pauli blocking for states 1, 2, 3 and 4, respectively. Here, g is spin-isospin degeneracy factor of the nucleon (g = 4 four delta states) and $\omega_{12,34}$ is the transition probability for binary collisions causing the transition $3,4 \leftarrow 1,2$, which is symmetric with respect to the exchange of 1 and 2 and $\omega_{34,12}$ is the transition probability causing the transition $1,2 \rightarrow 3,4$ (Abu-Samreh, 1991).

2.3.1 The transition probability

The transition probability from one microstate to another is needed for calculating the time rate of change of the distribution function. The main idea in this case is to relate the transition probability to the NN and ΔN cross sections in order to ensure that the collision effects are included. One way to determine the unknown cross sections for reactions involving resonance is the well-known detailed balance method by making use of the time-reversed reaction. This concept was widely used for the $N\Delta \rightarrow NN$ channel. The probability dω that two particles with four-momenta p_1, p_2 before the collision can be found after the reaction in the phase-space cell $d^3p_3d^3p_4$ around p_3 and p_4 , is given by (Wolf *et al*., 1993).

$$d\omega = (2\pi)^4 \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)|M_{fi}|^2 \frac{1}{4VE_1E_2} \frac{d^3\vec{p}_3 d^3\vec{p}_4}{(2\pi)^6 4E_3E_4}$$

$$= \frac{1}{16\pi^2} \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \; |^2 \frac{1}{VE_1E_2} \frac{d^3\vec{p}_3}{2E_3} \delta(p_4^2 - m_2^2) d^3\vec{p}_4 \qquad (2.7)$$

For particles of fixed mass, if one uses $p_4^2 = M^2$ and the identity $\frac{dp_4^\circ}{dM^2} = \frac{1}{2E_4}$, equation (2.7) can be rewritten as:

$$d\omega = \frac{1}{16\pi^2} \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)|M_{fi}|^2 \frac{1}{VE_1E_2} \frac{d^3\vec{p}_3}{2E_3} \frac{d^3\vec{p}_4}{2E_4} \delta(M^2 - m_4^2) dM^2 \qquad (2.8)$$

In the center of mass (c.m) frame we need to evaluate only four dimensional integral out of the six fold integral by making use of the delta function properties with

$p_f = p_3 + p_4$, $p_i = p_1 + p_2$, $p_f = |p_4|$, $p_i = |p_2|$, we get

$$\delta^4(p_i - p_f) d^3p_3 d^3p_4 = \delta^4(p_i - p_f) d^3p_f d^3p_4 = \delta(E_i - E_f) d^3p_4 \qquad (2.9)$$

In the c.m we can use $p_3 = -p_4$ *and* $E_j = \sqrt{m_j^2 + p_4^2}$ so

$$d^3p_4 = p_4^2 dp_4 d\Omega = p_f \frac{1}{2} dp_4^2 d\Omega = p_f \frac{E_3E_4}{E_f} dE_f d\Omega \qquad (2.10)$$

by evaluating integral over dE_f we get

$$d\omega = \frac{1}{16\pi^2} |M_{fi}|^2 \frac{1}{VE_1E_2} \frac{p_f}{E_i} \delta(M^2 - m_4^2) dM^2 \qquad (2.11)$$

The cross section can be then obtained by dividing by the incoming current

$$d\sigma = \frac{d\omega}{J} \tag{2.12}$$

where the current in the c.m can be written as

$$J = \frac{p_i}{V} \frac{E_i}{E_1 E_2} \tag{2.13}$$

Substituting equation (2.13) and equation (2.11) into equation (2.12) we get

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left| M_{fi} \right|^2 \frac{p_f}{p_i} \frac{1}{E_i} \delta(M^2 - m_4^2) dM^2 \tag{2.14}$$

If the final state is a resonance of finite lifetime, then instead of the Dirac delta function in mass one has to use a normalized probability distribution function for the mass of particle 4,

$$\delta(M^2 - m_4^2) \rightarrow F(M^2) \tag{2.15}$$

A useful and experimentally established parameterization is given by the Lorentz function (Wolf *et al.*, 1993):

$$F(M^2) = \frac{1}{\pi} \frac{m_4 \Gamma(M)}{\left(M^2 - m_4^2\right)^2 + m_4 \Gamma(M)^2} \tag{2.16}$$

That is all the mass dependence of the cross section is contained in $F(M^2)$. Finally the cross section can be rewritten (Wolf *et al.*, 1993) as:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left| M_{fi} \right|^2 \frac{p_f}{p_i} \frac{1}{E_i} \frac{m_4 \Gamma(M) dM}{4(M^2 - m_4^2)^2 + m_4 \Gamma(M)^2} \tag{2.17}$$

2.3.2 The collision terms

The general form for the two-body collision term is introduced in equation (2.6). This form can be approximated using equation (2.17) as (Abu-Samreh, 1991):

$$I\left[f\left(\vec{r}_1,\vec{p}_1;t\right)\right]=\frac{g}{(2\pi)^6}\int d^3\vec{p}_2 d^3\vec{p}_3\int d\Omega_4 \vec{v}_{12}\frac{d\sigma}{d\Omega}\delta^3\left(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4\right)\times \left(f_3 f_4 \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_3 \bar{f}_4\right) \quad (2.18)$$

Where $U(r)$ is the nucleon mean-field potential, $\frac{d\sigma}{d\Omega}$ is the differential cross section, f_1, f_2, f_3, f_4 is the abbreviations for the occupation probabilities of finding particles 1 , 2 ,3, and 4, respectively. Thus, $\bar{f}_i = 1 - f\left(\vec{r}_i,\vec{p}_i;t\right)$ are the Pauli-blocking factors and v_{12} is the relative velocity between 1 and 2 (Abu-Samreh, 1991).

The UU collision term includes the conservation laws of momenta and energy and takes full account of Pauli exclusion principle. The bracket contains the usual occupancy for the process $(\vec{p}_1,\vec{p}_2)\rightarrow(\vec{p}_3,\vec{p}_4)$ and the reverse process $(\vec{p}_3,\vec{p}_4)\rightarrow(\vec{p}_1,\vec{p}_2)$ (Engel *et al.*, 1994). Therefore the collision term can be written as:

$$I = I_{gain} - I_{loss} \quad (2.19)$$

The "gain" term describes events in which particles each with momentum $\vec{p}_3$ and $\vec{p}_4$ are scattered into empty states with momentum $\vec{p}_1$ and $\vec{p}_2$, while the "loss" term describes the scattering of particles with momentum $\vec{p}_1$ and $\vec{p}_2$ into states with momentum $\vec{p}_3$ and $\vec{p}_4$. Thus the collision term may have positive or negative values depending on values of the gain and loss terms. Hence, if the collision term is negative within a particular momentum state, nucleons will scatter out of that state. The collision term is zero (I = 0) when the system attains a thermalized state because the scattering of particles out of the state is exactly balanced by the scattering of other particles into that state. If this condition is met, the thermalized state of the system is described by a Fermi-Dirac distribution of nucleon at specific temperature (Abu-Samreh, 1991).

In order to develop expressions to be used calculating the transition rates of delta resonances, we have first to introduce the typical reactions in which delta may be produced. We are interested in elastic channels, among which.

$$\left.\begin{array}{l} N + N \to N + N \\ N + \Delta \to N + \Delta \\ \Delta + \Delta \to \Delta + \Delta \end{array}\right\} \tag{2.20}$$

The basic elastic scattering processes can be summarized as:

$$\left\{\begin{array}{l} \Delta + n \to p + n \\ \Delta + p \to p + p \\ n + \Delta \to n + n \\ p + \Delta \to p + p \\ n + p \leftrightarrow p + \Delta \\ p + p \to n + \Delta \\ n + n \to n + \Delta \end{array}\right\} \tag{2.21}$$

In this work, we shall start our discussion by developing the collision term for reactions in equation (2.20). In a similar way, the collision term can be reproduced in the same manner as in equation (2.7). For example, the collision term for the second part of equation (2.20) can be written as:

$$I\left[f\left(\vec{r},\vec{p}_N;t\right)\right]=\frac{4}{(2\pi)^6}\int d^3\vec{p}_\Delta d^3\vec{p}_N \int d\Omega_\Delta \vec{v}_{N\Delta}\frac{d\sigma}{d\Omega}\delta^3\left(\vec{p}_N+\vec{p}_\Delta-\vec{p}_N-\vec{p}_\Delta\right)\times \left(f_N f_\Delta \bar{f}_N \bar{f}_\Delta - f_N f_\Delta \bar{f}_N \bar{f}_\Delta\right) \tag{2.22}$$

Similar results can be written for the rest of channels.

2.3.3 The cross sections of free deltas

In this part of the study, we shall discuss the relevant cross sections needed for investigating the scattering of various channels in equation (2.14). It is worth mentioning here that while working in deriving the $N\Delta$ cross sections, a similar work has been submitted to Phys. Rev. C by Larionov and Mosel, 2004. We shall follow the same procedures developed previously (Larionov and Mosel, 2004). Thus, most of required equations were similar to those in Larionov and Mosel work.

