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M. Sc. Thesis

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BY

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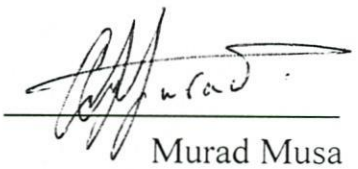
DECLARATION

I certify that this thesis submitted for the degree of Master of Science in Physics is the result of my own research, except where otherwise acknowledged, and that this thesis (or any part of the same) has not been submitted for a higher degree to any other university or institution.

Signed: 
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Date: November 15th, 2002

DEDICATION

To my mother, my brothers and Shiri

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It is a pleasure to acknowledge those who helped me during this study especially my supervisor Dr. Mohammad Abu-Samrah for his great efforts and support, my thanks also to my co-advisor Dr. Saker Darwish for his precious comments.

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ABSTRACT

The Intranuclear Effective Nucleon-Nucleon Interactions in Deformed Nuclides

There is no reason to believe that the mean-field should be spherically symmetric. In order to study other possibilities, the time independent particle model should be extended to include deformed potential by assuming angular dependence of nuclear radius. This can be achieved by introducing some deformed properties of nuclides.

In this study, various static deformations up to $\ell=5$ of nuclides have been included and investigated for ^{16}O and ^{40}Ca deformed nuclides using Brüeckner $\mathfrak{I}$-matrix formalism.

The developed model is tested against the deuteron nucleus and Nilsson model. An agreement within 15% is obtained. In general, we believe that nuclides should have some deformations to some extend and the spherically symmetric has to be considered as a special case from the general case, the deformed case.

Chapter One

Introduction and purpose

1.1 Introduction

The earliest theoretical studies of nuclei were concerned mainly with ground state and the low-energy properties. The liquid-drop model was historically the first model to describe nuclear properties (Weisskopf, 1961). The liquid-drop model provides an understanding of the ground state properties, such as angular momentum and quadrupole moments, as well as low-energy dynamics properties (Eisenberg and Greiner, 1970), such as vibrations that considered as standing waves on the liquid drop surface, rotations and single-particle excitations (Cohen, 1971).

Extensive microscopic nuclear structure calculations with different realistic nucleon–nucleon (NN) (nucleon is the generic name for protons and neutrons) interaction have been performed to produce the global ground state properties of nuclides (Bohr and Mottelson, 1975; Meyerhof, 1967). Conventional non-relativistic microscopic structure calculations for the nuclear many-body systems are based on modeling the nucleus as a collection of nucleons interacting through a NN potential (Krane, 1987). The natural framework for such a study is therefore the ordinary (non-relativistic) quantum many-body theory (Kadanoff and Baym, 1976). The

nucleons are treated as elementary structureless particles so that no internal excitation of a single nucleon is considered (Brown and Jackson, 1975). These models adequately describe the ground state properties (density, size, binding energy, etc.) of nuclei (Burchman, 1963). The shell model and other methods (Nilsson model, alpha cluster model, Fermi gas model, etc.) have also been developed for the same purpose (Jelley, 1990; Bllat and Weisskopf, 1952; Eisenberg and Greiner, 1997; Feynmann, *et al.,* 1965).

The microscopic treatment of nuclear many-body systems is made difficult by the strong NN interaction with a strong short-range repulsion (Wong, 1990). Early effects to reconcile the success of the shell model with this NN interaction were unsuccessful. The breakthrough came with the Brüeckner's reaction matrix formalism (Brüeckner, 1955). This theory was used for extracting out of the "**free**" NN interaction (independent-nucleon interaction) an "**effective**" interaction, that is, a NN interaction in the nuclear medium (Williams, 1991).

By analogy with atomic physics, one approximation is to apply the concept of the independent-particle or the Hartree-Fock (HF) approximation to the nuclei (Mahaux and Sartor, 1991). The nuclear HF method is an approximation for reducing the problem of many interacting particles to non-interacting particles in a mean field (Thouless, 1961; Villars, 1966). Accordingly, the interactions used in the calculations are

effective "quasi-particle" rather than "free" interactions. This has indeed assumed the shell model description of nuclear structure (Lawson, 1980; Talmi, 1984).

The independent-particle approximation (where the HF theory is used) consists of finding the solution of the Schrödinger equation governing the dynamics of the system in a restriction of the many-body Hilbert space to variationally determined single Slater determinant. Solving the Schrödinger equation for such an approximation is equivalent to searching for a Slater determinant (of a single-particle eigenfunction if the system wave function is antisymmetric), which yields a minimal expectation value for the effective Hamiltonian (Schiff, 1968). This model was modified to include the rearrangement potential by means of the effective interaction density dependence. The modified model is called the Density-Dependent Hartree-Fock (DDHF) model (Seimens and Jensen, 1987). It has been shown that one can indeed extract out of the free NN interaction an effective interaction giving a fair amount of the static properties of doubly magic nuclei such as binding energies and spatial nuclear densities (Preston and Bhaduri, 1975; Pandharipande and Wiringa, 1979).

