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The Calculation of Pion Width Resonances in the Intermediate Energy Regimes of Heavy-Ion Collisions using Non-Equilibrium Statistical Mechanics

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2005

DECLARATION

I certify that this thesis, which is submitted for the degree of master science in physics is the result of my own research, except where otherwise acknowledged, and that this thesis (or any part of the same) has not been submitted for a higher degree to any university or institution.

Signed:……………….

(Husain R. Samamreh)

Date:--------------------

DEDICATION

To the memory of my father, Rashad

To my wonderful wife, Samar

To my beloved family

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I am very pleased to express my gratitude and thanks to my supervisor Dr. Mohammad Abu-Samreh for his supervision, guidance and unlimited support through my long study journey even in an inconvenient times. I would like to thank my friends for the unlimited assistance. I would like to extend my thanks to my father, mother, brothers and sisters for their unlimited support. Special thanks will go to the center for theoretical and applied physics (CTAPS) in Yarmouk University, Irbid Jordan for allowing us to use their facilities to run the computer programs.

Abstract

The work presented in this thesis represents mainly the relaxation-time calculations of non-equilibrium nuclear systems during the collisions of two heavy-ions in the intermediate energy regimes (300-1000 MeV/nuclen). The relaxation-times (or the inverse of the resonance widths) in hot hadronic matter are calculated using a non-equilibrium microscopic statistical model that incorporates the Boltzmann-Uehling-Uhlenbeck collision term with the relaxation-time approximation. The relaxation-times were found to depend on both temperature and energy in the temperature range 20-180 MeV.

The temperature dependence of relaxation-time is found to decrease at low temperature, and to increase at high temperature before the quark matter is reached. For elastic processes at temperature about $T \sim 150$ MeV, a relaxation-time of 10 fm/c is obtained. This is comparable with typical time scales for a hadronic system generated in low energy heavy-ion collisions. The inelastic effects of baryons collisions on relaxation-times were also estimated and found to be negligible compared to elastic effects. The inelastic relaxation-times were found to be smaller than that of elastic relaxation-times by a factor of 10.

The massive pion resonances become important at temperatures about 100 MeV. As the temperature rises with the bombarding energy and reaches about T=160 MeV deconfinment will take place. This will continue up to $T \approx 200$ MeV.

The chemical relaxation-time estimated to be about 26 fm/c at $T \approx 150$ MeV. This is very large compared to the size of hot matter produced in nucleus-nucleus collisions. This implies that the chemical equilibrium would not be reached in an expanding hot hadronic matter consisting of pions only.

الملخص

إن العمل المقدم في هذه الرسالة يمثل معالجة إحصائية على المستوى الميكروسكوبي لحساب سعة الرنين لجسيمات البيونات والنيوكليونات مبنى على معادلة بولتزمان يولنج–يولنبك. وقد تم دراسة العديد من التفاعلات وفقا لهذه النظرية خلال عمليات الاتزان وتراخي الأنظمة النوويةغير المتزنة والتحول إلى أنظمة متزنة أثناء عمليات تصادمات الانوية الثقيلة عند مناطق الطاقة المتوسطة والتي تتراوح ما بين (300–1000) مليون الكترون فولت لكل نيوكليون.

لقد تم دراسة أزمنة التراخي من خلال نموذج ميكروسكوبي يدمج ما بين حد التصادم في معادلة بولتزمان – يولنج – يولنبك ونظرية تقريب أزمنة التراخي بدلالة درجة الحرارة.

وقد تم حساب أزمنة التراخي في المادة الهادرونية لدرجات حرارة تتراوح ما بين (20–180 مليون الكترون فولت). ووجد أن اعتماد أزمنة التراخي على درجة الحرارة كبير عند درجات الحرارة المنخفضة، بينما عند درجة الحرارة العالية يبدأ بالنقصان حيث يبدأ ظهور الجسيمات الأولية في حالة التصادمات المرنة، وعند درجة حرارة تقريبا 160 مليون الكترون فولت، وجد أن زمن التراخي 10 c/fm. وتم تقدير اثر التصادمات غير المرنة للباريونات ووجد أنه مهملا مقارنة مع تأثير التصادم المرن. كذلك وجد أن أزمنة التراخي للتصادمات غير المرنة

أقل بعشرة أمثال من التصادمات المرنة. ويبدأ تأثير ترددات البيونات الثقيلة يأخذ بعدا أكثر أهمية من بعد درجة الحرارة 120 مليون الكترون فولت، ويزداد مع ارتفاع درجة الحرارة. وعندما تصل درجة الحرارة 160 مليون الكترون فولت يبدأ تحويل الجسيمات إلى كواركات والعكس.

وقد تم تقدير أزمنة التراخي الكيميائية ووجد أنها تقريباً تساوي 26 c/fm عند درجة حرارة حوالي 180 مليون الكترون فولت. وهذا أكثر بكثير من حجم المادة النووية الساخنة الناتجة من خلال تصادم نواة مع أخرى. وهذا يدل على أن الاتزان الكيميائي لا يمكن الوصول إليه في حالة تمدد المادة الهادرونية الساخنة المكونة من البيونات فقط.

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CHAPTER ONE

Introduction and Motivations

1.1 Introduction

The meson's family consists of several particles, the lightest of them being the pion. The pion or the pi-meson (π–meson) was proposed as a hypothetical particle to interpret and understand the general behavior of nuclear potential in 1935 (Yukawa, 1935) and it was confirmed experimentally in 1947. This particle has a mass of approximately 270 times the mass of the electron ($m_\pi \sim 270\, m_e \sim 140$ MeV) (Perkins, 1987). The pi-meson family consists of a positively charged pion (π^+), a negatively charged pion (π^-) and a lighter neutral pion (π^0). The most general properties of pions are listed in Table 1.1 (Longo, 1973)

Table 1.1. The basic properties of pi-meson particles

Particle	Charge	Spin	Isospin T	Parity	Mass (MeV)	T_3	Mean Life(s)	Quarks
π^+	+e*	**0**	+1	Even	139.567	+1	2.603×10^{-8}	$u\bar{d}$
π^-	**-e***	0	+1	Even	139.567	-1	2.603×10^{-8}	$\bar{u}d$
π°	0	0	+1	Even	134.963	0	8.3×10^{-17}	$u\bar{u}$ or $d\bar{d}$

*$e=1.6\times 10^{-19}$ C.

Relatively speaking, mesons as well as nucleons inside nuclides interact among themselves by means of strong interaction. The strong interactions refer to processes involving baryons such as nucleons and pions inside the nucleus. These types of interactions give rise to nuclear forces between the nucleons inside nuclides and to the processes of formation and decay of mesons and baryons in nuclear interactions and high energies processes. The strong interaction is mediated by the exchange of mesons between nucleons. It gives rise to the longest range part of the strong interactions and is therefore commonly expressed as the absolute strength interaction. The exchange particles are either uncharged pions, π° , which leave the neutron and proton unchanged, or charged pions $\pi^{\pm}$, which will alter the identify of the nucleons. The forward neutron-proton (np) scattering represents the exchange of an uncharged pion and the backward represents proton-neutron (pn) scattering represents the exchange of a charged pion (Angelica, 2002). Processes in which strong interactions are manifested are said to be fast and their characteristic lifetimes vary from 10^{-23} to 10^{-22} sec. The nucleon-nucleon (NN) system interactions have been investigated extensively and the general properties of nuclear force have been well established in the low energy regime. Extensions of such investigations require going beyond medium and high energy regimes which can be achieved by means of heavy-ion (HI) collisions.

HI collisions made it possible to go beyond the nuclear matter density and the low energy regime. One of the main motivations for performing investigations with HI at medium and high energies is to produce nuclear matter at high density and excitation energy (Jaquman and Mekjian, 1985) by producing a nuclear system which is far from the ground state. Furthermore, this is resulted in producing other types of particles such as pions, mouns, leptons, --etc. to maintain conservation laws. Accordingly, a great number of new short lived formation, with a lifetime characteristics of strong interactions such as baryons and mesons have been observed in HI collision reactions. These types of particles (pions, meuns,---) are called resonance particles, or resonant states or Fermi resonances. Such particles have a definite properties as well as certain momenta and energies that enable resonances to be regarded as particles. Therefore, much more about nuclear potential can be learned by including the pion production in HI collisions. Moreover, the system produced in HI collisions is not immediately in thermal and chemical equilibrium. Consequently, interactions among the produced particles are necessary to achieve equilibrium. If equilibrium is reached, global observables such as transverse energy production can be related to thermodynamic variables, such as energy and entropy. Density are commonly used to characterize these collisions, in general.

1.2 Pion production in HI collisions

The study of pion production in nucleus–nucleus or HI collisions is of major concern of intermediate energy regime in high energy physics. A pion in the final state of a nuclear reaction is of interest for reasons ranging from the unique momentum transfer and quantum number matching possibilities to the simple fact that a pion must come from a rather energetic interaction in a nucleus (Giacomo and Clover, 1985). Provided a collision occurs, the two particles can scatter elastically or inelastically. If the beam energy is 150 MeV/nucleon or less, the inelastic channel can be suppressed and non relativistic kinematics used with considerable simplification (Gupta, 1988).

The medium energy range 100-1000 MeV/nucleon in HI collisions is interesting because it encompasses the threshold for significant pion production. The general picture of the threshold production can be figured out as a combination of the Fermi momentum of the two nuclei to permit the pion production in NN collisions. The pion produced has a relatively long mean free path (~5 fm) in general (Bertsch, 1976).

Meson production in HI collisions is considered to be as a sensitive probe of the reaction dynamics (Bertsch, 1988). Particularly interesting is the production of heavy mesons like the $\sigma, \rho, W\, and\, Z$ mesons at a such high kinetic energy that is in sufficient to create the respective meson in a free single NN collision. Secondary and cooperative processes such as $e^{-}e^{-}$ are needed to accumulate the energy for the reaction of the heavy meson. Therefore, the study of the subthreshold particle production is thought to reveal many aspects of the behavior of strongly interacting matter including the equation of state (Mao, 1996). However, the pion, as the lightest meson, plays exclusively important role in nucleon and nuclear structures as well in hadronic and nuclear reactions occurring on the hadron structural level. In particular, the problem of pionic constituents of nucleons has been discussing for an appreciable length of time from various view points (Baym, 1984).

The experimental investigations of pion interactions with nucleons and nuclei can divide the relevant directions of investigations into two groups. In the first case we are dealing with the pions as a probe of the nucleus and collective pionic modes in nuclei, and the role which the pion plays in hadronic structures. Another direction of investigation is focused on the influence of the nuclear medium on the $\pi\pi$ interaction using the pion induced pion-production reaction (Bonutti, 2000). Experiments on scattering of pions by nucleons indicate that at a kinetic energy approximately 160 MeV in the center of mass system, the scattering cross section processes pronounce a maximum. This maximum corresponds to the p state and to an isobaric spin equal to 3/2 (Perkins, 1987). These experiments imply that there is a strong interaction between the pion and the nucleon (Fritsch *et al*., 1974).

The interaction between a pion and a nucleon plays a vital role in low and medium energy physics for several reasons (Schutz *et al*., 1993). Firstly, it is being a prominent example of a strong hadronic interaction. Secondly, it is an important ingredient in many other hadronic reaction or scattering of a pion on a nucleus. During the past three decades, the pion-nucleon interaction has been often parameterized in separable terms in order to simplify its application in related few and many-body systems. One disadvantage of such an approach is that the underlying parameters which have no physical meaning and thus cannot be related to those occurring in other processes (Schutz *et al*., 1993).

Due to the low production threshold, pions are produced and reabsorbed quite frequently and thus provide a single for the whole dynamical evolution of the HI reaction. This implies in particular that a transport theoretical description of HI collisions has to produce the pion yields correctly before one can draw any further conclusion on more specific channels from the model (Teis *et al*., 1997).

Generally speaking, the πN reactions can be classified into three main categories, namely the elastic scattering, the inelastic scattering and the charge exchange one. Thus, systems such as: πN, ππ is of great importance as well as NN system. In all nuclear systems, the πN system becomes more and more considerably significant especially for nuclear energies higher than 100 MeV. Elastic scattering πN reactions that represents an elastic scattering between two-body systems consists of pions and nucleons are listed below:

$$\left.\begin{array}{l} nn \rightarrow n\pi \\ pp \rightarrow p\pi \\ np \rightarrow n\pi \\ pn \rightarrow p\pi \\ p\pi \rightarrow p\pi \\ n\pi \rightarrow n\pi \\ \pi p \rightarrow \pi p \\ \pi n \rightarrow \pi n \end{array}\right\} \qquad (1.1)$$

Typical pion-nucleon reactions can be written as:

$$\pi^{\pm} + p \rightarrow \pi^{\pm} + p \tag{1.2}$$

In fact, most of the attraction comes from two–pion exchange (TPE) processes, where nucleus resonance's may be excited in an intermediate states, the low range and intermediate range Paris potential is completely determined from πN and ππ interactions (Wiringa *et al.,* 1984). Dynamical equations for NN and πN scattering and derived by making the assumption that NN and πN processes can be described in a subspace of NN, πN, ππ reactions.

Inelastic scattering cases leading to pion production have a sizable threshold temperature 170 MeV, the other reactions will occur even for very low energies pion. A tremendous number of experimental data have been collected in order to pursue experimentally the scattering of π^+, π^-, π^0 pions on neutrons and protons to specify the πN interaction completely. In inelastic reactions, the target nucleus is left in an excited state; while the pion energy may be deposited through the creation of new pions. Accordingly,

$$\left.\begin{array}{l}\pi^+ + p \rightarrow \pi^+ + \pi^\circ + p \\ \pi^+ + p \rightarrow \pi^+ + \pi^+ + n\end{array}\right\} \tag{1.3}$$

The charge exchange reactions are similar to proton-neutron (pn) reactions in which the proton can be transformed into a neutron by loosing its charge. This reaction can be represented by:

$$\pi^- + p \rightarrow \pi^\circ + n \tag{1.4}$$

At high energy of the primary particles (greater than 5×10^9 eV) their collisions with the atom of air lead, as a role, to the initiation of electron-nuclear showers. Hence, the formation of a soft electron-photon component of the shower is expected (Krane, 1987). The result of interaction between the primary particles and the nucleus is disrupting the nucleon into separate nucleons or larger fragments, and the formation of unstable particles ($\pi^{\pm}$ and π^{0} mesons). The subsequent decays

$$\left.\begin{array}{l}\pi^{\pm} \rightarrow \mu^{\pm} \rightarrow e^{\pm} \\ \pi^{0} \rightarrow 2\gamma \rightarrow e^{+} + e^{-}\end{array}\right\} \tag{1.5}$$

investigations of such reactions are beyond the scope of this work.