The differential cross section in vacuum for $N_1N_2 \rightarrow N_3\Delta_4$ can be written as (Jänicke *et al*., 1992*)*:

$$d\sigma = (2\pi)^4 \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\overline{|M|}^2 \frac{(2m_N)^3 2M_\Delta}{4I} \times \frac{d^3\vec{p}_3}{(2\pi)^3 2\varepsilon_4} \frac{d^3\vec{p}_4}{(2\pi)^3 2\varepsilon_4} A_\Delta(M_\Delta^2) dM_\Delta^2 \tag{2.23}$$

where $\overline{|M|}^2$ is the square of the absolute value of the reaction amplitude, T the normalization of (Bjorken and Drell, 1965) the averaged over spins of incoming particles 1, 2 and summed over spins of outgoing particles 3,4. The flux factor $I = \sqrt{(P_1P_2)^2 - m_N^2}$ is the spectral function of the Δ resonance. The mass of the delta resonance, which is according to the probability function (Larionov *et al*., 2001; Ehehalt *et al*., 1993):

$$A_\Delta(M_\Delta^2) = \frac{1}{\pi}\frac{M_\Delta \Gamma_\Delta(\mathrm{M}_\Delta)}{(M_\Delta^2 - m_\Delta^2)^2 + M_\Delta^2 \Gamma_\Delta^2(\mathrm{M}_\Delta)} \qquad (2.24)$$

where Γ=110 MeV and $m_\Delta = 1.232\ GeV$ is the Δ pole mass, which satisfies the normalization condition (Larionov and Mosel, 2002):

$$\int_{(m_N+m_\pi)^2}^{\infty} dM_\Delta^2 A_\Delta(\mathrm{M}_\Delta^2) = 1 \qquad (2.25)$$

The mass dependent $\Gamma_\Delta(M_\Delta)$ of total Δ is parameterized according (Effenberger *et al.*, 1999):

$$\Gamma^*{}_\Delta(M_\Delta) = \Gamma_{sp}\frac{\rho}{\rho_0}\Gamma_\Delta^0\left(\frac{q(M^*{}_\Delta, m^*{}_N, m_\pi)}{q(m_\Delta, m_N, m_\pi)}\right)^3 \frac{m^*{}_\Delta}{M^*{}_\Delta}\frac{\beta_0^2 + q^2(m^*{}_\Delta, m^*{}_N, m_\pi)}{\beta_0^2 + q^2(M^*{}_\Delta, m^*{}_N, m_\pi)} \qquad (2.26)$$

Where $\rho_0 = 0.16\ fm^{-3}$ is the nuclear saturation density, Γ_{sp} is a constant (~ 80 MeV), $\Gamma_\Delta^0 = 0.118\,\mathrm{GeV}$ is the width at the pole mass, and $\beta_0 = 0.2\,GeV/c$ is the cut-off parameter. The first term in equation (2.26) represents the Δ spreading width, which describes the in-medium broadening of the Δ resonance due to scattering processes $\Delta\mathrm{N} \to \mathrm{NN}$, and $\Delta\mathrm{N} \to \Delta\mathrm{N}$. The second term in equation (2.26) is the $\Delta \to N\pi$ width taking into account the effective masses of the nucleon and Δ. In this term we have omitted the Pauli blocking of the decay nucleon. This is justified because in the real HI collision at the beam energy of 1 A GeV the nuclear matter at finite temperature of about 70 MeV is created, which reduces the Pauli blocking effect strongly with respect to the zero temperature nuclear matter. In equation (2.26), the q parameter can be written as:

$$q(m, m_1, m_2) = \sqrt{\frac{\left(m^2 + m_1^2 - m_2^2\right)}{4m^2} - m_1^2} \tag{2.27}$$

In the c.m frame of colliding particles with masses m_1 and m_2 (i.e. in the frame where $\mathrm{P}_1^* + \mathrm{P}_2^* = 0$) the differential cross section in nuclear medium after standard transformations reads as (Bertsch *et al*., 1988):

$$\frac{d\sigma}{d\Omega} = \frac{(2m_\mathrm{N})^3 2\mathrm{M}_\Delta}{64\pi^2 s} \overline{|M|^2} \frac{q(\sqrt{s}, \mathrm{M}_\Delta, m_\mathrm{N})}{q(\sqrt{s}, m_\mathrm{N}, m_\mathrm{N})} A_\Delta(\mathrm{M}_\Delta^2) \tag{2.28}$$

where $s = (p_1 + p_2)^2$. Comparing the cross sections for the reaction $N\Delta \to NN$ with the inverse channel $NN \to N\Delta$ and using $|M_{NN\to N\Delta}| = |M_{N\Delta\to NN}|$, we obtain

$$\frac{d\sigma^{N\Delta\to NN}}{d\Omega} = \frac{p_f^2}{N_f p_i^2} \frac{1}{F(M^2)} \frac{d\sigma^{NN\to N\Delta}}{d\Omega dM^2} \tag{2.29}$$

where p_f *and* p_i denote the c.m. three-momentum of the final and initial state of the reaction $N\Delta \to NN$ respectively. The quantity N_f is 2 as a consequence of spin averaging for particles of different isospin and N_f = 4 in case of identical particles in the exit channel. In such cases the following assumption were introduced:

$$\frac{d\sigma^{NN\to N\Delta}}{d\Omega dM^2} = \frac{d\sigma^{NN\to N\Delta}}{d\Omega} \frac{F(M^2)}{\int_{(m_n+m_\pi)^2}^{(\sqrt{s}-m_N)^2} F(M^2)dM^2} \tag{2.30}$$

and

$$\frac{d\sigma^{N\Delta\to NN}}{d\Omega} = \frac{p_f^2}{N_f p_i^2} \frac{d\sigma^{NN\to N\Delta}}{d\Omega} \frac{1}{\int_{(m_N+m_\pi)^2}^{(\sqrt{s}-m_N)^2} F(M^2)dM^2} \tag{2.31}$$

where

$$\frac{1}{\int_{(m_N+m_\pi)^2}^{(\sqrt{s}-m_N)^2} F(M^2)dM^2}$$

is always larger than 1 and especially large value close to threshold. $F(M^2)$ is given by equation (2.16) this leads to the physical effect, in which the $\Delta's$ are more strongly absorbed than in the conventional detailed-balance description.

For the reaction $\Delta^{++}n \to \mathrm{PP}$ the cross section can be calculated by the application of detailed balance to a hypothetical delta having fixed a mass. This leads to:

$$\sigma_{\Delta^{++}n\to \mathrm{PP}} = \frac{1}{4}\frac{\mathrm{P}_f^2}{\mathrm{P}_i^2}\sigma_{\mathrm{PP}\to n\Delta} \tag{2.32}$$

Here P_f , P_i are the final and initial momenta of the particles in the center of mass frame and the factor $1/4$ is used to represent the spin averaging and a symmetry factor for identical particles in the final state. Taking into account, however, that the delta has no fixed mass, the following expression can be written for $NN \to N\Delta$ cross section (Mosel, 1991; Wolf *et al*., 1993):

$$\sigma_{n\Delta \to PP} = \frac{4\pi}{4} \frac{P_N^2}{P_\Delta^2} \sigma_{PP \to n\Delta} \frac{1}{\int_{(m_N + m_\pi)^2}^{(\sqrt{s} - m_N)^2} f(M^2)\, dM^2} \tag{2.33}$$

Finally, by taking the isospin factors into account and taking symmetry factors for identical particles in the final state, we obtain the total cross sections of the other isospin channels as (Larionov, 2004):

$$\sigma^{tot}_{n\Delta^+ \to Pn} = \sigma^{tot}_{P\Delta^0 \to Pn} = \frac{2}{3} \sigma^{tot}_{n\Delta^{++} \to PP} \tag{2.34}$$

where

$$\sigma^{tot}_{P\Delta^+ \to PP} = \sigma^{tot}_{n\Delta^0 \to nn} = \frac{1}{3} \sigma^{tot}_{n\Delta^{++} \to PP} \tag{2.35}$$

and

$$\sigma^{tot}_{P\Delta^- \to nn} = \sigma^{tot}_{n\Delta^{++} \to PP} \tag{2.36}$$

2.4 The relaxation-time approximation model to the collision term.

The relaxation-time method has been widely used in HI collision because it provides a simpler context than UU collision term and gives many practical results with less work. By equating the relaxation-time collision term to UU collision term, the relaxation-times or the relaxation-rates can be calculated (Abu-Samreh, 1991). According to this approximation, the collision term approximation is as follows:

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = \text{Collision term} = -\nu\left[f(\vec{p},\vec{z},t) - f_{eq}(\vec{p},\vec{z},t)\right] \qquad (2.37)$$

where $f_{eq} = \left\{ e^{\left[\left(\frac{E_p-\mu}{T}\right)\right]} + 1 \right\}^{-1}$ is the local equilibrium distribution with local temperature T=T (t), chemical potential $\mu = \mu(t)$ and ν^{-1} is the relaxation-time. The relaxation to the equilibrium state is connected with elastic scattering collision frequency is density dependent (Emelyanov and Pantis, 1995). The relaxation-time can be calculated by equating the UU-collision term with the relaxation-time (UURX), and it may be calculated by using the following equation:

$$\frac{f - f_0}{\tau} = I_{UU}(f) \qquad (2.38)$$

where τ is the relaxation-time and f_0 is the thermalized distribution function (Abu-Samreh, 1991) .

