

Deanship of Graduate Studies



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Numerical Techniques for Systems

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Introduction

Numerical analysis is concerned with the development and investigation of constructive methods for the numerical solution of mathematical problems. The main topic of numerical analysis is to provide efficient numerical methods for the actual computation of the solution. We mean by a constructive method, a procedure that for any prescribed accuracy determines an approximate solution by a finite number of computational steps. In general, this number will depend on the required accuracy.

In the first chapter, we consider some preliminaries that will be needed later in our work.

In the second chapter direct techniques and matrix factorization will be considered, also in this chapter we consider some notations and remarks about the norms for their importance in measuring the speed of convergence of the numerical methods. The best known and most widely used direct method, is Gaussian elimination method which is attributed to Gauss, since it based on considerations published by Gauss in 1801 in his *Disquisitiones Arithmeticae*.

In principle, we have to distinguish between two groups of methods for the solution of linear systems:

- a. Direct techniques, or elimination methods, in which the exact solution, in principle, is determined through a finite number of arithmetic operations.
- b. Iterative schemes, that we considered in the third chapter which generate a sequence of approximations to the solution by repeating the application of the same computational procedure at each step of the iteration. Usually, they are applied for large systems with special structures that ensure convergence of the successive approximations. A key consideration for the selection of a solution method for a

linear system is its structure. In some problems, the coefficient matrix of the linear system may be a full matrix, i.e., it has few zero entries. And in other problems, the matrix may be very large and sparse, i.e., only a small fraction of the entries are different from zero.

Roughly speaking, direct methods are best for full matrices, whereas iterative methods often have decisive advantages over direct methods in terms of speed and demands on computer memory for very large and sparse matrices. In sparse problems, the nonzero elements of A are some times stored in a spares- storage format, while in other cases it is not necessary to store A at all!

In chapter four the solution of systems of nonlinear equations will be considered . As opposed to linear equations, no explicit solution techniques are, in general, available for nonlinear equations, and hence their solution completely relies on iterative methods.

Finally we introduced appendix containing software for the methods which we considered in chapter two, three, and four.