1.3 Resonance theory of pion particles

The fundamental force between nucleons in nuclei is dominated by the exchange of pi-mesons. When these particles are created in high energy proton reactions, they can be used to bombard nuclear targets. When pion interacts with nucleus, it forms a resonance with one of the bound nucleons. The resonance is shifted and broadened compared to the reaction on a free nucleon. These changes reflect the influence of the neighboring nucleons (Schmidt and Schukraft, 1993).

The precise definition of a resonance has been a matter of much debate in literature, the notion of a resonance not being so clear. Thus, different criteria are used for identifying resonances and determining their parameters (Kalkar, 2003). One of the most concepts in this context is the resonance region. In this region, an exchange particle is expected to appear on it at its threshold energy. These exchange particles appears from the excess energy of these reaction to keep it in the conservation laws. The delay time or the lifetime for that exchange particle in the reaction is too small. The baryon resonances were identified and investigated by analyzing the meson baryon scattering data using partial wave techniques. Moreover, pion production in HI collisions has been studied within the coupled resonance BUU model (Larionov and Effenberger, 2001)

In recent years, a new theoretical approach called the collision broadening of baryon resonances in a nuclear medium due to collisions of a resonance with nucleons has been developed. The resonance parameters are determined by fitting some energy-dependent functional form such as the Breit-Wigner to the scattering amplitude. In contrast to these conventional procedures we make use of one of the basic criteria for the existence of a resonance, namely a positive peak in the time delay in the collisions. However, the formation of a resonance which occurs as an unstable intermediate state in scattering processes introduces a time delay between the arrival of the incident wave packet and its departure from the collision region (Kalkar, 2003).

1.4 Theoretical background

According to Yukawa proposal, strong interaction between nucleons is mediated by the exchange of mesons in analogy with photon exchange in quantum electrodynamics. The meson theory of nuclear forces has been pursued in order to describe the NN interaction both qualitatively and quantitatively. Indeed, this picture is fully justified nowadays.

HI collisions of relativistic energies produce a unique possibility to study nuclear matter at high densities and high temperatures in the laboratory. These reactions last only several $10^{-23}s$ and within this time interval, the baryonic density varies between about three times normal nuclear matter density $\left(\rho = 0.17\frac{nucleon}{fm^3}\right)$ in the early phase (Blättel *et al*., 1993). Pions are considered to be a sensitive probe of the reaction dynamics. They are produced a abundantly and due to the large πN cross section pions are continuously trapped by forming baryonic resonances (e.g. $\pi N \rightarrow \Delta$) which then can decay by pion emission. Therefore, pions, especially these with momenta between 0.2 GeV/c and 0.5 GeV/c are expected to freeze out predominantly in the late stay of the collision. High energy pions, however, interact less strongly with nucleons and hence have a chance to decouple already in the early phase. Therefore, a detailed study of high energy pions may shed light on the hot and dense stage of the collisions. Therefore, the investigations of high-energy pions may open a new path to study the nuclear matter equation of state of high baryonic densities.

Pion production in nucleus-nucleus collisions has been proposed as an observable to test the nucleus equation of state. The pion production is an important field of nuclear research due to the pions important role in the overall reaction dynamics. The most elaborate threshold approaches for the description of pion production are microscopical kinetic models which include the propagation of pions and nucleon-resonances as well as their mutual interactions (Cassing *et al.*, 1990). Basically these are three different microscopical models.

Many models have been introduced and developed to understand the πN interaction in a step searching for a general form of nuclear potential. An approximate representation for the scattering amplitude of processes with two final particles, expected to be valid below the threshold for an elastic production of new particles, is deduced from the Mandelstam double integral representation (Cini and Fubini, 1960). Meson-nucleon and nucleon-nucleon scattering are then investigated by means of a model in which all particles are natural and spinless. For nucleon-nucleon scattering, the contribution of the two meson exchange to the scattering amplitude is expressed in terms of pion-pion and pion-nucleon amplitudes (Galitskii *et al.*, 1979).

There are numerous types of successful NN potentials based on meson exchange, among; the simple one-boson exchange (OBE) models have been used for constructing the NN potential by making use of very few parameters. The selected parameters were able to account reasonably well for the empirical NN data below pion production threshold. A realistic inclusion of such 2π-exchange contributions which replace not only the σ but also the ρ meson of OBE models can be done with the help of dispersion relations using πN data (Kim *et al*., 1994).

A simultaneous understanding of various hadronic reactions requires a microscopic treatment based on the fundamental theory of strong interactions. Hence, several investigations were conducted to present a meson exchange model for πN scattering. All models based on meson exchange have been very successful in describing the NN empirical data, up to nucleon kinetic energies of 1GeV or so. Therefore, it should be expected that the meson exchange concept works comparably well in the πN in system (Schutz *et al.*, 1993). The coupling of the πN in the reaction $\pi N \to \pi N$ and in the $NN \to NN$ interaction take into account nucleon, σ and ρ exchange processes, but one obtains no satisfactory description of the πN scattering data as the width of the resonance is too small (Schutz *et al*., 1995).

The boson exchange model has been introduced and developed not only to describe very well the NN data below pion threshold without the use of phenomenological form factors, but also can be applied consistently to analyze πN scattering.

The many-particle system, in the context of quantum field theory, is one for which the eigenvalue of the number operator is large (Martin and Schroinger, 1959). In such systems, the relativistic quantum field theory was found to be appropriate to describe particle production where the vacuum appears to as the natural reference state. The asymptotic evaluation that characterizes microscopic features were performed in terms of intensive parameters.

The time-dependent field correlation function, or Green's function, which describes the microscopic behavior of a multiparticle system, was found to be appropriate for cases of large energy and particle numbers. The full quantum approach or the finite-temperature Greens function formalism was applied to study the equation of state of a hot interacting pion gas at zero chemical potential and calculated the in-medium single pion self energy and the $\pi\pi$ scattering amplitude in the quasi particle approximation. The result obtained is that the higher energy range as well as higher partial waves in the $\pi\pi$ interaction is important for a reliable description of thermodynamic properties of an interacting gas of pions (Rapp and Wambach, 1995).

In most of HI collisions, the local wavelength of relative motion is small as compared to the nuclear interaction region, therefore, the classical approximations are useful in describing and understanding HI reactions. Many features of nuclear dynamics at lower energies, were the particles remain fairly degenerate, can be understood in terms of one-body nuclear models, such as the time dependent Hartree-Fock model or its classical analog (Randrup and Remond, 1990). But, there are two defects, namely, the neglect of quantal and absorption effects which limits the applicability of classical description of elastic scattering, however, these two effects don't destroy the classical picture completely and hence make semi-classical approximation to HI elastic scattering theory feasible and practicable (Mokherjee and Pandy, 1984).

The earliest nuclear dynamical models to describe HI collisions were based on hydrodynamic assumptions (Glassgold *et al.*, 1959). This model assumes a short mean free path and large number of degrees of freedom in order to reach local equilibrium (Danielewicz, 1979). However hydrodynamic models have been successfully employed for the simulation of high energy HI collisions. One can also use the hydrodynamic equations to study the evolution of the hadronic system at finite temperature and/or density. A hydrodynamic model of the high energy particle-nucleus and nucleus-nucleus collisions was introduced and developed to investigate the production of π-meson and resonance excitations at high temperatures and densities of nuclear matter (Lee and Matsuyama,1985). In the hydrodynamic picture, nuclear matter is always in thermal equilibrium and the density, temperature vary both in space and time (Hahn and Glendenning, 1987). Therefore, it cannot be applied at low energy regimes where the mean free path is large. It sometimes extended by introducing viscosity and heat conductivity concepts (Amsden *et al.,* 1977). The central high energy HI collisions are studied in a hydrodynamic model, they had devise an equation of state of the nuclear matter which contains a phase transition at low densities of the matter and includes the pion production and resonance excitations at high temperatures, they had use a numerical model calculation of the collision process exhibits two shock-waves propagating both in the target and in the projectile, they found that a substantial increase of the cross section for emitting higher energy fragments around 120° lab angle could serve as an experimental prove of the existence of a nuclear shock-wave (Danielewicz, 1979).

The Intra Nuclear Cascad model (INC) has been developed to study the collisional properties of the nuclear many-body system in the relativistic HI collisions regimes with a beam of energy ranging from 250 MeV to 2 GeV per nucleon (Serber, 1947). This model assumes multiple scattering but neglects the mean field effects and Pauli exclusion principle, for this approach to be valid, the inter particle distance must be larger than the size of the nucleon (Madey *et al*., 1983).

The Isospin Quantum Molecular Dynamics (QMD) follows the same scheme as the INC, but takes into account the nucleus potential which is calculated as the sum of all two-body potentials (Engle *et al*., 1994). Although quantum chrmodynamics (QCD), with quarks and gluons as fundamental degrees of freedom, is believed to be the underlying of the strong interaction, baryons and mesons have definitely retained their importance as relevant degrees of freedom for a realistic description of low energy nuclear phenomena (Kim *et al*., 1994).

The most elaborate theoretical approach for the description of pion production is based on the microscopic kinetic theories. This type of approach includes the propagation of pions and nucleon resonances as well as their mutual interactions (Engle *et al*., 1994). Each pion-nucleon resonance forms a structure as definite and real as an ordinary proton or neutron, and the fact that these resonances are extremely short-lived should not prejudice us against including them in a list of particles. The π^+p cross-section is dominated by a huge resonance at a pion energy of same resonance occurs in the π^-p elastic and charge-exchange cross sections as well.

A number of dynamical models have been developed with success at different levels, among the dynamical models, molecular dynamics (Aichelin, 1991). Dynamical models of mean field approach, represented by the Boltzmann /Vlasov /Nordheim–Uehling–Uhlenbeck models (BUU/VUU/BNV) (Bertsch and Gupta, 1988), (Baver *et al.*, 1987), (Stocher and Greiner, 1986) have been used to study the mechanism of particle productions.

The classical Boltzmann equation has been extended and modified by Uehling and Uhlenbeck (Uehling and Uhlenbeck, 1933) to include quantum effects by revising the collision term in the classical Boltzmann model. This model has been also called the Boltzmann-Uehling–Uhlenbeck (BUU) equation. The modified collision term is known as the Uehling- Uhlenbeck (UU) collision term. To emphasize that the mean field is included, it is often referred to as the well-known Vlasov-Uehling–Uhlenbeck (VUU) equation (Balescu, 1975). If the collision term is neglected the VUU equation is reduced to the Vlasov equation, these equations (BUU, VUU) provide a semi-classical approximation to

HI collision. During the past decade, such models have been widely used in studying the HI collisions in high energy regimes (Aichelin and Bertsch, 1985). The BUU is quite successful in describing the experimental data on pion production in proton-nucleus as well as nucleus –nucleus collisions (Engle *et al*., 1994).

The transport theories as BUU and QMD, IQMD have been very successful in describing the reaction dynamics of HI collisions. It has been found that the experimental meson spectra can be well understood when assuming the excitation and subsequent decay of nucleon resonances during the compressed stage. Since pions couple strongly to these resonances, the differential pion spectra provide a well suited probe for the dynamics of the baryonic resonances (Teis *et al*., 1997).

The isospin-dependent transport model has been introduced to study the isospin and momentum relaxation-times in the heavy residues formed in heavy-ion collisions at intermediate energies (Li and Ko, 1998). It was found that chemical and thermal equilibrium can be reached before dynamical instability is developed in heavy residues only at incident energies below the Fermi energy. Also the isospin relaxation-time was found to be shorter (longer) than that for momentum at beam energies (higher) than the Fermi energy (Li and Gross, 1993).

The structure of the nucleon resonance has been studied within the frame of coupled-channel meson exchange model for pion-nucleon scattering. It was found that the decay widths of unstable particles in the quasi-two-body states are important to explain the inelasticity of πN channel (Krehl *et al*., 2000).

1.5 Statement of the problem

We shall consider non-equilibrium nuclear system consisting of nucleons and pions. Our main interest will be focused on two-body collisions and only elastic collision processes between particles will be considered. The distribution function of the system in the overlapping region of nucleons and pions will correspond to a state of non-equilibrium. In order to investigate the effect of two-body collisions in HI collisions, each small volume can be studied separately in a local density approximation because of the short range of the nuclear forces. The collision processes between NN and nucleon pion (Nπ) will destroy the equilibration state of the system at a certain time known as the relaxation-time, τ. The time evolution of the pion distribution function was found to satisfy a kinetic equation of the form (Emelyanov and Pantis, 1994):

$$\left(\frac{\partial}{\partial t}+\frac{p_z}{E_p}\frac{\partial}{\partial t}\right)f(p_z,z;t)=\left(\frac{\partial f}{\partial t}\right)_{Coll} \quad (1.6)$$

where $\hbar = c = 1$, $\left(\frac{\partial f}{\partial t}\right)_{coll.}$ is known as the collision term (I_{Coll}) which represents the rate of change of f due to collisions, f , is the pion distribution function, p_z , is the longitudinal momentum and, E_p , is the energy of the projectile particle.

In this study, the relaxation-times were calculated by introducing the Boltzmann Uhleing-Uhlenbeck relaxation-time approximation (BUURTA) model. The BUURTA model provides a simpler context for the BUU collision term which gives many practical results with less work. In the proposed model, the relaxation-times can be calculated by equating the collision term with the relaxation-time collision term (Abu-Samreh, 1991). In general it was found that relaxation-times depend on density and temperature as well as the chemical potential. In order to give a realistic estimate of the relaxation-time, one thus needs to know the chemical potential as a function of temperature. The correct of the treatment, of course, requires the complete solution of all kinetic equations including the expansion of the system. This is best done in a transport approach that will be addressed in this work.

By knowing the relaxation-times, the width of the reaction and the cross section of each reaction can be calculated. This model will be used to study the HI reactions in energy rang 0.1–1.0 GeV and to calculate the distribution functions and the relaxation-times for these reactions.

CHAPTER TWO

The Kinetic Equations

2.1 Introduction

In this chapter, we shall introduce the kinetic equations to be used in calculating the relaxation-rates (width resonances). The particle collisions at the center-of-mass energies $E_{cen} \leq 1$ GeV are predominantly elastic and the pion number is conserved during the collision. The relaxation to the equilibrium state is connected with elastic scattering collision frequency which is density dependent (Emelyanov and Pantis, 1994). A theoretical model will be developed in order to meet our objectives. Such objectives are basically concerned with the calculations of relaxation-times and resonance widths.

2.2 The model

In order to investigate the pion resonance width in HI colisions theoretically, we shall start by considering a non-equlibrium nuclear system that consists of nucleons and pions. The statistical mechanics approach to the thermalization of non-equilibrium nuclear many-body system far from non-equilibrium state to equilibrium state can be made possible by means of the BUU microscopic model. In making use of the BUU model for studying the thermalization of the distribution function during HI collision processes and calculating the resonance widths, the following assumptions were made:

1- The non-equilibrium state of the nuclear system under study is the result of a collision between two heavy ions consists of free pions and nucleons. We shall be concerned mainly with only two-body systems and only elastic collision processes between particles will be considered. Accordingly, only binary collisions such as NN, $N\pi$ and πN will be considered, while the three and four collisions are neglected.