Generally speaking, more than one scattering mechanism may operate the same system at the same time find the collision term $\left(\frac{\partial f}{\partial t}\right)_{coll}$, we sum similar terms produced by different mechanism and each one is calculated in the absence of the other types of scattering mechanisms. In the RTA, each collision term is inversely proportional to the relaxation-time which a certain characterizes a certain scattering mechanism, thus, each mechanism contributes to the collision term by a sub-collision term. Accordingly, the total collision term can be written as (Al-Darabea, 2001):

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = -\frac{f - f_0}{\tau_2} - \frac{f - f_0}{\tau_3} - \tag{2.39}$$

The total relaxation-time for all channels contributing in nuclear processes can be written as:

$$\frac{f(\vec{r},\vec{p};t) - f_0}{\tau} = I_{tot}[f(\vec{r},\vec{p};t)] = I_{NN} + I_{N\Delta} + I_{\Delta\Delta}$$

$$= \frac{f(\vec{r}_N,\vec{p}_N;t) - f_{0N}}{\tau_{NN}} + \frac{f(\vec{r}_\Delta,\vec{p}_\Delta;t) - f_{0\Delta}}{\tau_{N\Delta}} + \frac{f(\vec{r}_\Delta,\vec{p}_\Delta;t) - f_{0\Delta}}{\tau_{\Delta\Delta}} \tag{2.40}$$

In case of considering a nucleon as incident particle and approximating, $f_{0N} \approx f_{0\Delta}$, the total relaxation-time can be written as:

$$\frac{1}{\tau} = \frac{1}{\tau_{NN}} + \frac{1}{\tau_{N\Delta}} + \frac{1}{\tau_{\Delta\Delta}} + --- \quad = \sum_{i=1}^{n} \frac{1}{\tau_i} \tag{2.41}$$

where i may be nucleon or delta and τ is the relaxation-time. By defining a combined relaxation time, equation (2.41) can be rewritten in the following form:

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} = \Gamma \tag{2.42}$$

where Γ is the resonance width at which pions are produced. The relaxation time τ_i governs the approach to equilibrium of a non-equilibrium distribution function describing the ith collision mechanism and its value depends on the specific collision mechanism.

If the relaxation-time for one mechanism is much shorter than all others, scattering takes place predominantly via other mechanisms (Al-Darabea, 2001).

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \frac{f(\vec{r},\vec{p};t)-f_0}{\tau}$$

$$= I[f(\vec{r},\vec{p};t)] = I^{p}_{pp} + I^{p}_{pn} + I^{p}_{p\Delta} + I^{n}_{nn} + I^{n}_{np} + I^{n}_{n\Delta} + I^{\Delta}_{\Delta p} + I^{\Delta}_{\Delta n} + I^{\Delta}_{\Delta\Delta}$$

$$= \frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{pp}} + \frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{pn}} + \frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{p\Delta}} +$$

$$\frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{nn}} + \frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{np}} + \frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{n\Delta}} +$$

$$\frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{\Delta p}} + \frac{f(\vec{r},\vec{p};t)-f_0}{\tau_{\Delta n}} + \frac{f(\vec{r},\vec{p};t)-f_\circ}{\tau_{\Delta\Delta}} \tag{2.43}$$

The resonance of Γ at which the deltas that are produced can be calculated according:

$$\Gamma = \frac{1}{\tau} \tag{2.44}$$

The calculation of Γ is addressed in this study. However, the calculated values of the transition rate will be compared to its calculated values by means of quantum and classical as well as the experimental values.

It is usually assumed that the lifetime of a resonance is given by its inverse total width $1/\Gamma$. This assumption is widely used in the transport simulations of nuclear collisions in order to describe the decays of various baryonic and mesonic resonances. For $\Delta E \ll \Gamma$ (broad resonance), where ΔE is the energy spread of the incoming particles and then the lifetime is given by the derivative of the phase shift with respect to the c.m energy (Leupold, 2001):

$$\tau = \frac{d\delta(E_{c.m})}{dE_{c.m}} \qquad (2.45)$$

This time represents the time delay in the transmission of the scattered wave in the case of scattering with only one partial wave.

Chapter Three

Results and discussion

3.1 Introduction

The purpose of the present work is to calculate the free $NN \leftrightarrow N\Delta$ cross sections on the basis of the Dirac-Brüeckner model starting from the bare NN interaction. The calculated cross sections are then used in the BUU model to study the delta production in HI collisions. It has been shown within the relativistic Dirac-Brüeckner approach that the $NN \rightarrow N\Delta$cross section is reduced at high densities (ter Harr and Malfliet, 1987).

The solution of the transport equation following the general procedure of numerical analysis that allows us to determine directly the temporal evolution of the particle distribution function, energy densities, temperature, and also the rate at which the field energy flows into the particle sector. All the calculations are performed in the center of mass of the colliding particles.

In this study, the Δ isobars are treated in essentially the same way as nucleons (Muo *et al.*, 1996). The dependence of beam energy on temperature is exhibited in Figure 3.1 as found by Muo and coworkers (1996).

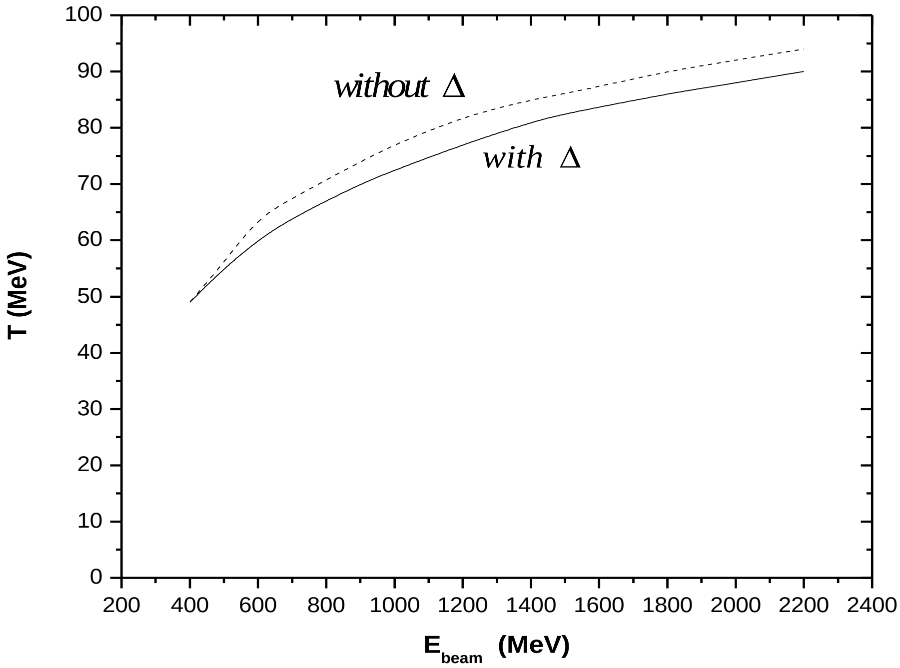


Figure 3.1 Influence of the Δ (1232) on the temperature of the fireball as a function of beam energy for the collision of equal mass nuclei as (Muo *et al*., 1996).

3.2 The numerical procedures

In this study, the delta-nucleons reactions in a microscopic transport model of the BUU type which propagates nucleons and delta resonances explicitly in space and time has been analyzed. We shall consider only the processes that lead to thermal equilibrium for deltas in hot hadronic matter.

Recently numerical solutions of the BUU equation were advanced and an agreement between the experimental data and the numerical results of BUU model has been achieved. The relaxation-times for the equilibration as a function of density and final temperature of the equilibrated system have been also investigated.

In this study the collision processes of various channels were followed for an initial system composed of two Fermi-spheres. The spheres radii are chosen in such a way to meet the intermediate energy regime. Typical values of Fermi momentum that vary from 1.6-5.0 fm^{-1} can be used.

The collision integral in equation (2.6) depends crucially on the phase-space distribution function $f(\vec{r},\vec{p})$. Traditionally, the Wigner phase-space distribution is applied in evaluating the collision integral. In order to simplify the problem, the dependence of the distribution function on space can be neglected without loosing any physics. This is because the Wigner distribution is equivalent to the Fourier transform of the one body density matrix over the relative coordinates. Besides, the energy depends mainly on Fermi momentum which represents the major part of the problem. Accordingly, one can take only the momentum dependence. Therefore, the initial distribution function can be chosen as:

$$f_1(\vec{p}) = \theta(\mu - \varepsilon(p)) \qquad (3.1)$$

where μ is the chemical potential and θ is the step function.

This choice is reasonable for Fermi system. Because of cylindrical symmetry of the two Fermi spheres, one can make use of cylindrical coordinated and choose a certain cylindrical box to put the spheres in. The time evolution of the distribution function is described by the collisional part of the BUU equation that can be solved numerically using a fortran computer program. The integrals at the right-hand side of equation (2.6) are integrated numerically in momentum-space and the new distribution function is evaluated on the grids. The angular integrations are calculated by using Gaussian numerical method (8 points Gaussian quadrature), while the integration over momenta are done using Simpson's rule. The ranges of momenta are as follows:

-5 fm^{-1} $\leq$ p $\leq$ 5 fm^{-1}. The distribution function is assumed to be zero outside the region bounded by the two Fermi-spheres.

The collision between nucleons will start when the two spheres collide. The pions collision is assumed at certain lab energies (E > 300 MeV/particle). The corner stone in these calculations is the particles cross section calculations. We shall start by testing the differential cross sections need for the calculations in order to evaluate the relevant collision integrals. The hadronic cross sections have been calculated from effective differential cross sections.

3.3 Results of the cross sections

We calculate the nucleon–delta (NΔ) elastic scattering cross sections up to four times that of nuclear matter density. The experimental determination of NΔ and the delta-delta (ΔΔ) scattering cross section is inaccessible even in free space. In almost, all transport models they are assumed to be equal to free NN scattering cross section (Muo *et al*., 1996) which was found to be density dependent. The BUU self-consistent equation can be used for the delta distribution function in the similar manner and depending the delta cross section using similar procedures that have been used for density nucleons cross section (Li and Gross, 1999).