The one-boson exchange potential (OBEP) model (Yukawa, 1935), or the meson-exchange model, is based on the hypothesis that the NN potential is meson mediated and the exchange particles are adequately

represented experimentally observed meson and meson resonances. The idea is due to Yukawa (1935), but we will follow a more development similar to that of Brown and Jackson (1975). Following the original Yukawa hypothesis, the range of the NN interaction is divided into classical long-range ($r > 1.5$ fm), dynamical medium-range ($0.5 < r < 1.5$ fm), and core short range regions ($r < 0.5$ fm). The pions play a special role in the forces between nucleons: The π- exchange yields the long-range part of the NN potential, the ρ- and ω- exchange are (Halzen and Martin, 1984) responsible for the short-range repulsion, the η- and ξ-exchange describe some fine structure (Iachello and Isacker, 1991). The forces calculated using the OBEP are found to depend not only on spin, isospin, and the distance between nucleons, but also on the nucleons relative momentum (spin-orbit interaction) (Jonson, 1995). When the momentum dependence is taken into account, the OBEP model is able to account for all the data on the scattering of nucleons up to energies where meson production becomes the dominant process (about 600 MeV laboratory energy) (Perey and Perey, 1976).

Major efforts to determine the parameters of the meson-exchange model by fitting two-nucleon data have produced phenomenological forces known as the "Paris" and "Bonn" potentials (Holinde, 1981). These potentials are sort of effective interactions like the OBEP.

Unfortunately, these forces are so complicated that they seldom been used in computations of nuclear structure. A similar and therefore more popular approximation for the phenomenological force is the static potential developed by Reid in the 1960's (Reid, 1968). This static potential depends only on spin, isospin, orbital angular momentum, and the radial separation distance between the nucleons (Negele, 1982; Rose, 1957). It was obtained by fitting two-nucleon scattering data to the phase-shifts and tensor coupling parameters obtained from Schrödinger equation, in which the potential is taken as the sum of terms of OBEP. The phase-shifts, and therefore the Reid potential, are well determined for orbital momentum $L \leq 2$, but information for higher partial waves are incomplete.

All models discussed above presumed a spherically symmetric potential. In general there is no reason to believe that the HF mean-field should be spherically symmetric. The mean-field, in many cases, deviate from spherical shape nuclides (deformed nuclides) (Nemeth and Ripka, 1972). For nuclides, which deviate from spherical shape, nuclear energy correction change in a specific way. In order to study this possibility, the independent-particle model may be generalized to have a deformed potential. The importance of nuclear deformation in many-body systems has been recognized for a long time (Bohr and Mottelson, 1975). In view of the intermediate number of degrees of freedom of a nucleus, the co-

existence and the interplay of both the individual-particle and the collective degrees of freedom in the nucleus is important (Maltman and Isgur, 1983). An exact microscopic quantum treatment of the many-body nuclear systems even as a system of structureless nucleons is not yet possible. Some approximations are thus necessary. For example, The Nilsson model (Nilsson, 1955) was developed to study the ground state properties of deformed nuclides. In the original Nilsson parametrization, a ℓ^2 term has the effect of making the potential well more like a deformed square well, as it makes higher ℓ-states have alower energy, which is a better approximation to the nucleon potential well (Jelley, 1990).

The extension of nuclear models to include the concept of shape, shape changes and rotations of shapes resulted from the single-nucleon motion is made possible by the "collective models" (Jelley, 1990). The clearest development is made possible using Nilsson potential, which gives rise to all of the salient features characteristic of deformed single-nucleon motion. It is also pointed out how the total energy of static deformed nuclides can be evaluated. Rainwater 1976 suggested that the motion of the single-particle deforms the whole nucleus and the observed quadrupole moment results from collective distortion of many orbits (Rainwater, 1976). Moreover, the nucleus can change its shape when rotating or vibrating (Simpson and Hime, 1989). This is known as the collective model and it was developed by Bohr and Mottelson since 1952.

The measure of deviation from a spherical shape can be expressed in terms of certain parameters called deformation parameters as in Nilsson Model (1955). Collective models were introduced to explain the excited states in nuclides can not simply described in terms of single-particle motion in a potential well (Liberman, 1977). The explanation for any of these excited states can be performed using the simple collective rotational and vibrational motions of all nucleons inside the nucleus. For deformed nuclides, vibrations about the deformed surface are important to describe the excited states of the nucleus (Jelley, 1990). The lowest excited states of nuclei with spherical ground states are vibrational collective states at excitation energy of a few MeV, described by the random-phase approximation (RPA) picture as coherent oscillations of the nuclear shape around its equilibrium (spherical shape) (Bhadhuri, 1988). Deformed nuclides have also vibrational states with energy comparable to these of spherical nuclei with nearby masses. Each deformed nucleus has a set of excited states with energies much less than 1MeV (Anastasio, Celenza, Pong and Shakin , 1983). The simple quantum mechanical picture of rotations is based on the linear response theory (Bertsch, Esbensen and Sustich, 1990).