2- The microscopic state of the system can be described in terms of a single-particle distribution function $f(\vec{r},\vec{p},t)$. The distribution function of the system in the overlapping region of nucleons and pions will correspond to a state of non-equilibrium. In this region the momentum distribution at each point in coordinate space is locally deformed. In order to investigate the effect of two-body collisions in HI collisions, each small volume can be studied separately in a local density approximation because of the short range of the nuclear forces.

3- The collision process between NN or Nπ in a microscopic picture and the distribution function of the system in the overlapping region will correspond to a state of non-equilibrium. The collision process between NN and nucleon with pion (Nπ) will destroy the equilibration state of the system at a certain time known as the relaxation-time. The collision between nucleons and pions will change the distribution function due to the transition of nucleons and pions into and out of states.

4- The relaxation-times can be calculated by equating the collision term with the relaxation-time collision term. This approximation is valid for special cases such as the case when the non-equilibrium distribution function is not far from the equilibrium one.

5- The system under considerations consists of nucleons and pions that at $t = 0$ occupy states described by two Fermi spheres and separated by the relative momentum of the two nucleons and pions in momentum space. When the two-body collisions starts, the occupation numbers will change and the Fermi spheres will be distorted, that means the equilibration process will start. For the system under considerations, only elastic collision between NN, $N\pi$, πN will be considered (energy and momentum is conserved). This restricts the possible number of collisions and only these reactions listed in equation (1.1) will be investigated.

Hence, the proton-proton (pp), neutron-neutron (nn), neutron-proton (np), proton-neutron (pn), proton-pion $(p\pi)$, pion-proton (πp), neutron-pion $(n\pi)$ and pion-neutron (πn) differential cross sections were required. Hence, these cross sections are chosen carefully in a way to reproduce the main features of the NN, $N\pi$ and πN scattering in the energy range (100-1000 MeV/ nucleon). Relatively speaking, the main features of πp, pπ, nπ, and πn are expected to be the same because of the common features between the proton and the neutron.

Since we are concerned mainly in the intermediate energy range where the NN collision occurred and the π-meson exchange the energy and momentum between nucleons. Moreover, in case of πN elastic scattering the σ and ω are vector mesons exchange the energy and momentum between the pions and nucleons. In such case, it will be too difficult to take these exchanged particles into account since the resonance width for πN is too small (Rapp and Wambach, 1995).

6- The quantal effects are included through the Pauli blocking factors $(1+f_i)$ for pions (Boson particles) and $(1-f_i)$ for nucleons (Fermion particles), where $i=1,2,3,4$. It is worth mentioning here that 1 and 2 refer to particles in initial states before collisions and 3 and 4 refer to particles in the final states after collisions.

7- The kinetic equation developed by Emelyanov and Pantis (1994) to investigate the time evolution of the pion distribution similar to equation (1.6) will be used in this study. In this case, the evolutions of particles (pions) to equilibration state in the collision volume are assumed to be driven by longitudinal expansion and in the absence of transverse flow. Thus, the distribution function depends on the longitudinal position, z, and momentum, p_z, as well as on t. According to the Boltzmann frame work, this represents the phase space time evolution of pion matter distribution function, $f_\pi(\vec{r},\vec{p},t)$, for the predominantly longitudinal expansion (Emelyanov and Pantis, 1994). We shall concentrate mainly on the central collision region which we assume to be essentially uniform space (Bjorken, 1983). Ideally, one would like to describe $f(\vec{p},t)$ by a set of transport equations (Wong, 1996). Assuming the dimensional expansion along the zaxis, equation (1.6) can be rewritten, according to the relaxation-time approximation, can be rewritten as (Heisenberg and Wang, 1996):

$$\frac{\partial f(p_\perp, p_z, t)}{\partial t} = \frac{f(\vec{p},t) - f_{eq}(\vec{p},t)}{\tau} \tag{2.1}$$

where f_{eq} is the thermalized distribution function.

The general solution of the pion distribution function presented in equation (2.1) can be written in the following form (Baym, 1984):

$$f = f_0\left(p_\perp, \frac{p_z t}{t_0}\right)e^{-x} + \int_{\infty}^{x} dx' e^{(x'-x)} f_{eq}\left(\sqrt{p_\perp^{2} + \left(\frac{p_z t}{t'}\right)}, \tau_{eq}\right) \tag{2.2}$$

where $x(t) = \int_{t_0}^{t} \frac{dt'}{\tau}$. The general solution contains the contributions from the initial state (at t = t_0) of pion matter f_0 and the thermalized part f_{eq}. Thus, information about the partial thermalization of pions can be extracted. The time t_0 is the time when pion–like excitations become well defined which depends on the geometry of collision. The pion distribution function near t_0 is defined according to the following equation:

$$f_0 = \left[\exp^{(E-\mu_0)/T_0} - 1\right]^{-1} \tag{2.3}$$

where $\mu = \mu(t_0)$ is the chemical potential and $T_0 = T(t_0)$ is the temperature initially. Given same initial distribution $f_0\, and\, t_0$, $f_0(p,t)$ is the free streaming, $I_{coll} = 0$. It is convenient to introduce the time dimensionless variable $\mathrm{s} = \frac{\mathrm{t}}{\mathrm{t}_0}$ to describe the time for pion system.

Similar kinetic equation can be introduced to describe the time evolution of nucleon's distribution function from non-equilibrium states to equilibrium ones. The difference between the pion kinetic equation and the nucleon kinetic equation is related to the collision term and to the distribution functions. This is because of the two different particle species, one is the fermion (nucleon) and the other is the boson (pion).

8- The collective variables like energy density ε and the number density n of pions will be decreased due to the expansion of nuclear matter. For each time t we have two self–consistent equations, for the local chemical potential $\mu(t)$ and local temperature T(t). The first equation is introduced to calculate the number of nucleons (pions) (Emelyanov and Pantis, 1994):

$$n(t) = \frac{g}{(2\pi)^3} \int \frac{d^3p}{\exp\left\{ [\sqrt{p^2 + m_\pi^2} - \mu(t)]/T(t) \right\} - 1} \qquad (2.4)$$

The second equation is developed by Emelyanov and Pantis (1994) for calculating the pion energy:

$$\varepsilon(t) = \frac{g_\pi}{(2\pi)^3} \int \frac{\sqrt{p^2 + m_\pi^2}\, d^3p}{\exp\left\{ [\sqrt{p^2 + m_\pi^2} - \mu(t)]/T(t) \right\} - 1} \qquad (2.5)$$

where $g_\pi = 3$ (three different values of the isospin). Particle energy and momentum are assumed to be conserved quantities during the collision process. Supposing that we have at some initial distribution function f_0 and the initial values n_0, we can find $\mu(t)$ for any $t > t_0$ and define the function $f(p_\perp, p_z, t)$.

2.3 The kinetic equations

The study of equilibration or thermalization processes of excited states of nuclear many-body systems is essential in chasing the time evolution of the distribution function of non-equilibrium systems where most calculations are performed by making use of BUU equation that has a collision term. The time evolution of the one-body phase-space distribution $f(\vec{r},\vec{p}_1;t)$ in its most general form can be written as (Engle *et al*., 1994):

$$\frac{\partial f(\vec{r},\vec{p}_1;t)}{\partial t}+\frac{\vec{p}_1}{m}.\vec{\nabla}_r f(\vec{r},\vec{p}_1;t)-\vec{\nabla}_r U(r).\vec{\nabla}_{p_1} f(\vec{r},\vec{p}_1;t)=I[f(\vec{r},\vec{p}_1;t)] \qquad (2.6)$$

The first term on L.h.s is the scattering term (particles are scattered into and out of the phase space volume element $d\vec{r}d\vec{p}$), where the second term represents the streaming term (particles pass in and out of the volume element around $\vec{r}$ because of their motion). The third term represents the change of the distribution function because of the applied external forces. In equation (2.6) $U(r)$ is the particle mean-field potential and the term on the R.h.s is known as the collision term. Since this term plays a vital role in this study, it needs to be discussed in more details.

2.3.1 The collision terms

The general form of the two-body collision term can be written as (Wannier, 1966):

$$\left(\frac{\partial f_1}{\partial t}\right)=\frac{g}{(2\pi)^9}\iiint d^3\vec{p}_2 d^3\vec{p}_3 d^3\vec{p}_4 \times$$
$$[W_{12,34} f_3 f_4 (1\pm f_1)(1\pm f_2)-W_{34,12} f_1 f_2 (1\pm f_3)(1\pm f_4)] \qquad (2.7)$$

where $g=3$ for pions (three different values of isospin)

$g=4$ for nucleons (two different values of spin and isospin)

The $W_{12,34}$ is the transition probability for binary collisions causing the transition $3,4\rightarrow 1,2$, which assumed to be symmetric with respect to the exchange of 1 and 2 and $W_{34,12}$ is the transition probability causing the transition $1,2\rightarrow 3,4$(Abu-Samreh, 1991).

It is of great importance that the transition probability is introduced properly in order to calculate the time rate of change of the distribution function when the system evolves from one microstate to another. The main idea in this case is to relate the transition probability to the $\pi N, \mathrm{NN}\,\mathrm{and}\,\mathrm{N}\pi$ cross sections to ensure that collision effects are included (Abu-Samreh, 1991). It has been shown that the transition probability can be related to the $\pi N, \mathrm{NN}\,\mathrm{and}\,\mathrm{N}\pi$ cross section by (Abu-Samreh, 1991):

$$W_{12,34} = \left(\frac{2\pi\hbar^2}{\mu}\right)^2 (2\pi\hbar)^4 \delta(E_1 + E_2 - E_3 - E_4)\delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\frac{d\sigma}{d\Omega}(12{,}34) \qquad (2.8)$$

where $E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$ and $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass, $\frac{d\sigma}{d\Omega}$ is the $\pi\mathrm{N}, \mathrm{NN}\,\mathrm{and}\,\mathrm{N}\pi$ differential cross sections, f_1, f_2, f_3, f_4 are abbreviations for $\mathrm{f}_1(\vec{\mathrm{p}}_1;\mathrm{t}), \mathrm{f}_2(\vec{\mathrm{p}}_2;\mathrm{t}), \mathrm{f}_3(\vec{\mathrm{p}}_3;\mathrm{t}), \mathrm{f}_4(\vec{\mathrm{p}}_4;\mathrm{t})$ used for the occupation probabilities. The inclusion of delta functions in the collision term is to ensure that the conservation laws of momenta and energy are implemented and takes full account of Pauli Exclusion Principle. Therefore, the brackets contain the usual occupancy for the loss process $(\vec{\mathrm{p}}_1, \vec{\mathrm{p}}_2) \to (\vec{\mathrm{p}}_3, \vec{\mathrm{p}}_4)$ and the reverse process (gain) $(\vec{\mathrm{p}}_3, \vec{\mathrm{p}}_4) \to (\vec{\mathrm{p}}_1, \vec{\mathrm{p}}_2)$ (Engle *et al.*, 1994). In the case of elastic scattering the nucleons and pions do not have the same transition probabilities and the condition of detailed balance can be written as:

$$\sum_{12} W_{12,34} = \sum_{34} W_{34,12} \qquad (2.9)$$

If equation (2.8) is substituted into equation (2.7) and by making use of equation (2.9), the collisional form of the BUU equation can be rewritten as:

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll} = \frac{g}{\mu}\left(\frac{\hbar^4}{2\pi}\right)^2 \iiint d\vec{p}_2 d\vec{p}_3 d\vec{p}_4 \frac{d\sigma}{d\Omega}(12{,}34) \times \upsilon_{12} \times \delta(E_1 + E_2 - E_3 - E_4) \times$$
$$\delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \times [\mathrm{f}_3 f_4 (1 \pm f_2)(1 \pm f_1) - f_1 f_2 (1 \pm f_3)(1 \pm f_4)] \qquad (2.10)$$

where $\vec{\upsilon}_{12}$ is the relative velocity between particles 1 and 2 (Balescu, 1975).

In the present study, more than one scattering mechanism is expected to operate on the same system and several collision terms are expected to contribute in different particle scatterings. Among, the collision term that represents the nucleon collisions, the one that represents the pion scatterings and others that represent the pion-nucleon scatterings. Thus, each scattering process has a similar collision term as in equation (2.10) and we expect that each one is calculated in the absence of the other types of scattering mechanisms. Accordingly, the total collision term which represents all channels contribute to the scattering mechanisms in the reactions is obtained by summing over all terms. In this case, the collision terms can be written as a combination of several sub-collision terms. Therefore, when all nuclear channels are included in the collision term, the general form of the collision term can be represented by:

$$I = I_{nn}^{n} + I_{n\pi}^{n} + I_{np}^{n} + I_{\pi n}^{\pi} + I_{\pi p}^{\pi} + I_{p\pi}^{p} + I_{pn}^{p} + I_{pp}^{p} \qquad (2.11)$$

We shall introduce each collision term separately.

The collision term represents the reaction $\pi n \to \pi n$ (or $n\pi \to n\pi$) which is resulted from the collision between the pion and the neutron. In this case, the total collision term can be written as a combination of two collision terms as:

$$I_{\pi}^{n} = I_{\pi n}^{\pi} + I_{n\pi}^{\pi} \qquad (2.12)$$

where

$$I_{\pi n}^{\pi}\left[f\left(\vec{r},\vec{p}_{\pi};t\right)\right] = \frac{g}{(2\pi)^3}\int d^3\vec{p}_n d^3\vec{p}_\pi \int d\Omega_n \vec{v}_{\pi n}\frac{d\sigma_{\pi n\to\pi n}}{d\Omega}\delta^3\left(\vec{p}_\pi + \vec{p}_n - \vec{p}_\pi - \vec{p}_n\right)\times$$
$$\times\delta\left(E_\pi + E_n - E_\pi - E_n\right)\times\left(f_\pi f_n \bar{f}_\pi \bar{f}_n - f_\pi f_n \bar{f}_\pi \bar{f}_n\right) \qquad (2.13)$$

and

$$f_\pi = \left[\exp^{(E-\mu)/T} - 1\right]^{-1} \quad \text{and} \quad \bar{f}_\pi = 1 + f_\pi$$

$f_n = \left[\exp^{(E-\mu)/T} + 1\right]^{-1}$ and $\bar{f}_n = 1 - f_n$

Similarly, for the reaction $\pi\,p \rightarrow \pi\,p$ and its inverse, one may write

$$I^{p}_{\pi} = I^{\pi}_{\pi p} + I^{\pi}_{p\pi} \tag{2.14}$$

where

$$I^{\pi}_{\pi p}\left[f\left(\vec{r},\vec{p}_{\pi};t\right)\right] = \frac{4}{(2\pi)^3}\int d^3\vec{p}_p d^3\vec{p}_{\pi}\int d\Omega_p \vec{v}_{\pi p}\frac{d\sigma_{\pi p\rightarrow\pi p}}{d\Omega}\delta^3\left(\vec{p}_{\pi}+\vec{p}_p-\vec{p}_{\pi}-\vec{p}_p\right)\times$$
$$\delta\,(E_{\pi}+E_p-E_{\pi}-E_p)\times\left(f_{\pi}f_p\bar{f}_{\pi}\bar{f}_p - f_{\pi}f_n\bar{f}_{\pi}\bar{f}_n\right) \tag{2.15}$$

and the distribution function for the proton is similar to that of the neutron.