The elastic cross sections for two particles channels provided a collision occurs, the two particles can scatter elastically or in-elastically if the beam energies 150 MeV or less. The inelastic channels can be suppressed and non–relativistic kinematics may be used with considerable simplification. Here we deal only with elastic channels that are commonly introduced as the delta state of the nucleon. Accordingly, the following channels are considered:

$$\left.\begin{array}{l} N+N \rightarrow N+N \\ N+N \rightarrow N+\Delta \\ N+\Delta \rightarrow N+N \\ N+\Delta \rightarrow N+\Delta \\ \Delta+\Delta \rightarrow \Delta+\Delta \end{array}\right\} \tag{3.1}$$

Since there is no delta beam or delta, target available the cross section for the $\Delta N \rightarrow NN$ process has to be derived by theory. The simple argument to understand this point is that in the NN collisions in which the deltas are formed, and the cross section has to be folded with the mass distribution to get the mass of the delta in the simulation. In the $N\Delta \rightarrow NN$ reaction the delta in the simulation has a definite mass and therefore the mass distribution is not present in the calculation of the $N\Delta \rightarrow NN$ cross section.

In this study, we present calculations of the cross sections of NN scattering as well as nucleon deltas scattering in nuclear matter within the BUU model taking into account Pauli principle. Starting from the bare NN interaction, in-medium NN cross sections have been calculated using relativistic (Larionov *et al.*, 2001) as well as non-relativistic (ter Haar and Malfliet, 1987) Brüeckner theory. The in-medium neutron-proton (np) cross sections were derived microscopically by ter Har and Malfliet (ter Haar and Malfliet, 1987). The derivation was based on the Boson-meson-exchange model for the NN interaction (Helgessaon and

Randrup, 1997; Ericson and Weise, 1988) and the Dirac-Brüeckner approach (Wu and Ko, 1989) for nuclear matter. We found that our microscopic results for in-medium (np) cross sections are in agreement of parameterization developed by Cugnon *et al.*, 1982, which is often used in transport model calculations. We know that the Cugnon parameterization of NN cross sections is, in fact, a fit of the free-space pp data; i.e., no difference is made between (np) and (pp) scattering. The elastic NN scattering angular distribution is taken to be (Cugnon *et al.*, 1981; Bass *et al.*, 1995; Ko and Li, 1996)

$$\frac{d\sigma_{elastic}}{d\Omega} \sim \exp(A(s)t) \tag{3.2}$$

where t is $-q^2$, the squared momentum transfer and

$$A_r(s) = 6\frac{\left(3.65\left(\sqrt{s}-1.8766\right)\right)^6}{1+\left(3.65\left(\sqrt{s}-1.8766\right)\right)^6} \tag{3.3}$$

where $\sqrt{s}$ is the c.m energy in GeV and A is given $(GeV/C)^{-2}$. The elastic cross sections of channels are taken to be in mb and momentum in GeV/c and $\hbar = c = 1$. Typical NN cross sections are displayed in Figure 3.2 and Figure 3.3.

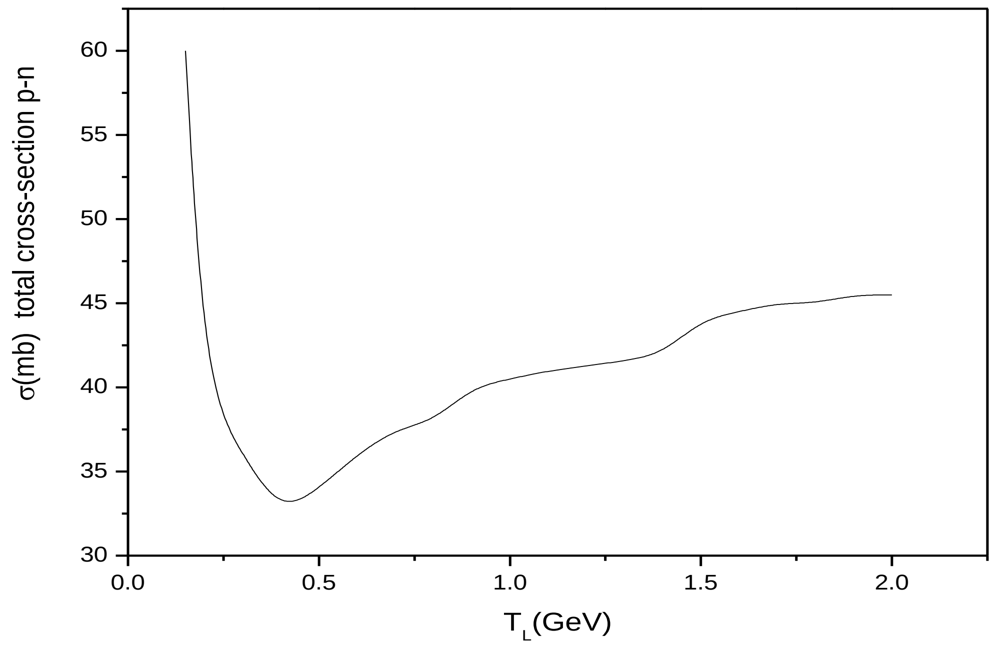


Figure 3.2 Typical *pn* total cross section dependence on the nucleon lab kinetic energy.

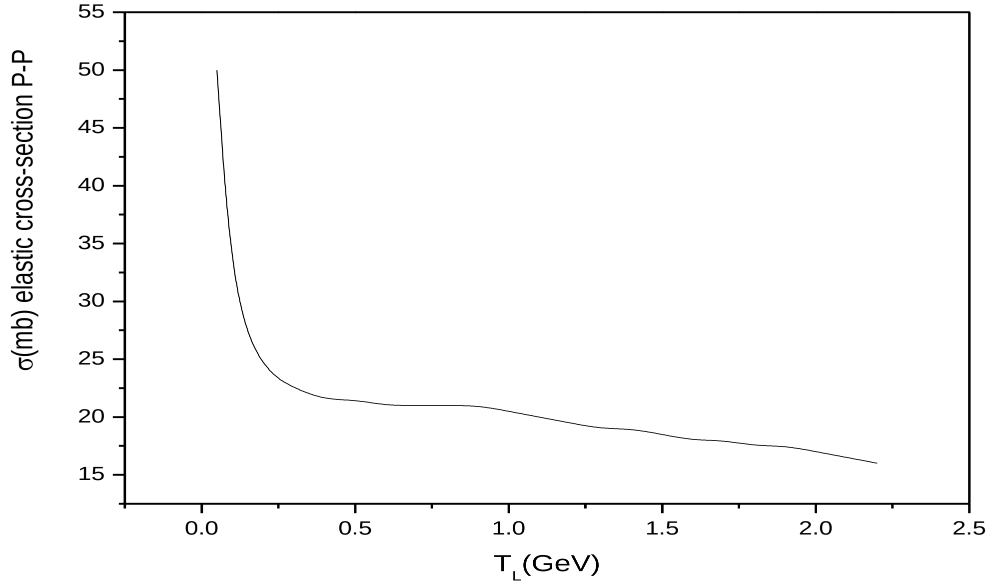


Figure 3.3 Typical *pp* elastic cross section dependence on the nucleon lab kinetic energy.

Relatively speaking, the reactions which are dominant in this energy domain $(\sim 1 GeV/u)$ are:

(a) $NN \rightarrow \Delta N$ (Hard-delta-production).

(b) $\Delta N \rightarrow NN$ (Delta-absorption).

For elastic scattering, the experimental cross sections delta are used for process (a) as well as for the elastic NN collisions. If the principle of the detailed balance is applied to the delta resonance, then its finite width has to be taken into account. On one hand, for the $NN \rightarrow N\Delta$ reaction cross section, in our simulations we used the approximations of $V_{\Delta} \approx -30\, MeV$ at $\rho_0 = 0.16\, fm^{-3}$ for the delta potential (Ehehalt *et al.*, 1993). The differential cross section for this channel is calculated according to equation (2.29) with the mass-dependence Δ-decay width has been taken from equation (2.23) (Teis *et al.*, 1997). The particle momenta were: 227, 238, 318, and 338 MeV/c, for the nucleon 1, nucleon 2, nucleon 3 and the delta particle, respectively. On the other hand, for the $N\Delta \leftrightarrow NN$ processes we use the VerWest–Arndt parameterization of the cross section (VerWest and Arndt, 1982). The mass of the delta resonance which is populated in our simulation in a NN collision is chosen according to equation (2.24) with $\Gamma(\mathrm{M})$ for deltas given by (Leupold, 2001):

$$\Gamma(\mathrm{M}) = \left(\frac{q}{q_r}\right)^3 \frac{\mathrm{M}_{\Delta}}{\mathrm{M}} \left(\frac{\upsilon(q)}{\upsilon(q_r)}\right)^2 \Gamma_r \qquad (3.4)$$

Where M is the actual delta mass, M_{Δ} is the peak delta rest mass (1232 MeV), q is the delta momentum in the rest frame of the delta, q_r the delta momentum and Γ_r = 110 MeV. The function

$$\upsilon(q) = \frac{\beta^2}{\beta^2 + q^2} \tag{3.5}$$

with β = 300 MeV cuts the width at high momenta.