The nuclear mean-field picture is capable of describing important features of both small and large deformation of nuclear shape. The collective vibrations of nuclides are introduced in a linear-response

picture (LRM) by allowing the mean-field to depend on time, leading to the RPA eigenvalue equation for frequencies of collective modes (Seimens and Jensen, 1987). Rotational motion is shown to require a viewpoint more sophisticated than the mean-field, because of its intimate relation to breaking of rotational symmetry. A better understanding of these features could be obtained on the mean-field basis by allowing the model to take into account the effective interactions that include both of the short-range correlation and the many-body effects (Detraz, 1995; Fesbach, 1995).

The simplified approximation of independent-particle model and the liquid-drop model have been developed for the same purpose. One way to unify the picture is to proceed with direct computations of DDHF model. While this model as described has solution of stable deformations, it can be extended by introducing an external one-body field adjusted to make other shapes stable. This procedure leads to some surprising results among these stable shapes of large deformations that can not be predicted by either the liquid-drop or independent-particle models (Brack, 1980).

While the independent-particle picture has insufficient treatment of self-consistency, the liquid-drop model is not good enough either. The energy differences responsible for ground state deformations in the independent-particle model are quite small, and depend on the details of which single-particle orbits are occupied. Indeed, simply by moving

particles among degenerate states in the spherical nucleus, we can obtain "oblate" or "prolate" deformation with almost the same energy. Typical examples of such deformations are shown in Figure 1.1.

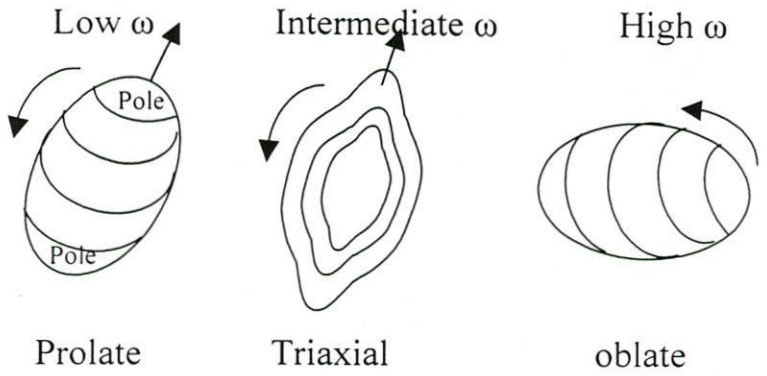


Figure 1.1. Typical nuclear deformation. (a) Prolate. (b) Triaxial. (c) Oblate

A nucleus can change its shape when rotating. At low spin (frequency), the nucleus has a prolate shape relative to the symmetry axis. At intermediate spin the rotation forces some matter into the rotating plane, but now the axis is no longer a symmetry axis and the nucleus becomes triaxial. At high spins, the matter is distributed in the plane of rotation axis and the nucleus flattens into an "oblate" shape. It is the latter set of shapes that was observed in ^{158}Er by Simpson and Hime (1989).

The liquid-drop model, like the Fermi-gas model, does not take account of these details. Thus we need a better understanding of how the deformed potential originates in the consistent mean-field. Strutinski (1968) has developed a method for calculating the deviations from the liquid–drop energy, using the mean-field picture (Strutinski, 1968).

The Strutinski method allows us to estimate the energy of the nucleus as a function of deformation, without solving the self-consistent mean-field equations. Instead, we only need a good approximation to the "smoothed" mean-field and its eigenvalues, as well as a good estimate of the liquid-drop energy. Strutinski method can also be applied to the case of density-dependent effective interactions. In this case, the rearrangement terms in the single-particle potential lead to corrections to $\delta\varepsilon$ which are the same order as $\delta\varepsilon$. Strutinski's approximation method has been tested by means of the DDHF method and gives the deviations from the liquid-drop within about 20% (Siemens and Jensen, 1987). The approximation would be almost exact in ordinary HF theory.

Nuclear spectra contains excited states in nuclei which are not simply described in terms of single-nucleon motion in a spherical or a deformed potential well. There are relatively simple motions of all the nucleons which do provide an explanation for many of these excited states; these are collective rotational and vibrational motoins. The cranking model was introduced to describe slow collective motion of nuclei where the adiabatic approximation was used (Seimens and Jensen, 1987). Unfortunately, this model is neither gives a fully satisfactory account of the observed rotational spectra nor rotational states from rotations about the mean field's symmetry axis were detected. Furthermore, this model can not explain the absence of odd values of

total angular momentum **J** for spin-zero (even-even) nuclides. Nor does it give any clue to understand the more complicated rotational spectra of odd nuclei, whose ground state spins are different from zero. The failure of the cranking model is due to its excessive reliance on the mean-field description, in which the ground state breaks the rotational symmetry of the Hamiltonian. To understand ground state angular momentum, as well as rotational motion, we have to go beyond the mean-field picture.