The collision term for the reaction $n\,p \rightarrow n\,\pi$ can be written as:

$$I^{n}_{np}\left[f\left(\vec{r},\vec{p}_{n};t\right)\right] = \frac{g}{(2\pi)^3}\int d^3\vec{p}_p d^3\vec{p}_{n}\int d\Omega_{\pi} \vec{v}_{np}\frac{d\sigma_{np\rightarrow n\pi}}{d\Omega}\delta^3\left(\vec{p}_{n}+\vec{p}_p-\vec{p}_{n}-\vec{p}_{\pi}\right)\times$$
$$\delta\,(E_{n}+E_p-E_{n}-E_{\pi})\times\left(f_{n}f_{\pi}\bar{f}_{n}\bar{f}_p - f_{n}f_p\bar{f}_{n}\bar{f}_{\pi}\right) \tag{2.16}$$

For the reaction $n\,n \rightarrow n\,\pi$, one may write for the collision term

$$I^{n}_{nn}\left[f\left(\vec{r},\vec{p}_{n};t\right)\right] = \frac{g}{(2\pi)^3}\int d^3\vec{p}_n d^3\vec{p}_{n}\int d\Omega_{\pi} \vec{v}_{nn}\frac{d\sigma_{nn\rightarrow n\pi}}{d\Omega}\delta^3\left(\vec{p}_{n}+\vec{p}_n-\vec{p}_{n}-\vec{p}_{\pi}\right)\times$$
$$\delta\,(E_{n}+E_n-E_{n}-E_{\pi})\times\left(f_{n}f_{\pi}\bar{f}_{n}\bar{f}_n - f_{n}f_n\bar{f}_{n}\bar{f}_{\pi}\right) \tag{2.17}$$

For the reaction $p\,p \rightarrow p\,\pi$

$$I^{p}_{pp}\left[f\left(\vec{r},\vec{p}_{p};t\right)\right] = \frac{g}{(2\pi)^3}\int d^3\vec{p}_p d^3\vec{p}_{p}\int d\Omega_{\pi} \vec{v}_{pp}\frac{d\sigma_{pp\rightarrow p\pi}}{d\Omega}\delta^3\left(\vec{p}_{p}+\vec{p}_p-\vec{p}_{p}-\vec{p}_{\pi}\right)\times$$
$$\delta\,(E_{p}+E_p-E_{p}-E_{\pi})\times\left(f_{p}f_{\pi}\bar{f}_{p}\bar{f}_p - f_{p}f_p\bar{f}_{p}\bar{f}_{\pi}\right) \tag{2.18}$$

Finally, for the reaction $p\,n \rightarrow p\,\pi$

$$I^{p}_{pn}\left[f\left(\vec{r},\vec{p}_{p};t\right)\right] = \frac{g}{(2\pi)^3}\int d^3\vec{p}_n d^3\vec{p}_{p}\int d\Omega_{\pi} \vec{v}_{pn}\frac{d\sigma_{pn\rightarrow p\pi}}{d\Omega}\delta^3\left(\vec{p}_{p}+\vec{p}_n-\vec{p}_{p}-\vec{p}_{\pi}\right)\times$$
$$\delta\,(E_{p}+E_n-E_{p}-E_{\pi})\times\left(f_{p}f_{\pi}\bar{f}_{p}\bar{f}_n - f_{p}f_n\bar{f}_{p}\bar{f}_{\pi}\right) \tag{2.19}$$

Clearly, one needs to know the differential cross sections in order to evaluate numerically each of the collision terms discussed above. These will be introduced in the next section.

2.3.2 The differential cross sections

In this section, we shall try to evaluate the kinematics part of the cross section for the reaction $1+2 \rightarrow 3+4$, where 1,2,3 and 4 represent particle species such as the nucleon and the pion. In general, the cross section formula can be written as (Malfliet, 1980):

$$d\sigma = \frac{1}{|\vec{\upsilon}_1 - \vec{\upsilon}_2|} \frac{1}{2E_1} \frac{1}{2E_2} |M|^2 \frac{d^3 p_3}{2E_3 (2\pi)^3} \frac{d^3 p_4}{2E_4 (2\pi)^3} (2\pi)^4 \delta^3 \left(P_{total}^{initial} - P_{total}^{final}\right) \qquad (2.20)$$

where M is the scattering amplitude of the two particle collisions. The "flux factor", $\frac{1}{|\vec{\upsilon}_1 - \vec{\upsilon}_2|} \frac{1}{4E_1 E_2}$, can be rewritten in a different way by expanding this factor in terms of collinear velocities as (Terrall, 1970):

$$\frac{1}{|\vec{\upsilon}_1 - \vec{\upsilon}_2|} \frac{1}{2E_1} \frac{1}{2E_2} = \frac{1}{4\sqrt{(\vec{p}_1 . \vec{p}_2)^2 - (m_1 m_2)^2}} = \frac{1}{4|E_2 p_1 - E_1 p_2|} \qquad (2.21)$$

In the laboratory, by making use of equation (2.21), the differential cross section for particles scattering becomes:

$$d\sigma = \frac{1}{4|E_2 p_1 - E_1 p_2|} |M|^2 \frac{d^3 p_3}{2E_3 (2\pi)^3} \frac{d^3 p_4}{2E_4 (2\pi)^3} (2\pi)^4 \delta^3 \left(P_{total}^{initial} - P_{total}^{final}\right) \qquad (2.22)$$

where we have made use of equation (2.22). The scattering amplitude squared M^2 is summed over spins of all particles (Rapp and Wambach, 1995). The final momenta can be determined by conservation of momentum (the delta functions) and are functions of the scattering angle cos θ and the incident momentum of the incident particle (or particle 1). Using the three momentum delta functions to eliminate the integration over particle 3 momentum, the differential cross section will be reduced to (Malfliet, 1980):

$$d\sigma = \frac{1}{16} \frac{1}{(2\pi)^2} \frac{1}{|E_2\, p_1 - E_1\, p_2|} \; |M|^2 \; \frac{d^3\vec{p}_4}{E_3 E_4} \tag{2.23}$$

By making use of

$$E_3 = \sqrt{p_3^2 + m_3^2} = \sqrt{p_4^2 - 2p_4 p_1 \cos\theta + p_1^2 + m_3^2} \tag{2.24}$$

equation (2.23) can be rewritten as:

$$\frac{d\sigma}{d\Omega} = \frac{1}{16} \frac{1}{(2\pi)^2} \frac{1}{|E_2\, p_1 - E_1\, p_2|} |M|^2 \frac{p_4^2 dp_4}{p_4 p_1 + p_4 E_2 - E_4 p_1 \cos\theta} \tag{2.25}$$

Equation (2.25) will be used to calculate the differential cross section of the scattering particles.

2.4 The relaxation-time approximation model

The relaxation-time approximation (RTA) model will be used as a simple approximation to the collision term. Consequently, estimation of the relaxation- rates or the resonance widths can be made possible. In the RTA each collision term is inversely proportional to the relaxation-time which characterizes that scattering mechanism. Thus,

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = -\nu\left[f(\vec{p},\vec{z},t) - f_{eq}(\vec{p},\vec{z},t)\right] \tag{2.26}$$

where $f_{eq} = \left\{ e^{\left[\left(\frac{E_\pi - \mu}{T}\right)\right]} \pm 1 \right\}^{-1}$ is the local equilibrium distribution with local temperature

T=T(t), chemical potential $\mu = \mu(t)$ and $\nu^{-1} = \tau$ the relaxation-time.

The relaxation-times can be calculated by equating the Uehling-Uhlenbeck collision term with the relaxation-time collision term (Wannier, 1966). This can be rewritten as:

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll} = \frac{g}{\mu}\left(\frac{\hbar^4}{2\pi}\right)^2 \iiint d^3\vec{p}_2 d^3\vec{p}_3 d^3\vec{p}_4 \frac{d\sigma}{d\Omega}(34,12)\delta(E_1 + E_2 - E_3 - E_4)\delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)$$
$$[\mathrm{f}_1 f_2(1 \pm f_3)(1 \pm f_4) - f_3 f_4(1 \pm f_1)(1 \pm f_2)] = -\frac{f_0 - f}{\tau} \qquad (2.27)$$

According to equation (2.27), each collision term is inversely proportional to the relaxation-time which characterizes that scattering mechanism,so each mechanism contributes to the collision term and the total collision term which represents all the elastic scattering mechanisms should be written in the following manner:

$$\frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_0}{\tau} = \mathrm{I}[\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t})] = \mathrm{I}^{\mathrm{n}}_{\mathrm{nn}} + \mathrm{I}^{\mathrm{n}}_{\mathrm{n}\pi} + \mathrm{I}^{\mathrm{n}}_{\mathrm{np}} + \mathrm{I}^{\pi}_{\pi\mathrm{n}} + \mathrm{I}^{\pi}_{\pi\mathrm{p}} + \mathrm{I}^{\mathrm{p}}_{\mathrm{p}\pi} + \mathrm{I}^{\mathrm{p}}_{\mathrm{pn}} + \mathrm{I}^{\mathrm{p}}_{\mathrm{pp}}$$

$$= \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\mathrm{n}}}{\tau_{\mathrm{nn}}} + \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\pi}}{\tau_{\mathrm{n}\pi}} + \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\mathrm{n}}}{\tau_{\mathrm{np}}} + \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\pi}}{\tau_{\pi\mathrm{n}}} +$$
$$\frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\mathrm{n}}}{\tau_{\pi\mathrm{p}}} + \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\pi}}{\tau_{\mathrm{p}\pi}} + \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\mathrm{n}}}{\tau_{\mathrm{pn}}} + \frac{\mathrm{f}(\vec{\mathrm{r}},\vec{\mathrm{p}};\mathrm{t}) - \mathrm{f}_{0\mathrm{n}}}{\tau_{\mathrm{pp}}} \qquad (2.28)$$

where τ is the relaxation-time and f_0 is the thermalized distribution function.

In particle physics, it is usually assumed that the lifetime of a resonance is given by the inverse of the total width $\frac{1}{\Gamma}$. This assumption is also widely used in the transport simulations of nuclear collisions in order to describe the decays of various baryonic and mesonic resonances. According, the combined relaxation-time can be defined as:

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} = \Gamma \qquad (2.29)$$

where i represents the scattering channel and Γ is relaxation-time inverse or the resonance width at which pions are produced. The relaxation-time τ_i governs the approach to equilibrium of a non-equilibrium distribution function describing the ith collision mechanism and its value depends on the collision mechanism. Thus, τ_i should be different for different collision mechanism. If the relaxation-time for one mechanism is much shorter than all others, scattering takes place predominantly via other mechanisms (Leupold, 2001).

In the transport simulations one deals with the partial lifetime τ_i of resonances with respect to decay into different channels including absorption and scattering. If one uses $\frac{1}{\Gamma_{\pi N \to NN}}$ as the lifetime of π with respect to the NN decay channel, this correspond to the use of standard cross section $\frac{d\sigma_{\pi N \to NN}}{d\Omega}$ for the absorption channel, conversely if the partial lifetime is changed then the cross section has to be changed accordingly.

Assuming that the overall lifetime is given by equation (2.29) and define a modified total width $\widetilde{\Gamma} = \tau^{-1}$. In general, $\widetilde{\Gamma}$ can be decomposed into modified partial widths according to $\widetilde{\Gamma}_i : \widetilde{\Gamma} = \sum_i \widetilde{\Gamma}_i$. One of the most important aspects in this case is that the modified branching ratios $\frac{\widetilde{\Gamma}_i}{\widetilde{\Gamma}}$ have to be the same as the original ones $\frac{\Gamma_i}{\Gamma}$. This implies that choosing $\widetilde{\Gamma}_i = \frac{\Gamma_i \widetilde{\Gamma}}{\Gamma} = \Gamma_i (\Gamma \tau)^{-1}$ ensures that the measurable cross sections for multi step processes are correct (Larionov and Effenberger, 2001). Accordingly, the probabilities of processes where the π resonance is present in the initial state are all multiplied by the same factor $\frac{\tilde{\Gamma}}{\Gamma} = (\Gamma\tau)^{-1}$ to keep the branching ratios constant:

$$\Gamma_{\pi N \rightarrow NN} = \tilde{\Gamma}_{\pi N \rightarrow NN} (\Gamma\tau)^{-1} \tag{2.30}$$

Thus, the differential cross section can be writes as:

$$\frac{d\,\tilde{\sigma}_{\pi N \rightarrow NN}}{d\Omega} = \frac{d\sigma_{\pi N \rightarrow NN}}{d\Omega} (\Gamma\tau)^{-1} \tag{2.31}$$

where $\frac{d\sigma_{\pi N \rightarrow NN}}{d\Omega}$ is the usual differential cross section represented by equation (2.27) (Larionov and Effenberger, 2001).

After the completion of introducing the differential cross sections needed for calculating the distribution functions from equation (2.27), by making use of numerical calculations. Finally, the obtained distribution function will be used for calculating the relaxation-times. This procedures will continue until the system reaches the equilibrated state. The general method will be discussed in the next chapter.

CHAPTER THREE

Results and Discussion

3.1 Introduction

The BUU equation describes the time evolution of the one-body phase space distribution under the influence of a self consistent mean field and hard core collisions which obey the Pauli principle. As this equation predicts well the experimental observables for light system, the application to heavier targets, however, is more demanding (Köhler, 1985). For the same bombarding energy we have to ensure the stability of the system for a large time duration needed to complete the reaction (Aichelin, 1991). Recently numerical solutions of the BUU equation were advanced and an agreement between the experimental data and the numerical results of BUU model has been achieved (Leupold, 2001). The nucleon-nucleon reactions have been investigated using the Uehling-Uhlenbeck-Nordheim modification in nuclear matter by Abu-Samreh and Köhler (1993). The relaxation-times for the equilibration as a function of density and final temperature of the equilibrated system have been also investigated by Abu-Samreh (1991).

In this study, the pion-nucleons reactions in a microscopic transport model of the BUU type which propagates nucleons and pion resonances explicitly in space and time has been analyzed. We shall consider only the processes that lead to thermal equilibrium for pions in hot hadronic matter. Elastic pion collisions were considered to be the principal thermalizing process. Then, we estimate the characteristic time scale for chemical equilibrium of pions in hadronic matter, first with pions, and then later including resonances.