The formula for free scattering $\Delta n \rightarrow \mathrm{PP}$ cross section as an example on process (a) is expressed in the following form (Cassing *et al.*, 1990):

$$\frac{d\sigma}{d\Omega} = \frac{1}{|v_1 - v_2|}\frac{m^2}{E_{i1}E_{i2}}\frac{1}{(2\pi)^3}\int \frac{dm^{*2}\frac{1}{2}\Gamma}{(m^* - m_{\Delta})^2 + \frac{1}{4}\Gamma^2}\int d\Omega_3 \frac{mp_3}{\sqrt{S}}\left|\overline{M}\right|^2_{\Delta n \rightarrow pp} \tag{3.6}$$

where m^* denotes the mass of the pion and nucleon forming a delta and p_3 is the momentum of the other nucleon, E_1 *and* E_2 are the energies of the incoming nucleons. The spin averaged matrix elements calculated as the particle states normalized according to (Effenberge *et al.*, 1999):

$$\left|\overline{M}\right|^2 \cong \frac{4}{3}[\frac{2}{3}q^4[\frac{f^{\pi}_{NN} f^{\pi}_{N\Delta\pi}}{m^2_{\pi}}\frac{1}{m^2_{\pi} - t} - \frac{f^{\rho}_{NN} f^{\rho}_{N\Delta}}{m^2_{\rho}}\frac{1}{m^2_{p} - t}]^2 + \frac{1}{3}[(1-3g^{/})^2[\frac{f^{\pi}_{NN} f^{\pi}_{N\Delta}}{m^2_{\pi}}]^2 +$$

$$\frac{1}{3}[\frac{f^{\pi}_{NN} f^{\pi}_{N\Delta}}{m^2_{\pi}}\frac{m^2_{\pi} - \omega^2}{m^2_{\pi} - t}]^2 - \frac{2}{3}(1-3g^{/})\left[\frac{f^{\pi}_{NN} f^{\pi}_{N\Delta}}{m_{\pi}\pi^2}\frac{m^2_{\pi} - \omega^2}{m^2_{\pi} - t}\right] + t \leftrightarrow \varepsilon \tag{3.7}$$

Where ω^2 denote the invariant mass squared in the direct channel and f^{π}_{NN} are the form factors having the following forms

$$\left.\begin{aligned}
& f^{\pi}_{\mathrm{N\,N\Delta}}(t) = f^{\pi}_{\mathrm{N\,N\Delta}} \frac{\Lambda_{\pi}^2 - m_{\pi}^2}{\Lambda_{\pi}^2 - t} \\
& f^{\pi}_{\mathrm{N\,N}} \cong 1.0 \\
& f^{\pi}_{\mathrm{N\,\Delta}} \cong 2.0 \\
& \\
& \mathrm{M}_{\Delta} \cong 1220 MeV \;, \Lambda_{\pi} \cong 780\ MeV \;, g^{/}_{\mathrm{N\,\Delta}} \cong \frac{1}{3} \;\;, m_{\rho} \cong 770\ MeV
\end{aligned}\right\} \qquad (3.8)$$

The in-medium modified $pp \rightarrow n\Delta^{++}$ cross section has been implemented in the BUU model somewhere else (Effenberger *et al.*, 1999). More details can be found in Effenberger, PhD thesis. The cross sections for another isospin channels are related to the $pp \rightarrow n\Delta^{++}$ cross section by the Clebsch-Gordan coefficients given as:

$$\sigma^{med}_{pp\rightarrow p\Delta+} = \sigma^{med}_{pn\rightarrow p\Delta^0} = \sigma^{med}_{pn\rightarrow n\Delta^+} = \sigma^{med}_{nn\rightarrow n\Delta^0} = \frac{1}{3}\sigma^{med}_{pp\rightarrow n\Delta^{++}} \qquad (3.9)$$

and

$$\sigma^{med}_{nn\rightarrow p\Delta^-} = \frac{1}{3}\sigma^{med}_{pp\rightarrow n\Delta^{++}} \qquad (3.10)$$

In this study we are mainly concerned in free particle cross sections.

The systems Ca+Ca, Ru+Ru and Au+Au were studied at the beam energies of 400, 1000 and 1500 MeV. The parameterization for the $NN \rightarrow N\Delta$ cross section to be used in the BUU was found to fit well the formula (Efenberger, 1999):

$$\sigma_{NN\to N\Delta} \cong \sigma^{free}{}_{NN\to N\Delta}[1+0.24(\frac{\rho}{\rho_\circ})^2 - 0.07(\frac{\rho}{\rho_\circ})] \quad (3.11)$$

Typical results of the differential cross sections for pp scattering were exhibited in Figure 3.3. Similar results were reported by Larionov and Mosel (Larionov and Mosel, 2004).

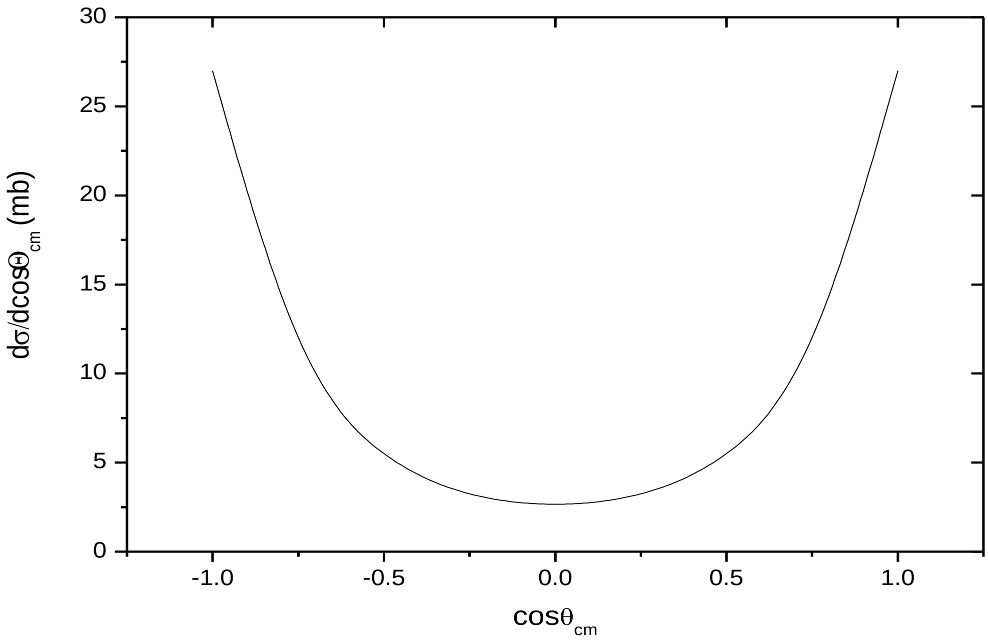


Figure 3.4 Out going neutron c.m polar angle distribution in the reaction $pp \to n\Delta^{++}$ at $p_{lab} = 1.66 GeV/c$.

The next step is to use the tested relations for differential cross section discussed above in the BUU equation and solve such equation for single particle distribution function. A typical distribution function for the delta particle in the NΔ→NN reaction using equation (3.2) is shown in Figure 3.5.

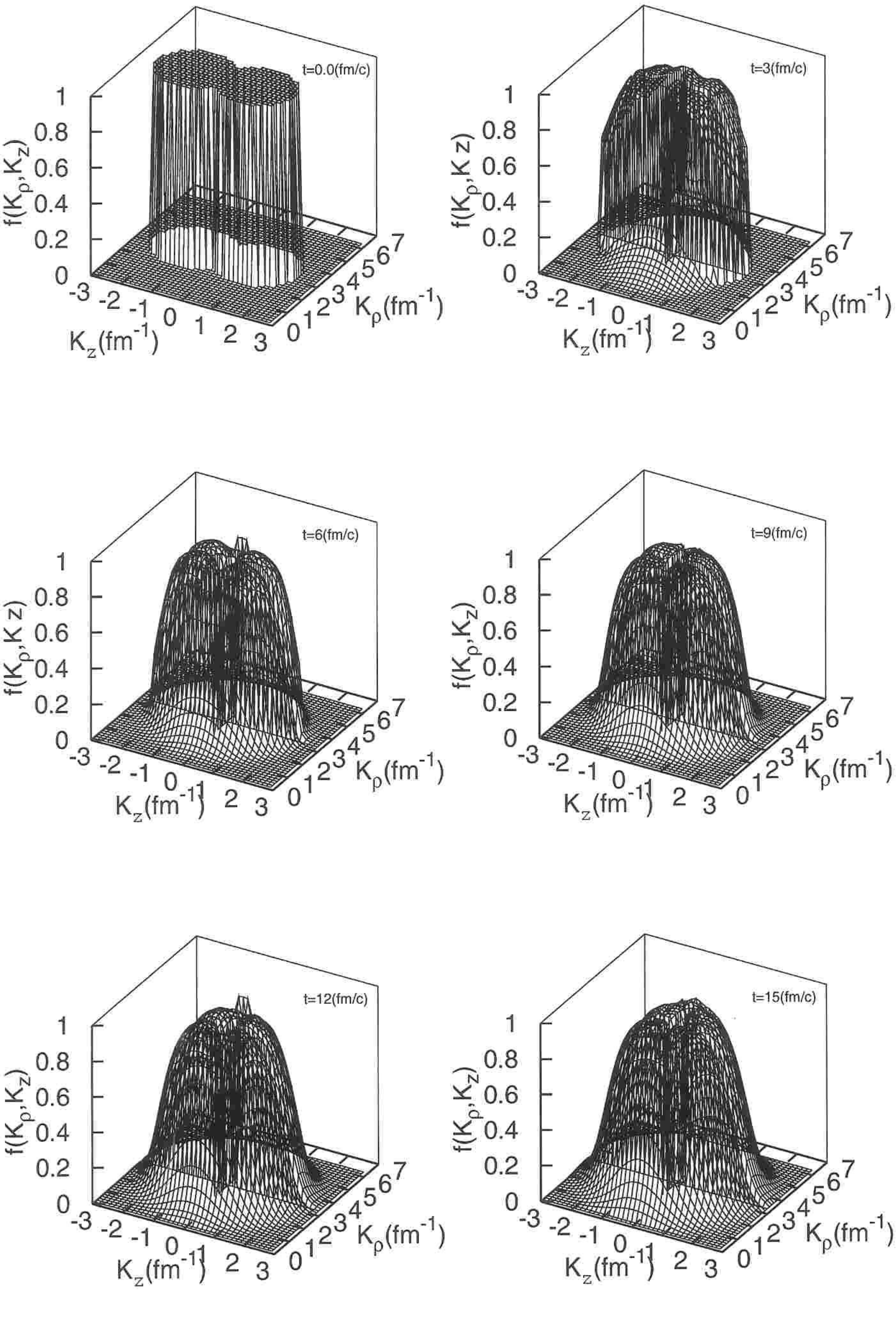


Figure 3.5 Typical distribution functions for the delta particle in the reaction $N\Delta \rightarrow NN$ at lab energy 338 MeV per nucleon.