Thus far, all deformed nuclides investigations were performed on quadrupole (Collective models) and monopole (Nilsson model) deformations using free NN cross sections (Rowe, 1970). Non of them reproduced the correct empirical root-mean square radius and the empirical binding energy simultaneously. The calculated results show a discrepancy of 15% between theoretical and experimental results (Hofmann and Jensen, 1984). We expect higher deformations play a certain role in reducing the difference between the experimental and the theoretical NN interaction energies results. In this study, we shall consider various static deformations up to ℓ=5, of nuclides and try to describe and develop some effective interaction results from deformed nuclei, motivating them with the simple deformed mean-field picture. Thus, we expect to understand what happens when two nucleons scatter from each other inside deformed nuclides.

Two major improvements for the NN interaction are still possible. On the one hand one may argue that one may yet find an NN potential which gives the right saturation density and binding energy. On the other hand, one may argue that we must extend OBEP of nuclei to include dynamical of nucleon excitation. Here our motive is to investigate the first point view by including the nucleus deformation in the OBEP model.

In the present study, we advocate Brüeckner $\mathfrak{I}$-matrix formalism. This choice is motivated for several reasons. Firstly, the $\mathfrak{I}$-matrix considers deformed states leads to a rotational band, similar to that of the ground state, but where the lowest value of **J** is not zero. Secondly, the excitation energies of these states are displaced from the ground state energies by the excitation energy of the mean-field excited state above the mean-field ground state. Thirdly, the effective interaction between nucleons can be developed upon replacing the scattering matrix by the in-medium scattering cross section to take into account the nuclear matter effect. The $\mathfrak{I}$-matrix formalism in which scattering amplitude depends explicitly on energy shall be used for this purpose.

1.2 Statement of the problem

To describe the bound nuclides (a collection of A strongly interacting nucleons) in terms of the NN interaction, one has to make use of the quantum mechanical methods that govern both the bound ($E < 0$) and unbound ($E > 0$) nuclear regime.

We shall consider a nuclear system of A-nucleons (nucleon mass = m, spin s = ½, isospin τ = ½, each) at zero temperature where the Coulomb interaction has been switched off. First, if we assume a fixed number of nucleons A, then the total Hamiltonian operator of the system is approximated by the sum of terms for each particle individually (Ring and Schuck, 1980):

$$H = \sum_{i=1}^{A} \frac{P_i^2}{2m_N} + U(r_i, s_i, \tau_i) \qquad (1.1)$$

Here P_i is the single-nucleon momentum operator, m_N is the nucleon mass, and $U(r_i, s_i, \tau_i)$ is the single-nucleon potential. The single-nucleon potential is related to the NN interaction V_{ij}^{NN} through (Siemens and Jensen, 1987)

$$U(r_i, s_i, \tau_i) = \sum_{i \neq j}^{A} V_{ij}^{NN} \qquad (1.2)$$

Therefore, the Hamiltonian that contains the two-body interaction can be written as (Dobaczewski and Nazarewics, 1995):

$$H = \sum_i \frac{P_i^2}{2m_N} + \frac{1}{2} \sum_{i \neq j} V_{ij}^{NN} \qquad (1.3)$$

The NN interaction is generally of a short range attraction for r>0.5 fm, and for r<0.5 fm it has a repulsive core part as illustrated in Figure 1.2.

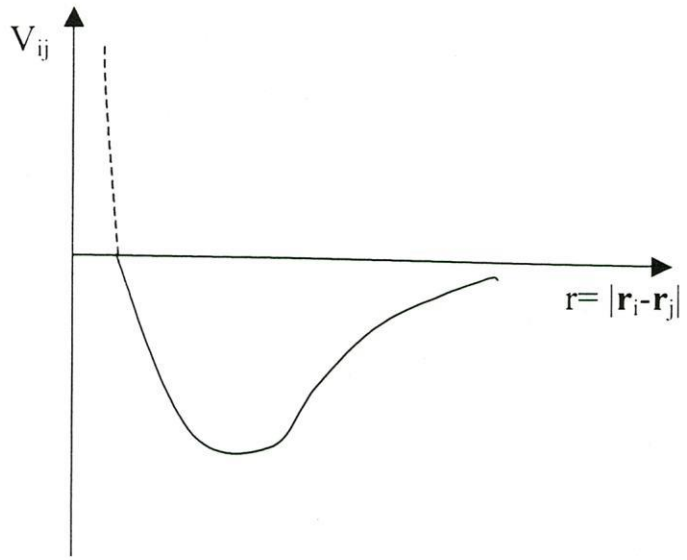


Figure 1.2. Illustration of typical NN two-body interaction $V\,|\mathbf{r}_i-\mathbf{r}_j|$