3.2 Results and discussion

In this study the collision processes of various channels were followed for an initial system composed of two Fermi-spheres. The spheres radii are chosen in such a way to meet the intermediate energy regime. Typical values of Fermi momentum that vary from 1.36-5 fm^{-1} can be used.

The collision integral depends crucially on the phase-space distribution function $f(\vec{r},\vec{p})$. Traditionally, the Wigner phase-space distribution is applied to the evaluation of the collision integral. In order to simplify the problem, the dependence of the distribution function on space can be neglected without loosing any physics. This is because the Wigner distribution is equivalent to the Fourier transform of the one body density matrix over the relative coordinates. Besides, the energy depends mainly on Fermi momentum which represents the major part of the problem. Accordingly, one can take the momentum dependence. The initial distribution function can be chosen as:

$$f_1(\vec{p}) = \theta(\mu - \varepsilon(p)) \tag{3.1}$$

where μ is the chemical potential and θ is the step function. This choice is reasonable for Fermi system. Because of cylindrical symmetry of the two Fermi spheres, one can make use of cylindrical coordinated and choose a certain cylindrical box to put the spheres in.

The time evolution of the distribution function is described by the Collis- ional part of the BUU equation, which is solved using a high speed computers. The integrals at the right-hand side of equation (2.10) are integrated numerically in momentum-space and the new distribution function is evaluated on the grids. The angular integrations are calculated by using Gaussian numerical method (8 points Gaussian quadrature), while the integration over momenta are done by using Simpson's rule. The range of momenta are as follows: $-5 \leq p \leq 5$. The distribution function is assumed to be zero outside the region bounded by the two Fermi spheres.

The collision between nucleons and pions will start when the two spheres collide. The pions collision is assumed at certain lab energies (E > 100 MeV/nucleon). The corner stone in these calculations is the particles cross section. We shall start by testing the differential cross sections to be used in the calculations in order to evaluate the relevant collision integrals. The hadronic cross sections have been calculated from effective differential cross sections. The elastic differential cross sections for the $\pi N, \mathrm{NN} \, \mathrm{and} \, \mathrm{N}\pi$ collisions are calculated using equation (2.25) by assuming the principle of isospin invariance (Acherstaff *et al*., 2002) as:

$$\left.\begin{aligned}
\frac{d\sigma}{d\Omega}(p\,p \rightarrow p\,p) &= \frac{d\sigma}{d\Omega}(nn \rightarrow nn) \\
\frac{d\sigma}{d\Omega}(\pi^{+} p \rightarrow \pi^{+} p) &= \frac{d\sigma}{d\Omega}(\pi^{-} n \rightarrow \pi^{-} n) \\
\frac{d\sigma}{d\Omega}(\pi^{-} p \rightarrow \pi^{-} p) &= \frac{d\sigma}{d\Omega}(\pi^{+} n \rightarrow \pi^{+} n) \\
\frac{d\sigma}{d\Omega}(\pi^{0} p \rightarrow \pi^{0} p) &= \frac{d\sigma}{d\sigma}(\pi^{0} n \rightarrow \pi^{0} n) \\
\frac{d\sigma}{d\Omega}(\pi^{+} n \rightarrow \pi^{0} p) &= \frac{d\sigma}{d\Omega}(\pi^{-} p \rightarrow \pi^{0} n) \\
&= \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\pi^{0} p \rightarrow \pi^{+} n \\
&= \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\pi^{0} n \rightarrow \pi^{-} p)
\end{aligned}\right\} \qquad (3.1)$$

All processes included in the calculation are summarized and results for invariant amplitudes are presented in equation (3.1) which follows from the isospin – invariance of the pion – nucleon interaction.

The differential cross section of a resonance formation R with mass m in particle-particle collision averaged over all spins of initial particles and summed over the spins of final particles is given as (Larionov and Effenberger, 2001):

$$\frac{d\sigma_{12 \rightarrow 34}}{d\Omega} = \frac{1}{64\pi^{2}} M^{2} \frac{p_{34}}{p_{12} E^{2}} A(M^{2}) \times 2\left(2J_{R} + 1\right) \qquad (3.2)$$

Where $|M|^2$ is a spin averaged matrix element squared, p_{12} *and* p_{34} are center-of-mass momentum of incoming and outgoing particles, E is the total center of mass energy $A(M^2)$ is the spectral function of the resonance and J_R is the spin of the resonance.

The elastic scattering cross sections used in calculating Γ for energies up to $T_{lab} \approx 100$ –1000 MeV (T_{lab} is the kinetic energy for incident particles in the lab system) were assumed to be isotropic and were approximated on the basis of the experimental total cross sections. We examined several πN channels (systems listed in equation (3.1)), by assuming a low pion concentration in the range $\frac{N_\pi}{N_N} \approx 0 - 0.2$ to all channels.

The $\pi^{\pm} p$ cross sections were obtained using a computer program written in Fortran language that recovers the cross sections from the experimental elastic scattering at energies above 500 MeV. The calculated differential cross sections for various channels are displayed in Figures 3.1 all the way up to Figure 3.7.

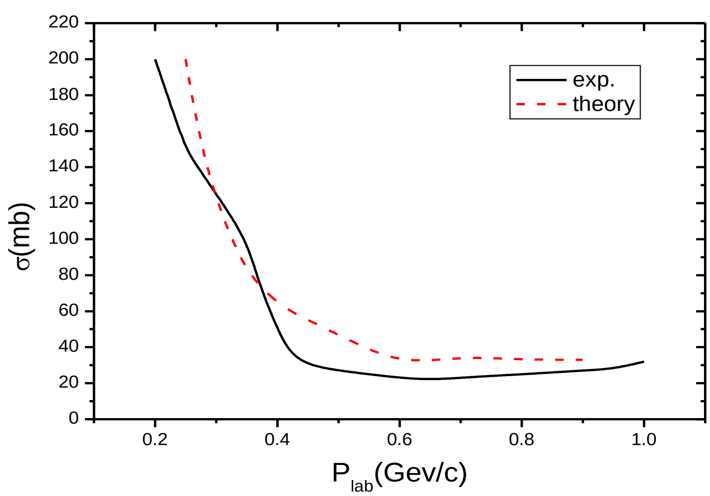

Figure 3.1. The total cross section for (pp) reaction where:

$\frac{d\sigma}{d\Omega}(pp \to pp) = \frac{d\sigma}{d\Omega}(nn \to nn)$. (The data were taken from Acherstaff *et al*. (2002)).

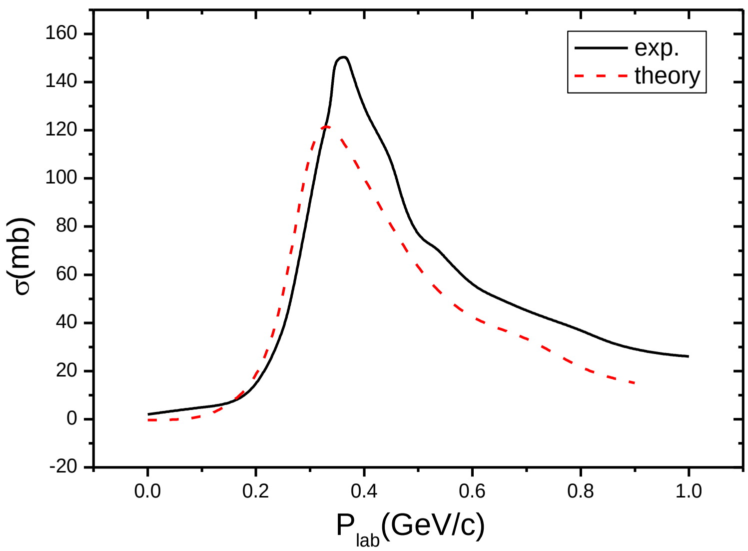

Figure 3.2. The total cross section for $np \rightarrow np$ reaction. (The data were taken from Acherstaff *et al.* (2002)).

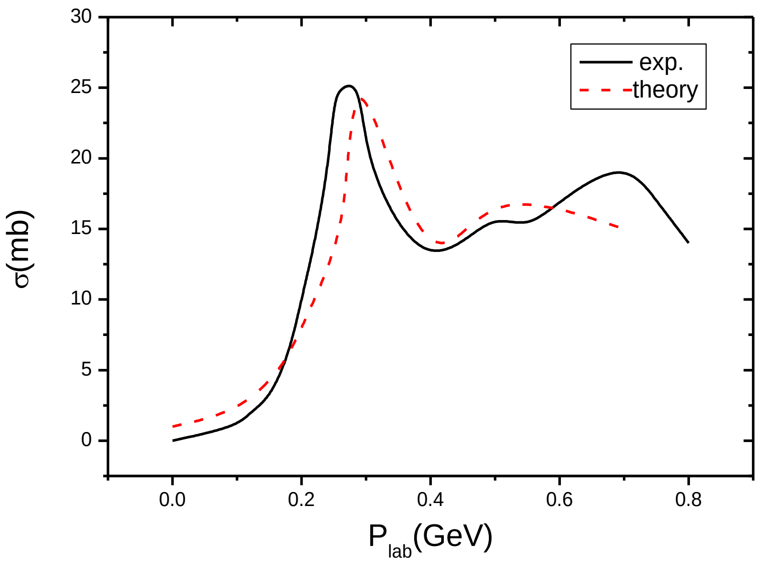

Figure 3.3. The total cross section for $\pi^- p \rightarrow \pi^- p$ reaction. The total cross section for $\pi^- p$ reaction where: $\frac{d\sigma}{d\Omega}(\pi^- p \rightarrow \pi^- p) = \frac{d\sigma}{d\Omega}(\pi^+ n \rightarrow \pi^+ n)$. (The data were taken from Effenberger *et al.* (1999)).

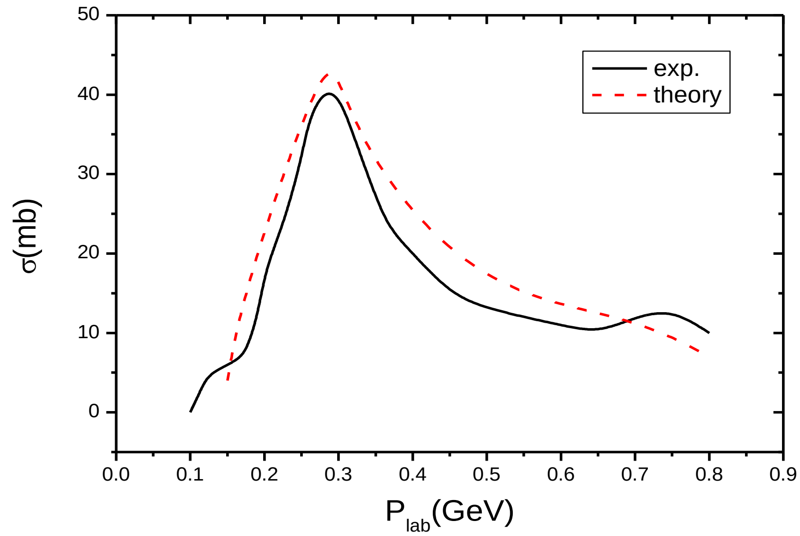

Figure 3.4. The total cross section for $\pi^- p \rightarrow \pi^0 n$ reaction. (The data were taken from Effenberger *et al.* (1999)).

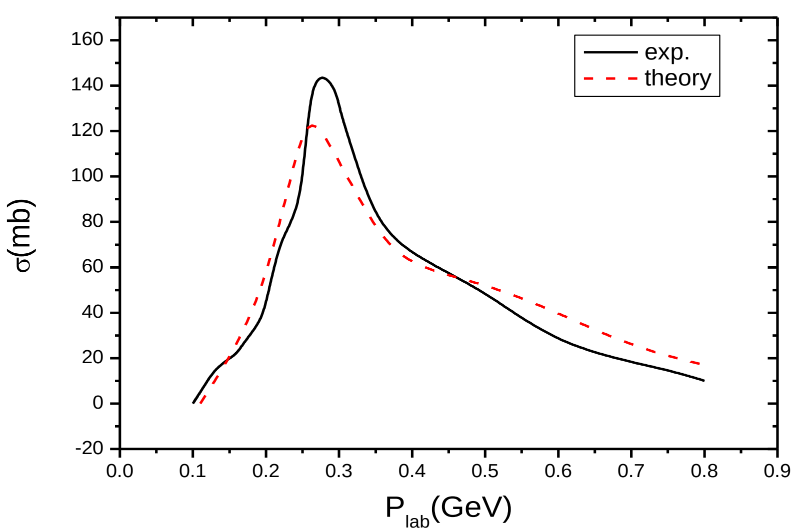

Figure 3.5. The total cross section for $\pi^+ p \rightarrow \pi^+ p$ reaction. (The data were taken from Effenberger *et al*. (1999)).

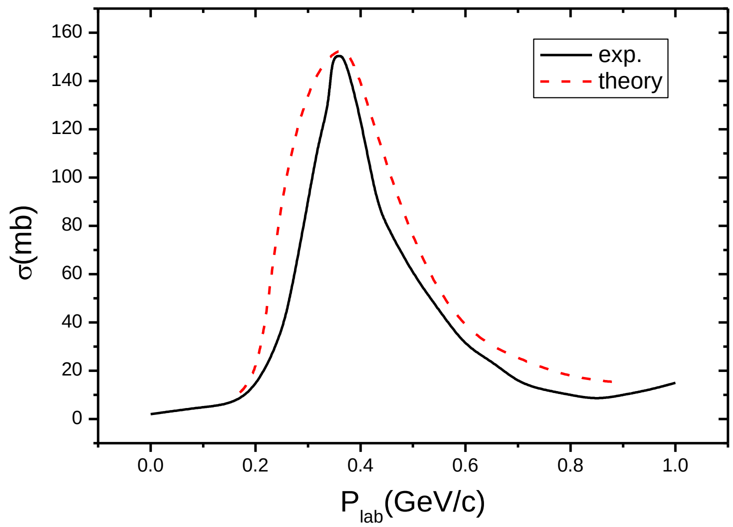

Figure 3.6. The total cross section for $\pi^+ p$ reaction where:

$\frac{d\sigma}{d\Omega}(\pi^+ p \rightarrow \pi^+ p) = \frac{d\sigma}{d\Omega}(\pi^- p \rightarrow \pi^- p)$. (The data were taken from Acherstaff *et al*. (2002)).