3.4 The relaxation-time results

Using the BUU transport model, the relaxation-times in the HI collisions at intermediate energies have been calculated. It is found that only at incident energies below the Fermi energy, thermal equilibrium can be reached before dynamical instability is developed in the heavy residues. Also, the relaxation-time is shorter (longer) than that for momentum at beam energies lower (higher) than the Fermi energy.

The single particle relaxation-time is defined as (Boym and Pejhick, 1991):

$$\frac{1}{\tau} \cong \frac{g}{(2\pi)^3}\int d^3\vec{p}_2 d^3\vec{p}_3 d^3\vec{p}_4 \;\frac{g}{2m^2}\;\frac{d\sigma}{d\Omega_m}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\times [f_1 f_2 \overline{f}_3 \overline{f}_4 + \overline{f}_1 \bar{f}_2 f_3 f_4] \qquad (3.12)$$

The symbol g represents the degeneracy of the single –particle state with a given value of linear momentum $\vec{p}$. Considering the protons and neutrons identical particles occupying the same orbital, with $g = 4$. The estimated NN relaxation-time as a function of temperature is shown in Figure 3.6. The relaxation-time decreases rapidly with energy in the low regime while it increases in the intermediate one. The maximum points appeared in Figure 3.6 is an indication of creation or production of certain particles. In this study, the delta particle is expected to be mostly favored.

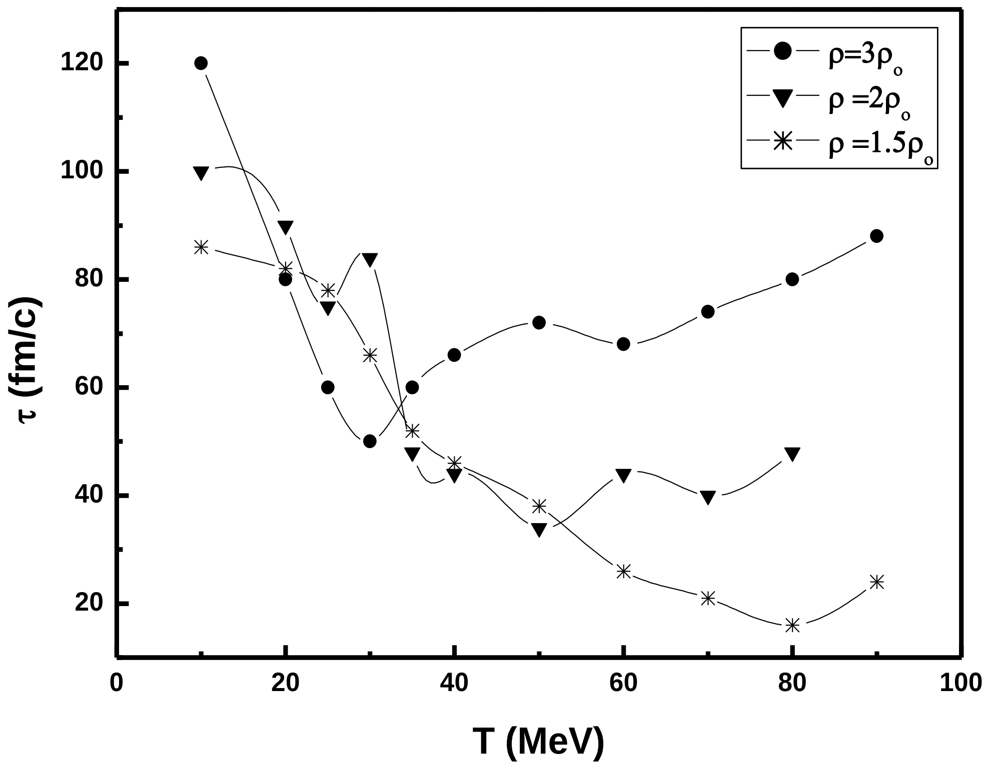


Figure 3.6 The NN relaxation-time dependence on temperature.

The three dimensional surface plots for the $\Delta N \rightarrow NN$ channel is plotted in Figure 3.7. The relaxation-time of 100 fm/c or more was considered to be infinite. Thus, most states were found to have high relaxation-times especially at earlier times. Therefore, we have chosen some selected plots to show various relaxation-times and resonance widths. For instance, three dimensional surface plots for the relaxation-times of the reaction $\Delta N \rightarrow NN$ were displayed in Figures 3.8.

A contour plots for the resonance widths of the particle are plotted in Figure 3.9.

Although thermal equilibrium is not completely established at beam energies higher than Fermi energy, it is still interesting to compare their relaxation-times. The comparison is made for $Ca^{40} + Sn^{124}$ collisions at beam energies from 500 to 1000 MeV/nucleon and at an impact parameter of 1 fm. The momentum relaxation-time is found to decrease with increasing beam energy. This is in qualitative agreement with that found by Florewski and Abu-Samreh (1996). On the other hand, the relaxation-time decreases slowly with the beam energy. The shorter relaxation-time at incident energies below about 500 MeV/nucleon is in agreement with what was found in deep inelastic heavy ion collisions (Bertsch *et al.*, 1988).

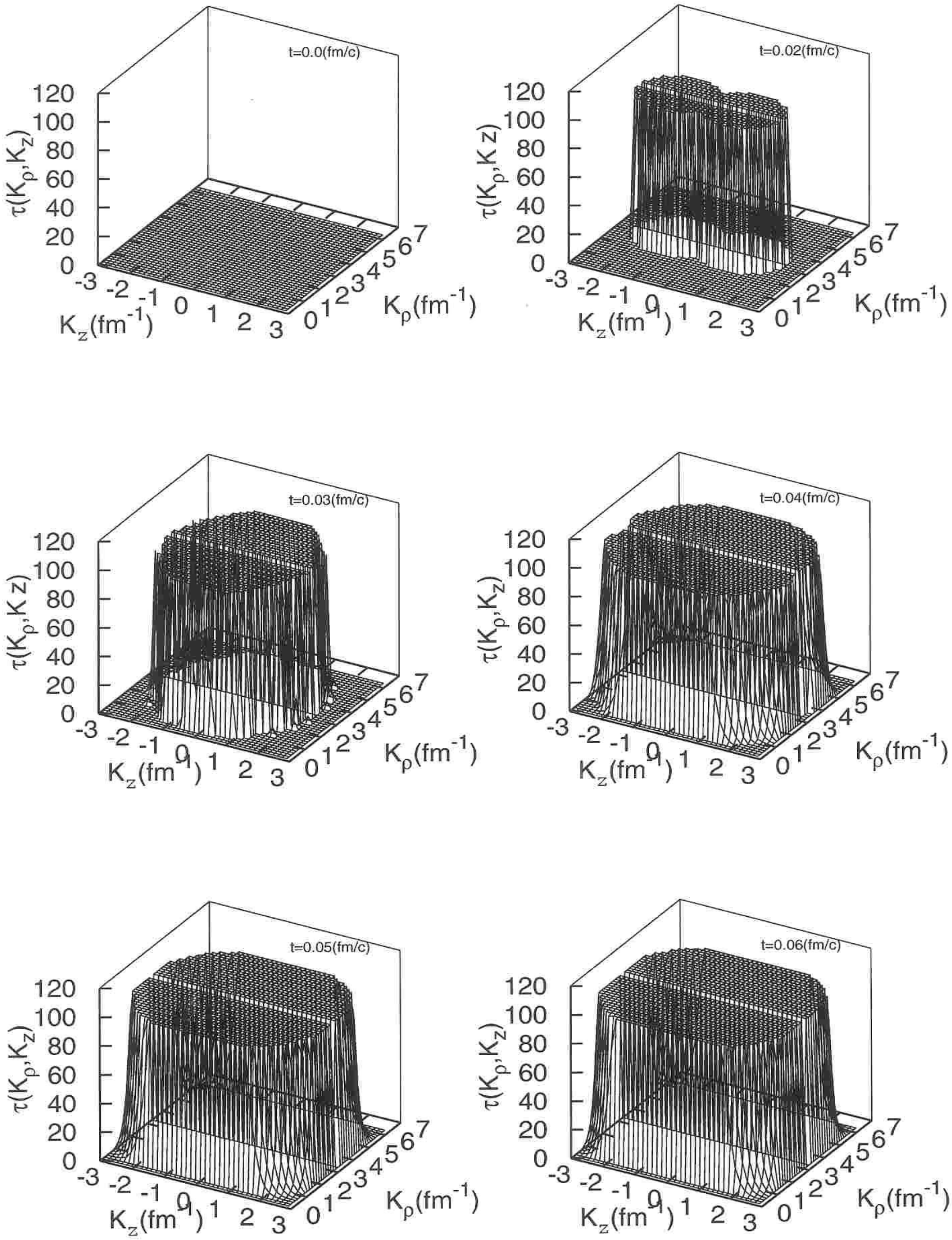
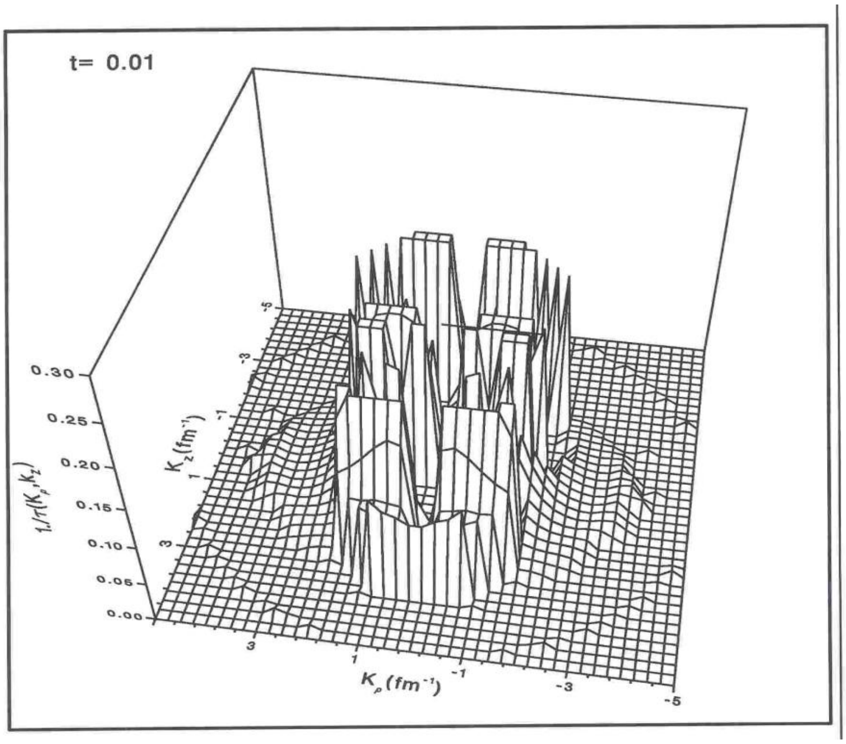