Second, we consider a full dynamical treatment by introducing the isospin operator τ_{op}, as the projection operator in the $\tau = \frac{1}{2}$ isospin space as follows (Mar'i, 1987):

$$\tau_{op}^{\frac{1}{2}}|\tau\rangle = \left|\frac{1}{2}\right\rangle = 1 \qquad (1.4)$$

will produce a single nucleon. The average value of the isospin operator over any normalized state $|\alpha\rangle$ will produce the total number of nucleons, A. Thus

$$\langle\alpha|\sum_{i=1}^{A}(\tau_{op}^{\frac{1}{2}})_i|\alpha\rangle = A \qquad (1.5)$$

The eigenvalue equation for any state $|\alpha\rangle$ can be written as:

$$H|\alpha\rangle = E_\alpha|\alpha\rangle, \qquad (\alpha = 1,2,3\text{----------}\infty) \qquad (1.6)$$

In order to construct the effective interaction between nucleons, consider the following equation for the $\mathfrak{I}$-matrix (Ring and Schuck, 1980)

$$\mathfrak{I}_E = V + \lim_{\varepsilon \to 0} V \frac{1}{E - H_o + i\varepsilon} \mathfrak{I}_E \tag{1.7}$$

Since we are trying to describe the scattering of two nucleons by their interaction V, we ought to choose H_0 so it describes their motion if they don not scatter. Thus we will expect to contain something like the mean-field potential $V(\vec{r})$. We will not specify precisely how to compute it until later, for now we will suppose that H_0 is a known one-body operator

$$H_0 = \sum_{i=1} H_0(\vec{r}_i, \vec{s}_i, \vec{\tau}_i) \tag{1.8}$$

and introduce a basis of its single-particle eigenstates with wave function in coordinates, spin, isospin (Flügge, 1974)

$$\langle \vec{r} S_z \tau_z | \alpha \rangle = \Phi_\alpha(\vec{r}) \chi_\alpha(S_z \tau_Z) \tag{1.9}$$

and eigenvalues

$$H_0 |\alpha\rangle = \varepsilon_\alpha |\alpha\rangle \tag{1.10}$$

For the two-body states, analogous to the scattering states we take product states $|\alpha\rangle|\beta\rangle$ as a basis (they are not anti-symmetric, but we can always find the matrix elements of anti-symmetric states if we know the product state matrix). The Green function $(E - H_o + i\varepsilon)^{-1}$ of equation

(1.7) is diagonal in this basis, its matrix element being (Beausang and Simpson, 1996)

$$\langle\alpha\beta|\frac{1}{E-H_o+i\varepsilon}|\gamma\delta\rangle=\delta_{\alpha\gamma}\delta_{\beta\delta}(E-\varepsilon_\alpha-\varepsilon_\beta+i\varepsilon))^{-1} \qquad (1.11)$$

Thus equation (1. 7) becomes, in this basis

$$\langle\alpha\beta|\Im_E|\gamma\delta\rangle=\langle\alpha\beta|V|\gamma\delta\rangle+\sum_{\mu\nu}\langle\alpha\beta|V|\mu\nu\rangle\frac{1}{E-\varepsilon_\mu-\varepsilon_\nu}\langle\mu\nu|\Im_E|\gamma\delta\rangle \qquad (1.12)$$

In the basis of two-body states, we can see how to introduce the Pauli principle in the two-body scattering: by requiring that the intermediate states $|\mu\rangle$ and $|\nu\rangle$ be unoccupied in the nucleus. Formally, we write (Breit, 1962; Bonche, Levit and Vautherin, 1984)

$$\langle\alpha\beta|\Im_E|\gamma\delta\rangle=\langle\alpha\beta|V|\gamma\delta\rangle+\sum_{\mu\nu}\langle\alpha\beta|V|\mu\nu\rangle\frac{\langle\mu\nu|Q|\mu\nu\rangle}{E-\varepsilon_\mu-\varepsilon_\nu}\langle\mu\nu|\Im_E|\gamma\delta\rangle \qquad (1.13)$$

Equation (1.13) is the well known Brüeckner equation (Brüeckner, Locket and Rotenberg, 1961) which takes into account of the Pauli exclusion principle in nuclei through the Pauli operator Q. The Pauli operator Q is given in terms of the occupation probabilities n_μ and n_ν of states $|\mu\rangle$ and $|\nu\rangle$ by (Berne, 1977):

$$\langle\mu\nu|Q|\mu'\nu'\rangle=\delta_{\mu\mu'}\delta_{\nu\nu'}(1-n_\mu)(1-n_\nu) \qquad (1.14)$$

In a nucleus ground state, $n_\mu=1$ if the state $|\mu\rangle$ is occupied, and $n_\mu=0$ if it is unoccupied. Thus H_0 can be evaluated using the $\Im$–matrix

interaction instead of the bare two-nucleon interaction V. In our basis, we have

$$\langle\alpha|H_0|\beta\rangle = \langle\alpha|\frac{P^2}{2m_N}|\beta\rangle + \sum_{\gamma=1}^{A}[\langle\alpha\gamma|\Im_E|\beta\gamma\rangle - \langle\alpha\gamma|\Im_E|\gamma\beta\rangle] \qquad (1.15)$$

The set of equations (1. 13) and (1. 15) are known as the Brüeckner Hartree-Fock (BHF) equations.