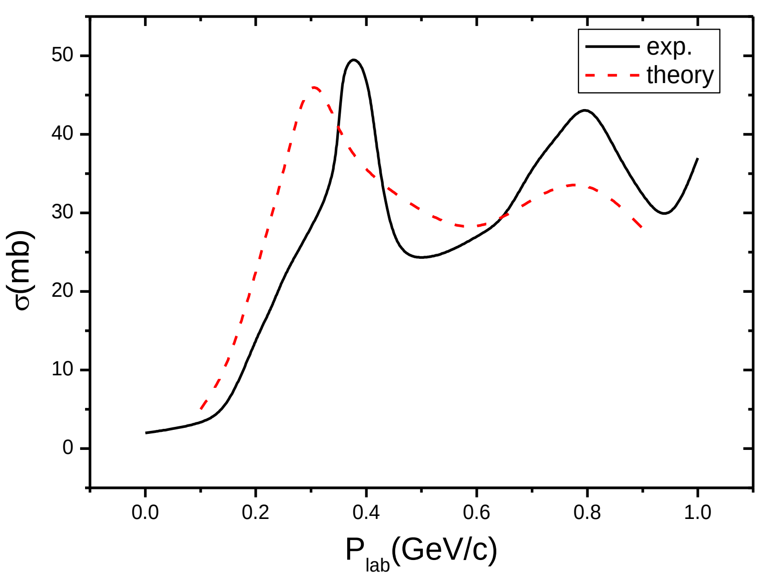

Figure 3.7. The total cross section for $\pi^{-}p$ reaction where:

$\frac{d\sigma}{d\Omega}(\pi^{-}p \rightarrow \pi^{-}p) = \frac{d\sigma}{d\Omega}(\pi^{+}n \rightarrow \pi^{+}n)$. (The data were taken from Acherstaff *et al*. (2002)).

In this study, initial distribution is represented by two Fermi spheres. In momentum space the particles (pions and nucleons) are assumed to stream each other until collisions thermalize the system around a time τ. The free streaming allows the distribution in momentum-space to change drastically. This is exhibited in Figure 3.8. Initially, the pions have large longitudinal momentum due to high relative energy of the incoming nucleons in the nucleus-nucleus collisions. However, in the one-dimensional free streaming expansion of the system at later times only those pions with similar longitudinal velocity will travel together locally in space and time. The phase space separates the longitudinal momenta and the distribution function changes from a wide to narrow one in p_z, locally in space and time as seen from Figure 3.8.

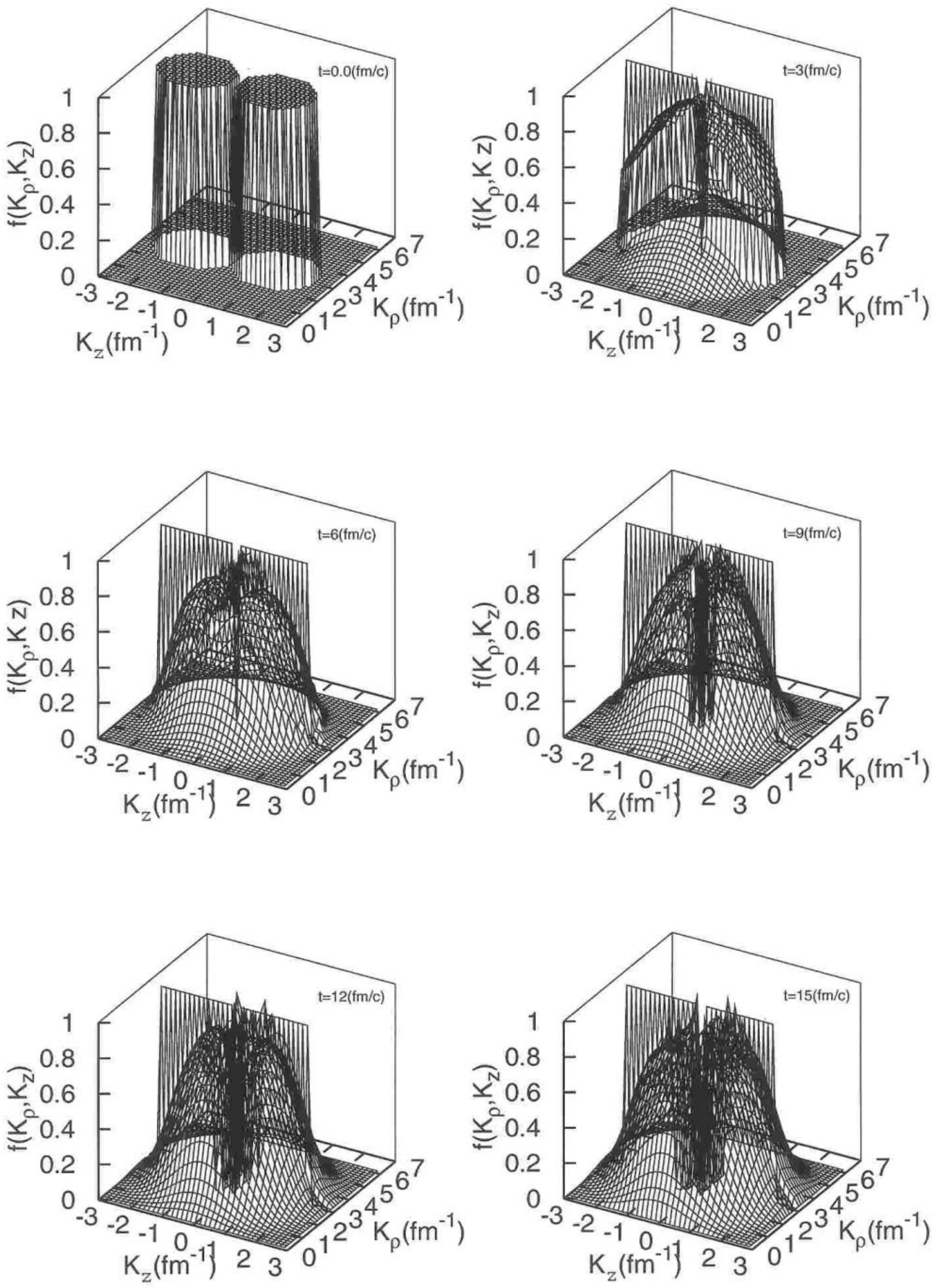

Figure 3.8. Three dimensional surface plots of the distribution function f_1 of two equal Fermi spheres of radii 3 fm^{-1}.

Collisions between particles will then attempt to thermalize the system towards an isotropic distribution. High energy nucleon–pion collision offers a unique opportunity to explain the macroscopic scale properties of nuclides. Hence, secondary interactions among the produced particles are necessary to achieve equilibrium. Particles attain thermal equilibrium in a time of 13-18 fm/c in hot hadronic matter where the distribution is expected to be of the form of equation (2.3). We shall consider the processes that lead to thermal equilibrium for pions in hot hadronic matter and elastic pion collision, $\pi + n \rightarrow \pi + n$, turn out to be the principle thermalizing process.

It was found that the longitudinal expansion extended the system to a size similar to the transverse size at a time of order the nuclear transverse dimension τ~R where the three-dimensional expansion takes over, the densities will decrease rapidly and this is resulted in reducing collisions drastically and a free streaming scenarios is again likely. Around, the same time, however, the system may breakup, freeze out and fragment.

In the ideal case, a strong narrow resonance in the πN cross section fairly well isolated from other resonances should manifest itself as fairly sharp minima in the temperature dependence of the relaxation–time. In the actual case of πN systems, the proximity of particle overlapping with other resonance's, and the width of the resonance complete with the NN cross section and a minimum in the temperature dependence curve becoming somewhat smoothed out or even degenerating into inflection point is expected. The resonance character of the cross section is more clearly manifest in the $\pi^+ N$ system than in the $\pi^- N$ system. This is due to the fact that the resonance's in the $\pi^- N$ cross sections are fewer in number and better separated than these in the $\pi^+ N$ cross sections and don't overlap one another. In $\pi^- N$ scattering the only possible intermediate states are these with isospin $T = \frac{3}{2}$ resonance's. On the other hand, $\pi^{+}N$ scattering goes via two channels with $T = \frac{1}{2}$ and $T = \frac{3}{2}$, so in the $\pi^{+}N$ cross sections, we see that not only the $T = \frac{3}{2}$ resonance, but also the $T = \frac{1}{2}$ resonance's (Gasiorowicz, 1966). The considerable discrepancy between the $\pi^{+}N$ and $\pi^{-}N$ curves at low temperatures is due to the fact that at low energies, $\pi^{+}N$ scattering gives mainly via $T = \frac{3}{2}$ state (Gasiorowicz, 1966).

Pions are produced dominantly by the baryon resonance decays therefore the pion multiplicity is very sensitive to the number of resonances present in the system. The two-pion production model was applied in the pion-nucleon reaction, especially in the vicinity of the reaction threshold. In this case, both baryonic and mesonic intermediate resonances have been considered explicitly (Mueller, 1993). The model is good enough to reproduce the total cross sections in the intermediate vicinity of the threshold and for pions having incident kinetic energies above 250 MeV. The low energy pion-nucleon data were investigated for isospin invariance by comparing charge exchange amplitudes derived from charge-exchange data with those predicted from recent $\pi^{\pm}p$ elastic scattering through the application of isospin invariance and a clear indications of isospin breaking data in the pion-nucleus interaction has been observed(Gibbs and Kaufmann, 1994).

The calculated distribution functions, displayed in Figure 3.8 were used in calculating the average kinetic energy of particles, especially the pions. The average kinetic energy of pions is calculated using:

$$\langle \varepsilon \rangle = \frac{V}{N} \frac{g}{2\pi^2 \hbar^3} \int_0^\infty \varepsilon(p)\, f(p.t)\, p^2 dp \qquad (3.3)$$

where g is the spin isospin degeneracy factor, and

$$\varepsilon(p)=\sqrt{p^2+m_\pi^2}-m_\pi \tag{3.4}$$

is the kinetic energy of the nucleon, and $f(p,t)$ is the distribution function given by Bose-Einstein for pions and Fermi-Dirac distribution for nucleons.

The pionic total energy can be written as:

$$E_\pi=\frac{3V}{2\pi^2}\int_0^\infty \frac{\left(\sqrt{p^2+m_\pi^2}-m_\pi\right)p^2dp}{e^{\frac{\sqrt{p^2+m_\pi^2}-m_\pi-\mu_\pi}{T}}-1} \tag{3.5}$$

Expanding the above equation, the following equation can be obtained (Danielewicz, 1979)

$$E_\pi=\frac{1}{\pi^2}\frac{3}{2\beta}m\pi^2\sum_{l=1}^\infty\left[\frac{3}{l^2\beta}K_2(l\beta m\pi)+\frac{1}{l}K_1(l\beta m\pi)\right] \tag{3.6}$$

Where K_1, K_2 are the modified Bessel functions.

The chemical potential is determined from the number density. In this case, the number of pions does not change, and, after kinetic equilibrium has been established, the pions are distributed according to a Bose-Einstein distribution. Thus,

$$\rho_\pi \approx \frac{N}{V}=\frac{g}{2\pi^2}\int_0^\infty f(p,t)\,p^2dp \tag{3.7}$$

by assuming normal nuclear matter density, $\rho\approx\frac{N}{V}=0.17\,nucleon\,fm^{-3}$. A suitable expansion and integration of right-hand side of the above equation gives (Danielewicz, 1979):

$$\rho_\pi=\frac{1}{\pi^2}\frac{3}{2\beta}m\pi^2\sum_{l=0}^\infty\frac{1}{l}K_2(l\beta m\pi) \tag{3.8}$$

Where K_2 is the modified Bessel function.

A plot between the ratio of pion to nucleon numbers versus temperatures is shown in Figure 3.9. There will be 5 pion per nucleon at T= 100 MeV, and then flattens out nuclear matter is gradually transformed into a hadron (Stöcker and Greiner,1986). At low temperatures of the order of 50 MeV or less, most of the pions reside in the condensed zero momentum state (Han and Stöcker, 1986). At higher temperatures, the pion yield is due to nuclear resonance's. The massive pion resonance's become important at temperatures about T≈ 100MeV. As the temperature rises smoothly with the bombarding energy, reaches about T≈ 100 MeV and T≈ 200 MeV deconfinment will take place. At energies in the range of relativistic heavy ion collisions. At high temperatures, the production of resonance's can be achieved explicitly using the statistical mechanics approach. The resonance's can be viewed as excited nucleons (resonance pair production) and a Boltzmann distribution for the excitation probability of the ith resonance can be assumed at temperatures above 100 MeV.

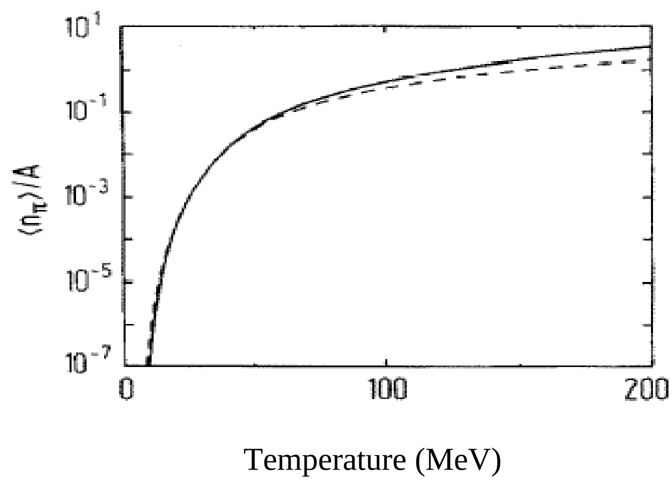

Figure 3.9. Pion multiplicities versus the temperature for baryon densities two times (solid line) and four times (dashed line) normal nuclear matter density. The curves describe the properties of a hot and dense piece of infinite nuclear matter (Hahn and Stöcker, 1986).

The pion number as a function of temperature is shown in Figure 3.10.

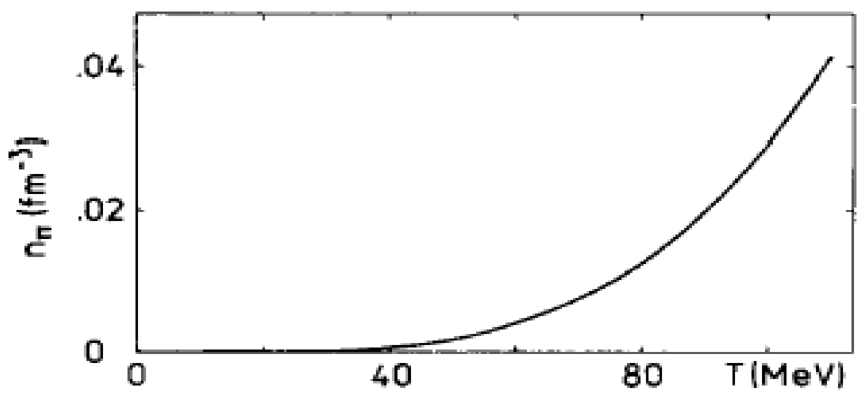

Figure 3.10. Density of pion for hot nuclear matter, (Danielewics, 1984).