Figure 3.7 Three dimensional surface plots of the relaxation-time for $\Delta N \rightarrow NN$



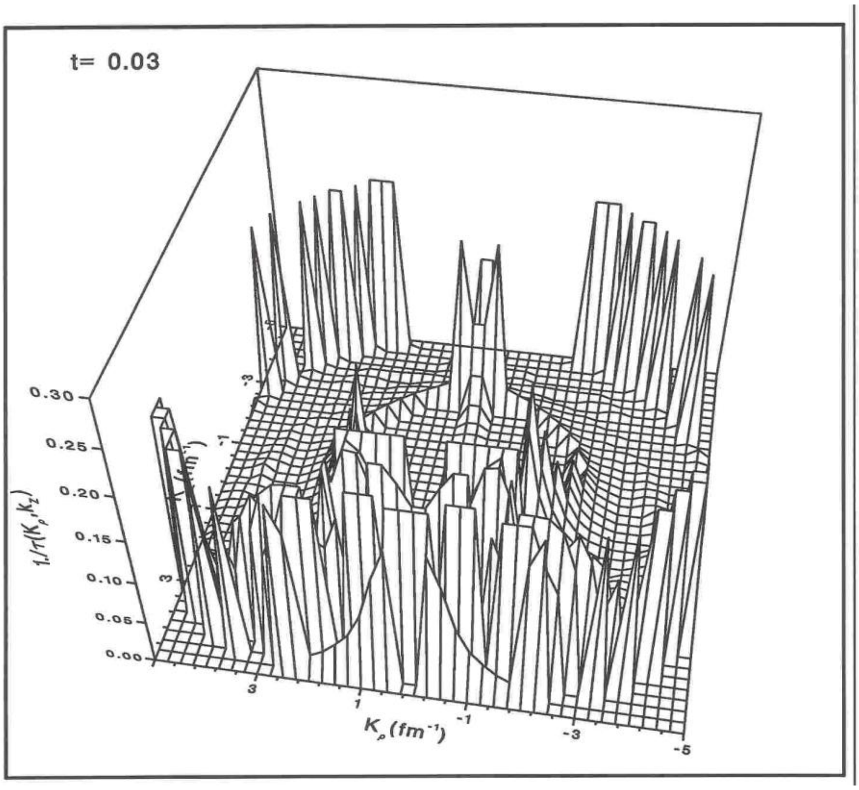


Figure 3.8 The resonance width of the delta particle in the reaction $\Delta N \rightarrow NN$.

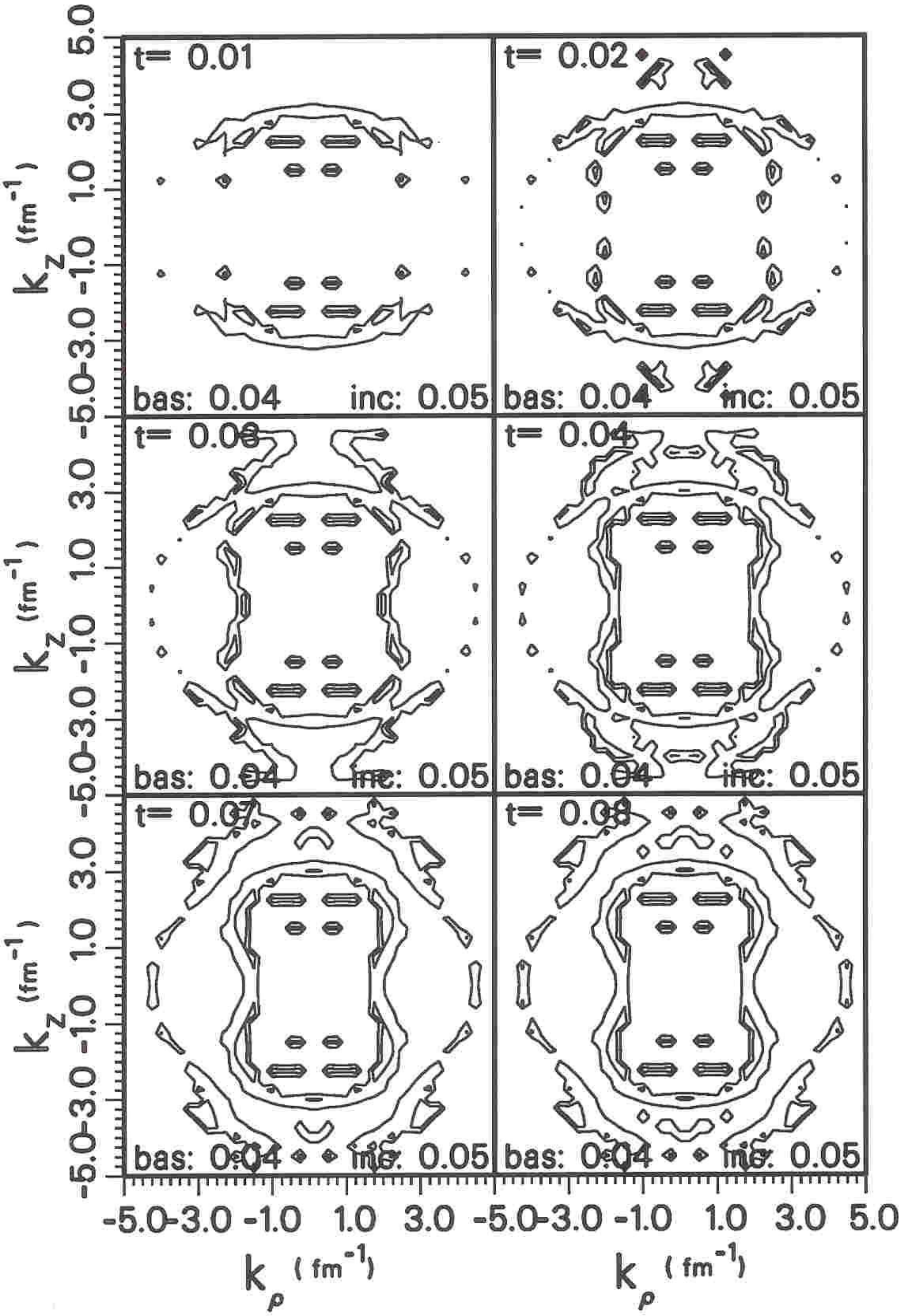


Figure 3.9 Contour plots of the resonance width (relaxation rates) of the delta particle in the reaction $\Delta N \rightarrow NN$.

For the $\Delta N \rightarrow NN$ channel occurring at normal nuclear density ($\rho_\circ = 0.17\, fm^{-3}$) and low temperature, the relaxation-time is found to fit the following equation:

$$\tau_{\Delta N} = \alpha \frac{1-\exp[-(\varepsilon-\mu)/T]}{(\pi T)^2 + (\varepsilon-\mu)^2} \tag{3.13}$$

Where μ is the Fermi energy of incident nucleon (= 38 MeV) and $\alpha \approx 1150 \quad fm\, Mev^2$. Figure 3.10 displays the relaxation-time dependence on temperature for various channels.