We approximate the effective non-core Hamiltonian with V_{eff}^{NN} replacing V_{ij}^{NN} in H by

$$H_{eff} = T_{rel} + V_{eff}^{NN} \qquad (1.16)$$

where V_{eff}^{NN} is the sum of the Brüeckner $\Im$-matrix and T_{rel} is the relative kinetic energy operator given by

$$(T_{rel})_{ij} = \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} \qquad (1.17)$$

The underlying NN interaction is the OBEP potential. For evaluating matrix element of H_{eff} we choose the harmonic oscillator basis with $\hbar\omega$ = 14 MeV in order to compare results to be obtained in this study and other results given elsewhere, for example as in (Cohen, 1971; Jelley, 1990; Heyde, 1999).

Chapter Two

General properties of NN potential in deformed nuclides

2.1 Introduction

The nuclear force between any two-nucleons is assumed to hold neutrons and protons inside the nucleus despite the Coulomb repulsion of protons (de Shalit and Feshbach, 1974; Jelley, 1990). Some aspects of this force are still incompletely understood, but several qualitative features can be described (Krane, 1987; Cohen, 1971). Normally discussion of the force $F(\vec{r})$, is in terms of the related interaction potential $V(\vec{r})$, where $\vec{r}$ is the vector separation between two nucleons and where force and potential are related by the usual expression $\vec{F}(\vec{r}) = -\vec{\nabla}V(\vec{r})$.

There are several models that have been proposed and developed to understand the nature of nuclear force between nucleons among these the quantum model, the independent particle model, the mean-field model or the Brüeckner model (Bhadhuri, 1988). According to the mean-field model, each nucleon moves in a field created by all other nucleons (Heyde, 1991). This mean-field is assumed to be the same for all the various nucleons and it determines the motion of the nucleons, which in

turn determines the field. The mean-field approximation breaks translational, rotational symmetries and Galilian invariance, giving a degenerate set of wave functions with different center-of-mass coordinates (Ring and Schuck, 1980).

The NN potential was developed for spherically symmetric nuclear systems and found to be a complicated one and turns out to have several contributions such as the central term, the spin-spin term, spin-orbit term in addition to other terms that have been already introduced to include isospin interactions. This conclusion is drawn from experiments using polarized nucleons (i.e, with their spins oriented in a particular direction). Several forms of NN potential such as Yukawa potential, harmonic oscillator potential, Woods-Saxon potential (Mayer, 1949), Hamada Johnston potential (Hamada and Johnston, 1962), Reid potentials (Reid, 1968) and Skyrme potentials (Skyrme, 1956) have been developed and introduced to gain a full picture about the NN general behavior and nuclear structure. The Nilsson model was developed to include a deformation type corresponding to the spin-orbit interaction.

There is no reason to believe that the mean-field should be spherically symmetric. In order to study other possibilities, the time independent particle model should be extended to include deformed potential by assuming angular dependence of nuclear radius (Bohr and Mottelson, 1975).

In this chapter we shall extend previous investigations to include other nuclides deformations in a sufficient depth to extract the main features of the nuclear potential. Some information can be obtained from simple qualitative considerations, but generally it is necessary to carry out very detailed theoretical analysis of experimental data on the two nucleons system.