The π-meson density dependence on temperature is shown in Figure 3.10, when the densities are high enough, the Fermi energy starts to approach the lowest resonance. In such case, the resonance was described by Boltzmann statistics. In the presence of excited states, i.e. resonances, the number of pions is changed by the decay process of resonances (and vice versa).

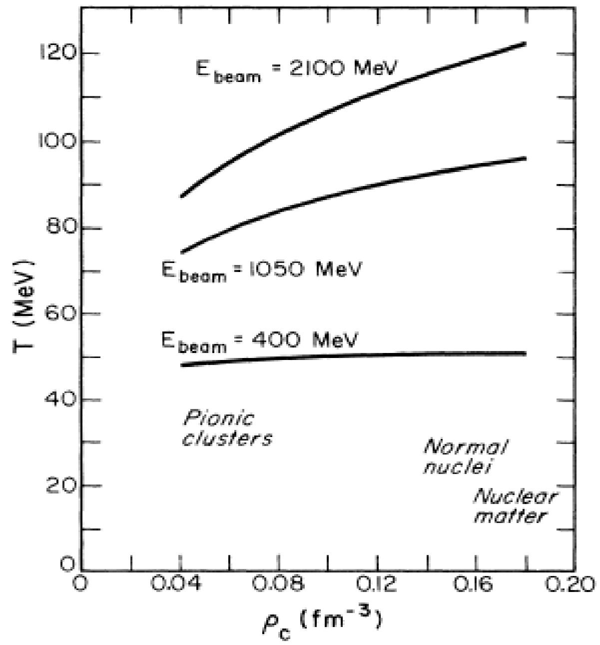

Figure 3.11. Pion to nucleon ratio of the fireball as a function of critical density for the collision of equal mass nuclei (Kapusta, 1977).

Calculations have been also extended to study the behavior of a single state relaxation-time on a microscopic basis. The states relaxation-times are carried out in two-steps. First the BUU collision integral for the distribution function representing two Fermi-spheres is calculated exactly by numerical methods. Second, calculation of $f_1 - f_1^0$ is made after both the distribution function f_1 and the thermalized distribution function f_1^0 are calculated. The relaxation-times is then followed using the expression $\tau_p = \frac{f_1^0 - f_1}{I(f_1)}$ were this equation represents a state described by a distribution function f_1 , by which the system has been started, and which decays the characteristic relaxation-time τ_p . The calculated relaxation-times are measured in fm/c units and all relaxation-times of $100\, fm/c$ or higher are considered to be infinite. The best way to read these relaxation-times is to make contour plots of the relaxation-time inverse. In this case infinite relaxation-times can be represented by no contour lines when $\frac{1}{\tau}$ is plotted. We present in Figure 3.12 three dimensional surface plots of the relaxation-times of NN scattering processes. Similar results were obtained for the rest of channels in equation (1.1).

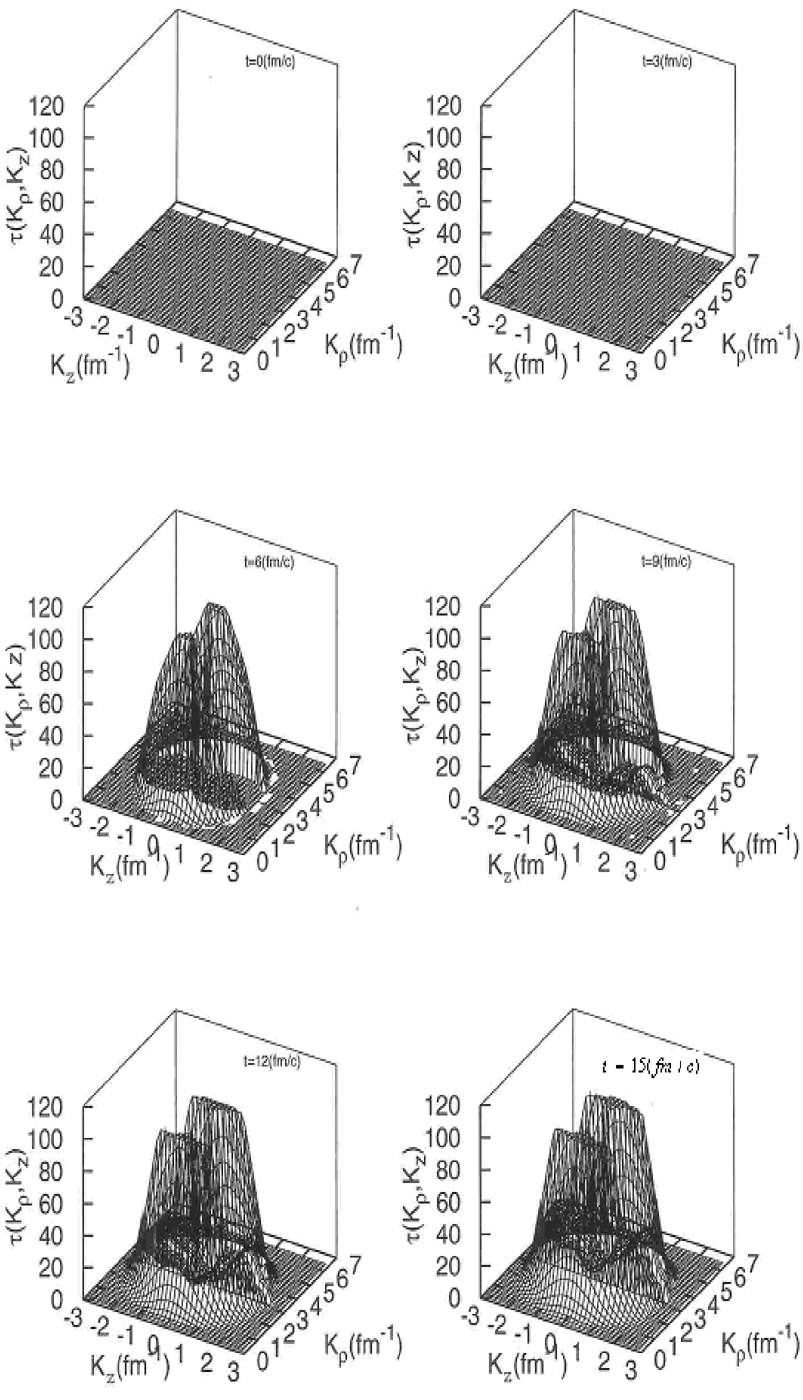

Figure 3.12. Three dimensional surface plots of the time evolution of the relaxation-time τ in the scattering plane (k_ρ, k_z) resulting of two equal Fermi spheres.

Figure 3.13 displays contour plots of the relaxation-time inverse of states belonging to the same two equal Fermi-spheres system discussed above. The lowest collision rate value is represented by the outer contour line and is indicated by “bas”, while the distance between the contour lines is indicated by “inc”. That is, the contour line values are increased by “inc” when going from outside to inside in each contour plot. The inside contour line represents the peak of the collision rate. The time steps represented by t are taken to be a multiple of 10^{-21} sec. At t = 0.01 sec, before the collision starts, nucleons are occupying states to $T = 0 MeV$ Fermi spheres, hence the relaxation-times are infinite and no contour plot appears in this case. When the collision starts and the system start to disassemble, nucleons make transitions into states of finite relaxation-times. Apparently, these states are located above the Fermi surface close to the contact zone. Initially, rare collisions between nucleons from each polar region being scattered into the equatorial region may contribute to the relaxation. Therefore, contour lines of $\frac{1}{\tau}$, which represents states of short relaxation-times, appear Figure 3.8 as solid line. As the time continues to evolve, nucleons will continue to scatter in and out of states. This in turn creates a number of short relaxation-time states below the Fermi surface as well above it, and the nucleons are then allowed to scatter into these unoccupied states. Hence the contour lines quickly spread uniformly over the whole system. At the later stages, say at $t = .15 \times 10^{-21} \sec$, when the system has developed a thermalized distribution, the states have long relaxation-times and the contour lines start to disappear. The reason that states may have an infinite relaxation-time is because the scattering of a given nucleon out of the state is exactly balanced by the scattering of other nucleons into the state. This means that the probability of scattering “out” a nucleon from that state remains the same in equilibrium and may never change.

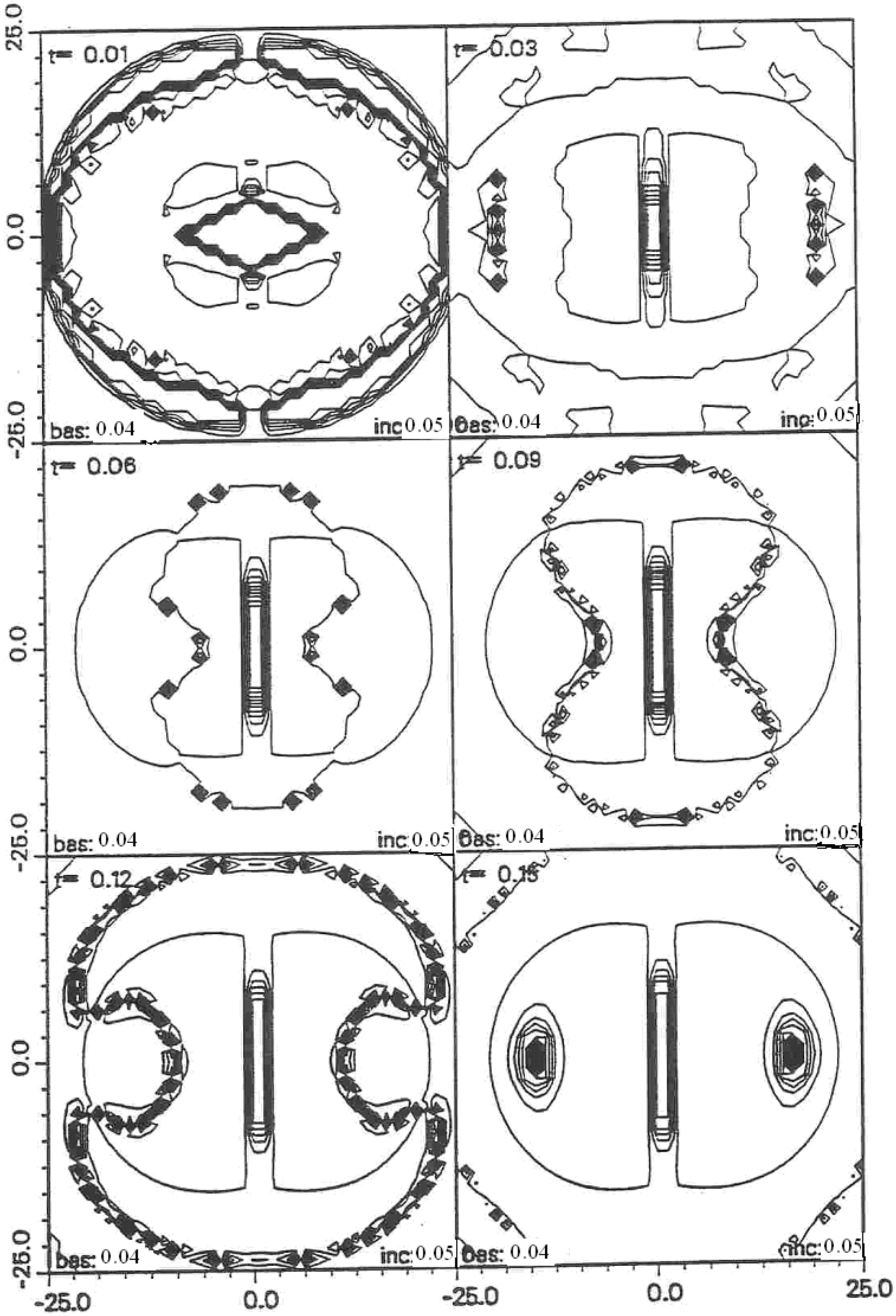

Figure 3.13. Contour plots of the time evolution of the distribution function f_1 in the scattering plane (k_ρ, k_z) resulting of two equal Fermi spheres.

The average relaxation-time of a nucleon in nuclear matter dependence on temperature in the temperature range 20-80 MeV and different nuclear matter densities were displayed in Figure 3.14. The curves show almost the same behavior. The presence of the curvature is associated with the production of resonance particles, namely, the pions.

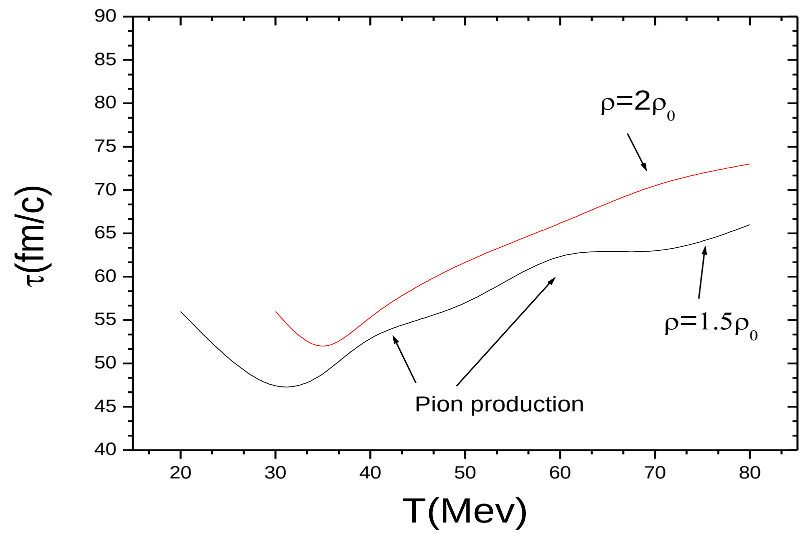

Figure 3.14. The dependence of nucleons relaxation-time on temperature and density.

Let us now discuss hadronic matter relaxation-times at low temperatures which mainly consists of pions (Figure 3.15). The relaxation-time estimated to be about 14 fm/c at T = 30 MeV (about 20 fm/c with $\mu_\pi = 138 MeV$ at the same temperature) for such state. This is very large compared to the size of the hot matter produced in nucleus-nucleus collisions. This implies that the thermal equilibrium would not be reached in the expanding hot hadronic matter consisting of pions only. Generally speaking, the πN collision at the center-of-mass energies $E_{\pi\pi} \leq$ 1GeV are predominantly elastic, the pion number is conserved during the expansion in A +A collision. The relaxation from non-equilibrium state to the equilibrated state is characterized by a collision frequency that is density dependent (Emelyanov and Pantis, 1994).