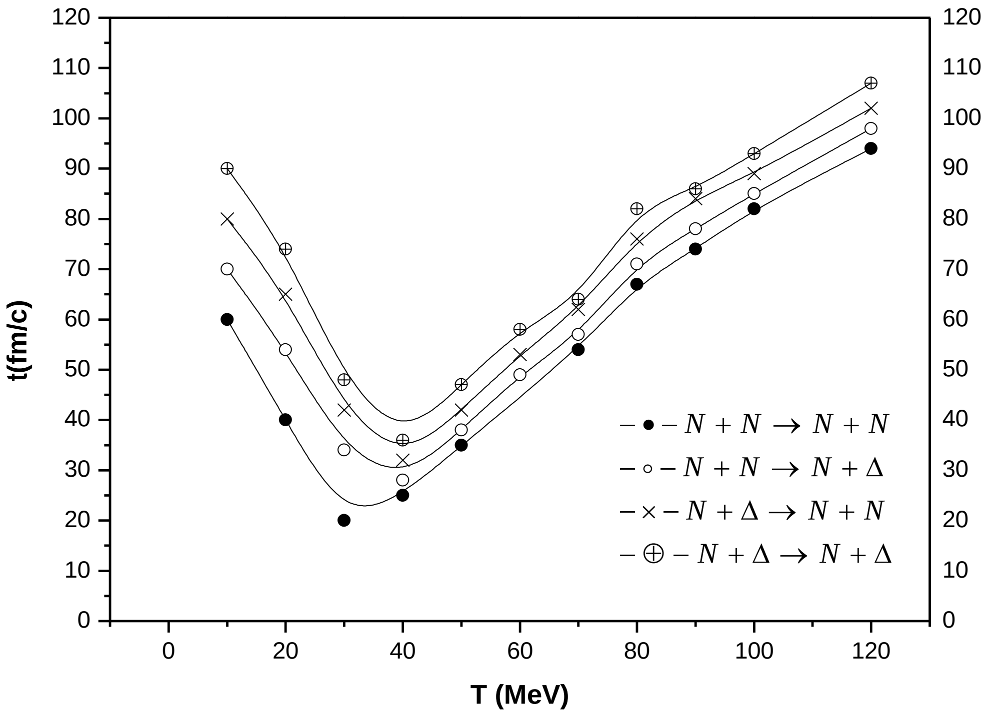


Figure 3.10 Typical relaxation-time dependence on temperature lab 338 MeV/nucleon for delta channels as displayed in equation (3.1).

The lifetime of the delta resonance especially the one treated as the intermediate state of the two-body (e.g. NΔ) scattering depends crucially on the relation between its width Γ and the energy spread of the incoming particles ΔE. In the BUU transport model, the collision particles (by definition) have fixed energies and momentum. Within the transport BUU model we study the influence of the choice for the Δ resonance lifetime on HI collision at 1GeV/nucleon. The width of the Δ production is calculated theoretically using Brüeckner model according to (Zhang and Wilets, 1992; Oset and Salced, 1987):

$$\Gamma_{\Delta}(k,\omega) = 2\pi \int < \Delta | H_{\pi} | N >^{2} \frac{1}{\pi} \operatorname{Im}\left(\frac{1}{\omega_{\Delta}^{2} - k^{2} - m^{2} - \prod(k,\omega)} \right) \frac{d^{3}k}{(2\pi)^{2}} \qquad (3.14)$$

When $w_{\Delta} \rightarrow M_{\Delta} - M_{N} + \frac{k^{2}}{2M_{\Delta}}$.

By integrating equation (3.14) over m^{*}, the inelastic collision effects will be eliminated. Thus we may assume that the width Γ is small compared with m_{Δ} and thus concentrate on the production of Δ at resonance. Comparing the results obtained in this study with those obtained when equation (3.14) is used, an agreement between the two results was obtained within 85%.

In transport simulations one deals with the partial lifetimes τ_{i} of resonances with respect to decay into different channels, including absorption and rescattering processes. If one uses $1/\Gamma_{N\Delta\to NN}$ as the lifetime for the Δ with respect to the Δ N decay channel, this corresponds to the use of the "standard" cross section $d\sigma_{N\Delta\to NN}/d\Omega$ for such channel. Conversely, if the partial lifetime is changed, then the cross section has to be changed accordingly. Notice that the probabilities of the processes with the Δ resonance in the final state, like e.g. NN → NΔ, are not modified. In order to do that, we have assumed that the overall lifetime is given by equation (2.43) and define a modified total width $\tilde{\Gamma} \equiv \tau^{-1}$, $\tilde{\Gamma}$ can be decomposed into modified partial widths $\tilde{\Gamma}_i : \tilde{\Gamma} = \sum_i \Gamma_i$. There is one important aspect: the modified branching ratios, $\tilde{\Gamma}_i/\Gamma$, have to be the same as the original ones, Γ_i/Γ, i.e. $\tilde{\Gamma}_i = \Gamma_i\tilde{\Gamma}/\Gamma = \Gamma_i(\Gamma\tau)^{-1}$ This ensures that the measurable cross sections for multistep processes are correct.

The standard BUU calculation employing the Δ lifetime $1/\Gamma$ predicts the data at the intermediate energies. However, the peak position is slightly changed by the Fermi motion. The physical reason for increasing absorption with the lifetime according to equation (3.12) is due to the long living Δ resonances (see Figure 3.11). This increases the probability that a Δ will be absorbed in the collision with a nucleon.

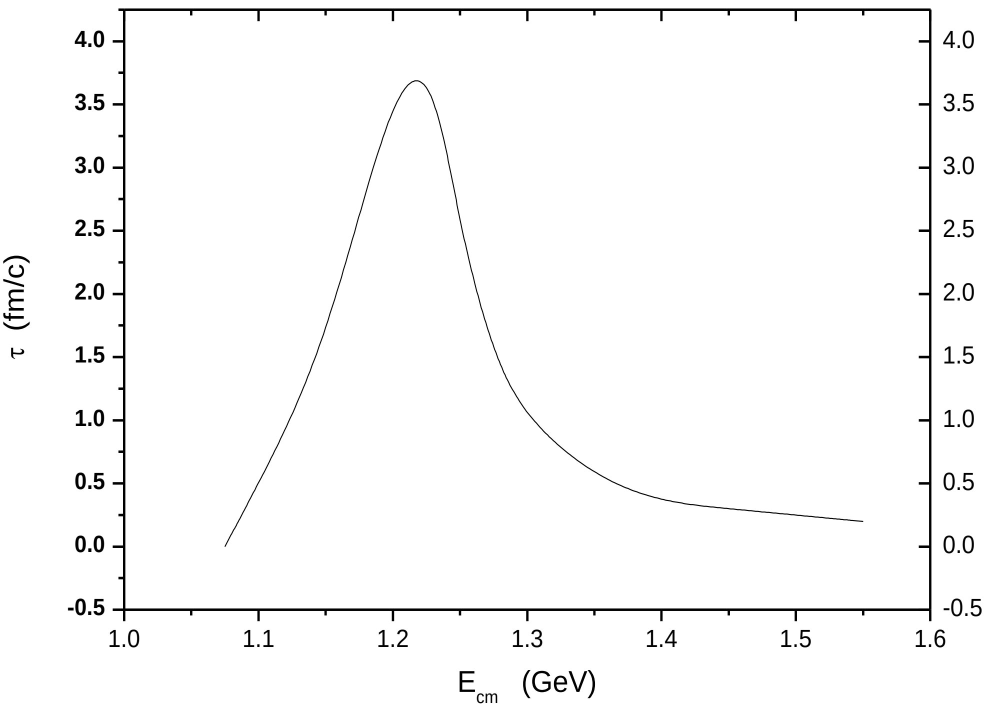


Figure 3.11 The lifetime of the Δ resonance as functions of the total c.m energy of the pion and nucleon as found by Larionov (2002).

Chapter Four

Conclusions and future work

In this work we have calculated the free $NN \rightarrow N\Delta(1232)$ cross section within the frame of Brüeckner model taking into account the nuclear interactions and the effective masses of the nucleon and Δ resonance. It has been observed that the cross section decreases with the nuclear matter density at high densities. The reason could be due to strong nonequilibrium momentum distribution at the beginning of collision. Indeed, the nonequilibruim momentum distribution reduces the scalar density, and therefore, increases the effective mass. This effect is expected to be stronger for lighter systems, since smaller baryon density is reached in this case. Momentum conservation is presented as a delta-function in the BUU collision term.

We have investigated the effect of the presence of deltas in the nuclear matter system. The temperature dependence of relaxation-times for the ΔN systems were found to differ from NN ones in the irregularity of their behavior, they have inflection points and some evident of maxima and minima. Such a temperature dependence of the transport relaxation–time is associated with nucleonic resonances in the ΔN elastic interactions. It has been found that the delta entrance channels are especially sensitive to the pion and delta dynamics and a good agreement for differential inelastic scattering and the exclusive (Δ, NP)

reactions in the delta resonance region have been obtained (Engle *et al*., 1994).

We have studied the thermal and chemical relaxation-time scales of deltas in hot nuclear matter using the non-equilibrium statistical mechanics. From the explicit calculation, it has been found that deltas in hot nuclear matter are in a phase where elastic collisions rates are very fast compared to typical expansion rates of the system. For a temperature of $T \approx 180$ MeV, a relaxation-time of $\tau \approx 5$ fm/c was obtained.

Future work will concentrate on a transport theoretical calculation of the thermal equilibration. We also plan to extend the present study to include the reactions involving strange particles. In addition, several effects have been neglected such as the coulomb effects, the higher order collision effects, such as the triple collision effects. Therefore, in order to improve this model, we suggest the following:

1- Investigations should be extended to include triple and higher order collisions.

2- Investigations that include the viscosity dependent in order to exhibit their effect on the relaxation-times.

3- This model can also be modified to include the electric and magnetic field effects on thermalization and relaxation-times.

4- For complete study, the inelastic collision process between particles should be study. A consistent transport theory must include also the corresponding modifications of the Δ resonance absorption and rescattering probabilities.

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