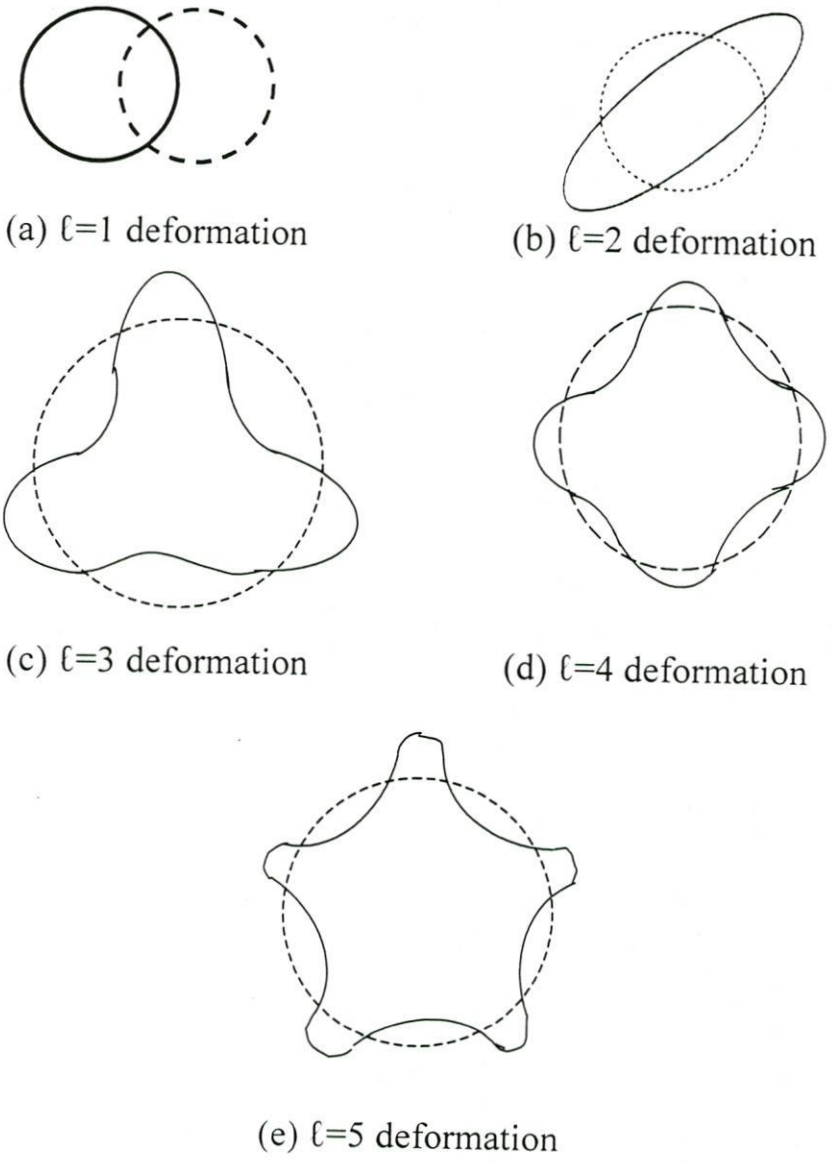
2.2 Deformed nuclides

Before proceeding in developing the NN interaction of complex deformed nuclides composed of many neutrons and protons, it is of great importance to introduce the deformation concept. We shall consider an arbitrary deformation described by $R(\theta,\varphi)$ of the surface of the nucleus. The quantity $R(\theta,\varphi)$ is the distance from the center of the nucleus to its surface in direction of polar angle θ, measured in laboratory frame. If deformation effects are allowed, the angular dependence of the deformed nuclear surface can be described by (Bohr and Mottelson, 1975):

$$R = R(\theta,\varphi) = R_0[1 + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta,\varphi)] \qquad (2.1)$$

where $\alpha_{\ell,m}$ are the deformation parameters characterizing the shape of the deformed surface and $Y_{\ell,m}(\theta,\varphi)$ are the spherical harmonics of order ℓ. The expansion in equation (2.1) allows us to describe very general shapes

as illustrated in Figure 2.1 for nuclear deformations corresponding to $m_\ell=0$ and $\ell=1$, 2,3, 4, and 5.



(a) $\ell=1$ deformation

(b) $\ell=2$ deformation

(c) $\ell=3$ deformation

(d) $\ell=4$ deformation

(e) $\ell=5$ deformation

Figure 2.1. Illustration of some nuclear shape changes corresponding to (a) dipole deformation ($\ell=1$); (b) quadrupole deformation ($\ell=2$); (c) octupole deformation ($\ell=3$); (d) hexadecapole ($\ell=4$); and (e) fifth pole deformation ($\ell=5$).

If the deformation parameters $\alpha_{\ell,m}$ are allowed to vary completely without constraints, the resulting average nucleon density could depend significantly on deformation (Burcham, 1963). This is not allowed, if we want to approximate calculations with a saturating nuclear force, which leads to almost the same nuclear density for all nuclei (Cohen, 1971). This requires a constant nuclear volume independent of deformation and a volume conservation of the equipotential surface corresponding to the nuclear surface. Usually this surface potential is approximated by $U(\vec{r}) = \frac{1}{2} U_o$ (where U_o=(-51+33(N+Z)/A) MeV) for $|\vec{r}| = R(\theta, \varphi)$. The condition would be different if another surface has been chosen (Seimens and Jensen, 1987).

Another problem one has to face is that the nuclear center-of-mass becomes a function of deformation. To fix the position of the center-of-mass, another constraint on the deformation parameters is needed. In the case of axial symmetry, all $\alpha_{\ell m_\ell}$ vanish except for $m_\ell = 0$, i.e, $\alpha_{\ell m_\ell} = \delta_{0 m_\ell} \alpha_{\ell 0}$. Hence, there are two constraints can easily be shown to determine $\alpha_{\ell m}$ (Siemens and Jensen, 1987)

$$\alpha_{00} = -\frac{1}{\sqrt{4\pi}} \sum_{\ell} \alpha_{\ell 0}^2 \qquad (2.2)$$

where

$$\alpha_{\ell 0} = -\frac{3}{2}\sqrt{\frac{3}{4}}\sum_{\ell=2}\frac{(\ell+1)\alpha_{\ell 0}\alpha_{\ell+1.0}}{(2\ell+1)(2\ell+3)} \qquad (2.3)$$

The second order of $\alpha_{\ell 0}$ in equation (2.2) includes the volume conservation condition; while equation (2.3) ensures that the center-of-mass remains at r=0 for all deformations.