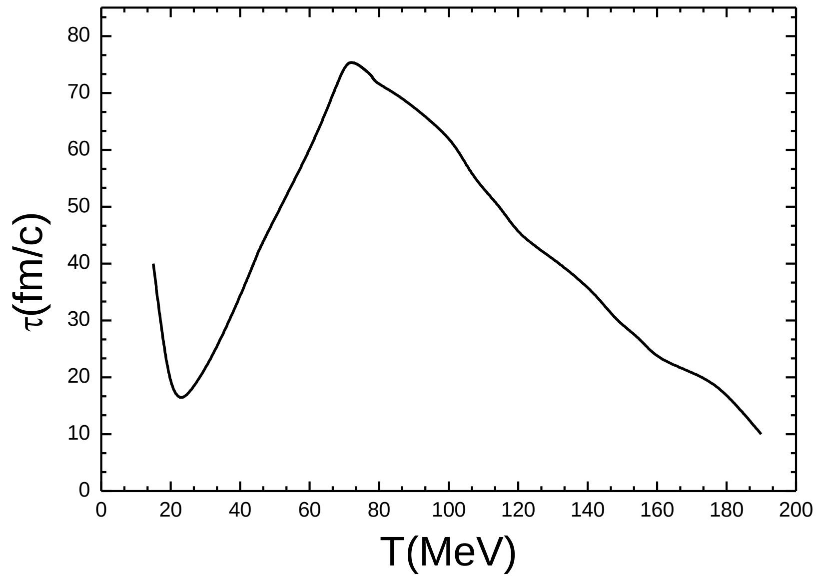

Figure 3.15. The dependence of hadronic matter relaxation-times at low temperatures.

The thermodynamic equilibration in a hadronic system is driven by multiple collisions among particles in the system. Generally the equilibration time is directly proportional to the collision rate. In an expanding system, therefore, the collision rate should be much larger than the expansion rate of the system in order to maintain thermodynamic equilibrium. As long as the expansion velocity of the system does not exceed the most probable velocity of the thermal particles in the system, this condition is satisfied if the mean free path of a particle is shorter than the size of the system. In the following we determine the relaxation-times of pions assuming that the system is slightly deviated from the equilibrium state, i.e. in the relaxation-time approximation.

Since the system still remains in thermal equilibrium, the state can be characterized by a maximum of the entropy consistent with the conservation laws of energy, momentum and of relevant particle numbers. Since we neglect dissipative, i.e. pion number changing, such processes implies that entropy as well as the pion number is conserved in the expansion. As a result of keeping the pion number constant, the chemical potential will rise. This is due to the overpopulation of pionic states due to the decay of the resonances. It has been estimated that pion chemical potential is about 86 MeV at freeze-out temperature (Song and Koch, 1993).

In Figure 3.15 the thermal relaxation-times for each process as a function of temperature was displayed. The total relaxation-time is determined by including all true pion number changing processes. Due to the hadronic resonances, the resulting relaxation-time is almost an order of magnitude shorter than that previously obtained within chiral perturbation theory (Negele, 1982). With μ_π= 100 MeV we find that τ ~ 10 fm/c at T ~ 150 MeV. A relaxation-time of about 5 fm/c at T ~ 180MeV was found and such value increases to about 10 fm/c at T = 150 MeV. At a temperature of T = 150 MeV the total relaxation-time is about 10 fm/c, which is comparable to typical system sizes created in ultrarelativistic heavy ion collisions involving heavy nuclides. However, the corresponding temperature, for which the thermal relaxation-time assumes the same value of 10 fm/c is considerably low, namely about 110 MeV. From these numbers we expect that even with the inclusion of the resonance chemical freeze-out takes place before the thermal one. Moreover, the above estimate has assumed that the resonances are formed instantaneously. Thus, the chemical relaxation-times may be slightly larger once the formation of the resonances is taken into account properly.

First we consider resonance decays. In this case the relaxation-time, which is a measure of how fast the relative chemical equilibrium between pions and nucleons is reached, will be inversely proportional to the decay width. The general behavior of chemical relaxation –time can be expressed as (Song and Koch, 1993).

$$\tau_{chem} = \frac{1}{\Gamma}\left(\frac{\rho_\pi}{\rho_\pi + 4\rho_N}\right) \tag{3.9}$$

where ρ_π is the equilibrium density of pions and ρ_N is the nuclear matter equilibrium density. It was found that τ_{chem} values range from 0.2 to 0.8 fm/c. This is very short, and to a good approximation we can assume that pions are always in relative chemical equilibrium.

The elastic πN and ππ scattering have been extensively studied by means of chirial symmetry (Robilotta, 1985). An agreement with experiment is good both below threshold and for pion energy up to 350 MeV, actual calculations require the knowledge of various parameters entering equation (3.9) (Robilotta, 1985). For Nb+Nb collision up to 400 MeV, $\tau \approx 1.4 \times 10^{-22} \sec\,(1\sec = 3 \times 10^{23}\,\frac{fm}{c})$. Let us also assume, for a moment, that the time scale for the collision processes are long compared to the lifetime of the system, whereas the elastic processes, which are responsible for the kinetic equilibration, are fast. In this case, the number of pions does not change, and, after kinetic equilibrium has been established, the pions are distributed according to Bose-Einstein distribution.

The thermal equilibration of pions at low temperatures will be governed by elastic two body collisions. As long as we consider elastic two-body collisions, the thermal relaxation time τth is given by (Song and Koch, 1993)

$$\begin{aligned}\frac{1}{\tau_{th}} = \frac{1}{4}\left(1-e^{-\beta E_a}\right)\frac{1}{2E_a}\int \frac{d^3 p_2}{(2\pi)^3 2E_2}\frac{d^3 p_3}{(2\pi)^3 2E_3}\frac{d^3 p_4}{(2\pi)^3 2E_4} \\ \times (2\pi)^4 \delta^4\left(p_a + p_2 - p_3 - p_4\right) \\ \times \sum_{2,3,4}\left|M\left(\pi_a \pi_2 \to \pi_3 \pi_4\right)\right|^2 f_2\left(1+f_3\right)\left(1+f_4\right)\end{aligned} \tag{3.10}$$

where f_i's are Bose-Einstein distribution function. The factor 1/4 in front comes from identity of particles at initial and final states and the sum is over the spin and isospin degeneracy of particles 2, 3 and 4. The second term (~ $e^{-\beta E}$) indicates the contribution from inverse reaction. Note that this result is obtained in the relaxation-time approximation. The mean relaxation-time is defined as:

$$\bar{\tau}_{th} = \frac{1}{\int d^3p_a f_a(p_a)} \int d^3p_a \tau_{th}(p_a) f_a(p_a) \qquad (3.11)$$

When we using a suitable numerical calculations and simplifying algebra the mean relaxation-time is simplified to (Song and Koch, 1993):

$$\bar{\tau}_{th} \approx \frac{12 f_\pi^4}{T^5} \qquad (3.12)$$

With this approximation it has been shown that the mean relaxation-time is about 5 fm/c at T = 150 MeV (Song and Koch, 1993). Since the mean free path $\lambda \approx \tau$ in the relativistic limit, the result can be compared to the size of the system. Here we assume that the radius of the hot matter would be 5 ~ 10 fm. This implies that pions are in thermal equilibrium in hadronic matter, mainly due to the two body elastic collisions.

At temperatures close to the phase transition, other heavy mesons like kaons, vector mesons, etc., become increasingly abundant. Thus, a reduction of thermalization time becomes more and more noticible. Consequently, pions can maintain thermal equilibrium in hot hadronic matter even at comparatively low temperatures. From formula equation (3.12) we expect a freeze-out temperature ≤ 130 MeV for a small system such as S + S or S + Au. This is somewhat lower than the values extracted from experiment, which are ~150 MeV. The reason for this slight discrepancy is most likely due to the flow generated in the reaction, which reduces the effective system size. We also expect that light vector mesons, ρ, ω, φ, reach thermal equilibrium in hot hadronic matter. For ρ, ω the dominant reactions will be the collisions with thermal pions through heavy resonances such as $\pi + \rho \rightarrow \pi + \rho$ and $\pi + \omega \rightarrow \pi + \omega$, (Song and Koch, 1993). In this reference it has also been shown that the various collisions in hot hadronic matter make it possible for phi mesons to be in thermal equilibrium.

Relatively speaking, in a hadronic system consisting of only pions, the number of pions cease to change at the early stage of the expansion even if the system is out of equilibrium at the beginning of the evolution. Once we include resonances, additional processes help to change the pion number. Again we assume that the decay processes such as $\pi\pi$ and $N\pi$ are fast enough to maintain the relative chemical equilibrium even if pions have a finite chemical potential.

We show the relaxation-time with μ_π = 100 MeV becomes short compared to the size of the system. Thus the number of pions would be changed in hot hadronic matter even near the thermal freeze-out temperature. Especially axial vector mesons such as ρ-meson are easily interact with pions and annihilate into two pions. However, we should note that the thermal freeze-out temperature is also reduced when we include a finite pion chemical potential (Song and Koch, 1993). The above result, of course, implies that also the number of vector and axial vector mesons changes as quickly as that of the pions. Moreover, the total chemical relaxation-time is changed with pion chemical potential at fixed temperature. Even at T =150 MeV, the pion relaxation-time is comparable to the size of the hot system as long as the pions have a finite chemical potential; $\tau \sim$ 5 fm/c at μ_π = 50 MeV. If the hadronic system is produced out of equilibrium, for example with μ_π = 100 MeV, the excess of pions will be reduced by the inelastic reactions involving vector and axial vector mesons. These number changing processes will lead to the decrease of pion chemical potential and finally cease to be effective as the chemical potential is reduced below a certain value. In this study, the pions were found to freeze out within 13 fm/c in Au +Au collisions. Experimental results predicts a similar time scale for the emission of high energy pions (Bass *et al.*, 1995). According to calculations, the nuclear matter density exceeds twice saturation value in central Au +Au collisions at 1 GeV/nucleon within the first 15 fm/c. Therefore, given a freeze-out temperature of about 150 MeV we would expect the pion chemical potential not to exceed a value of 50 MeV.

As the system expands and pions develop the chemical potential the relaxation-time becomes shorter than that obtained ones near equilibrium obtained from Figure 3.12. We find that the τ is reduced by half at T = 150 MeV (τ ~ 5 fm/c) due to the pion chemical potential. However, even with the induced pion chemical potential the chemical relaxation-time is considerably larger than the thermal relaxation-time at the same temperature.

The dependence of all the rates on the pion concentration (i.e the ratios $\frac{N_{\pi^+}}{N_N}$ *and* $\frac{N_{\pi^-}}{N_N}$) can be approximated to same degree of accuracy by a straight line. Generally speaking, the relaxation–times were found to increase with increasing the temperature only as $\tau \approx \alpha\sqrt{T}\, e^{BT}$ in the temperature range T ≈ 20 – 150 MeV (Gupta, 1988). The lifetime τ is given with the sum of the pionic decay rate as $\tau = \frac{h}{c\Gamma}$, where, Γ, is the width of the reaction. The presence of positive pions was found to be more effective than that of negative pions in altering the relaxation–times of nuclear matter.

Comparing with previous calculations (Song and Koch, 1993) based on chiral perturbation theory, the inclusion of the resonances has reduced the chemical relaxation-time by about a factor of 10. When we neglect the formation time of these resonances, the resulting chemical relaxation time of pions is 10 fm/c at temperature, T = 150 MeV. This value is comparable to the size of the hot system produced the collision of large nuclei. Given a system size of 5 ~ 10 fm we obtain a thermal freeze-out temperature which is small compared to those extracted from experiments (Song and Koch, 1993). This might be due to flow effects which lead to smaller effective system sizes. If we take the thermal freeze-out temperature be about 150 MeV, then the freeze-out size of the system would be 2 ~ 3 fm. On the other hand the chemical relaxation time for a system of this size would be at temperature, T = 170 MeV. This implies that chemical freeze-out of pions happens at considerably higher temperatures than thermal freeze-out. Where pion spectra and particle abundances could be reproduced assuming the same freeze-out conditions. In order to properly assess the magnitude and importance of this discrepancy, a detailed transport calculation including all the number changing processes presented here is needed.

CHAPTER FOUR

Conclusions and Future Work

We have investigated the effect of the presence of pions in the nuclear matter system. The mean characteristic features of differential cross sections of $\pi^+ N$ and $\pi^- N$ have been investigated, the $\pi^+ N$ and $\pi^- N$ curves differ from NN curve in the irregularity of their behavior they have inflection pionts and some evident maxima and minima. It has been found that the pionic entrance channels are especially sensitive to the pion dynamics and a good agreement for differential inelastic scattering and the exclusive (π^+, NP) reactions in the delta resonance region have been obtained (Engle *et al*., 1994).

We have studied the thermal and chemical relaxation-time scales of pions in hot hadronic matter using the non-equilibrium statistical mechanics. From the explicit calculation, it has been found that pions in hot hadronic matter are in a phase where elastic collisions rates are very fast compared to typical expansion rates of the system. For chemical equilibration the dominant contribution comes from the inelastic collision involving a1l mesons. For a temperature of $T \approx 150$ MeV, a relaxation-time of $\tau \approx 10$ fm/c was obtained, this is a typical time of hadronic matter.

The temperature dependence of relaxation–times for the (NN) systems were displayed in Figure 3.14. Such a temperature dependence of the transport relaxation–time is associated with nucleonic resonance's in the πN elastic interactions.

We have also studied the effect of baryons on the chemical relaxation-time of pions. At temperature, T = 150 MeV with a chemical potential, μ_π = 100 MeV the relaxation-time is about 5 fm/c which is certainly comparable to the system size at freeze-out. However, we should note that the thermal freeze-out temperature is also reduced when we include a finite pion chemical potential (Song and Koch, 1993). We found that in this case the chemical relaxation time is about 5 fm/c at temperature, T =150 MeV. For systems which are larger than 5 fm we, therefore, expect that the chemical potential at freeze-out will be considerably smaller than the value of 86 MeV, which has been obtained without taking number changing processes into account.

In conclusion, as long as the effective system size is not considerably smaller than 5 fm, a buildup of a pion chemical potential larger than 100 MeV would be very difficult to understand. At the same time, we also predict a considerable difference between the chemical and thermal freeze-out temperatures. To which extent that is reflected in the data needs to be investigated within a transport calculation.

Furthermore, Pion scattering provides a good result the neutron-proton (or equivalently isospin) character of a transition because of a useful property of the pion-nucleon interaction., the $\pi^+ p$ or $\pi^- n$ cross sections exceed the $\pi^- n$ or $\pi^- p$ cross section by a factor of 9.

Future work will concentrate on a transport theoretical calculation of the chemical equilibration. We also plan to extent the present study to include the reactions involving strange particles. Therefore in order to improve this model, we suggest the following:

1- In addition several effects have been neglected such as the coulomb effects, the higher order collision effects, such as the triple collision effects.

2- Investigations should be extended to include triple and higher order collisions.

3- Investigations that include the viscosity dependent in order to exhibit their effect on the relaxation-times.

4- This model can also be modified to include the electric and magnetic field effects on thermalization and relaxation-times.

5- For a complete study, the inelastic collision process between particles should be studied.

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