With these constraints in mind, let us determine the surface area of a deformed nucleus. When the nucleus assumes a deformed shape, its surface area must be larger, since the sphere has the smallest area for a given volume (equation (2.6)). The surface area element of the deformed nuclear shapes is given by (Arfken, 1997):

$$dS = R\left[R^2 + \left(\frac{dR}{d\theta}\right)^2\right]^{\frac{1}{2}} \sin\theta\, d\theta\, d\varphi \qquad (2.4)$$

The total surface area can be obtained by integrating equation (2.4) over a complete solid angle. The result can be written as follows:

$$S = 2\pi\int_0^{\pi} R^2(\theta)\sin\theta\sqrt{1+\frac{1}{R^2}\left(\frac{dR}{d\theta}\right)^2}\, d\theta \qquad (2.5)$$

Which to a second order in the deformation parameters $\alpha_{\ell m}$ results in (Bohr and Mottelson, 1975)

$$S = 4\pi\ R^2\left[1+\frac{1}{8\pi}\sum_{\ell=1}(\ell-1)(\ell+2)\alpha_{\ell 0}^2\right] \qquad (2.6)$$

The deformation parameters can have any value and we shall take small deformation values only in the present study.

2.3 General properties of nuclear forces

The study of the two nucleons system has told us a lot about the force between nucleons. In general, its range is short, (~1 fm), it is attractive and strong, producing potential energy up to 100 MeV, it is repulsive at still shorter distances, less than 0.6 fm, and it depends on the nucleons spins and isospin (Cohen, 1971). Most of this information is obtained from studying the scattering (continuum) states of the two-nucleon system, rather than its bound state (Merzbacher, 1996; Villars, 1966).

2.3.1 General properties of the NN force and potential

The nuclear shell model (or single-particle model) assumes that nucleons mainly move independently from each other in an average field with a long mean-free path. The basic non-relativistic picture that one has to start from, however, is that one where A nucleons are moving in the nucleus with given kinetic energy $\frac{P_i^2}{2m_i}$ and interacting with the two-nucleon force $V(i,j)$.

In attempting to describe bound nuclei in terms of the NN interaction, one has to make constant use of quantum concepts that govern both the bound ($E<0$) and unbound ($E>0$) nuclear regime. The

starting point is the nuclear A-body Hamiltonian (Ring and Schuck, 1980)

$$\hat{H} = \sum_{i=1}^{A} \frac{P_i^2}{2m_i} + \sum_{j>i}^{A} V(\vec{r}_i, \vec{r}_j) \tag{2.7}$$

And the one-body phase residual interaction Hamiltonian (Siemens and Jensen, 1987)

$$\hat{H} = \sum_{i=1}^{A} \left(\frac{P_i^2}{2m_i} + U(\vec{r}_i) \right) + \hat{H}_{res} \tag{2.8}$$

Evaluating the above interaction is one of the tasks in understanding bound nuclear structure physics. If, as in many cases, the residual interaction $\hat{H}_{res}$ can be left out initially, an independent-particle (nucleon) motion shows up and it is quite well verified experimentally. If we consider the nucleus to consist of a number of separated point particles at positions $\vec{r}_i$ interacting through the two bodies potential $V(\vec{r}_i, \vec{r}_j)$, then a total potential acting on a given nucleon at $\vec{r}_i$ is obtained by summing the various contributions, that is, (Brown and Jackson, 1975)

$$U(\vec{r}_i) = \sum_j V(\vec{r}_i, \vec{r}_j) \tag{2.9}$$

The general properties of NN force in equation (2.9) are investigated and the general behaviors are summarized through the following points (Cohen, 1971):

1. It contains a central term, attractive in S-states, short range. That is, the interaction potential energy between a pair of nucleons is characterized by a short range and large amplitudes. For higher angular momenta values, some tensor features may appear this implies a certain deformation, such as a quadrupole deformation.
2. It is spin dependent. This property can be followed through the study of the singlet ($1S_0$) and triplet ($3S_1$) states of the deuteron.
3. It contains spin-orbit interaction terms (polarization experiment), which implies polarization is not equal to zero.
4. It is velocity dependent, because of change in phase shifts with energy. Accordingly, the nuclear force is not completely attractive but exhibits some repulsion.
5. It is an exchange force, the force mechanism is exchanging a particle (or more) between the interacting nucleons. This exchanged particle called the field particle or the meson.
6. It is charge symmetric and charge independent force, this implies that $F_{np}=F_{pn}$.
7. It is isospin dependent, this concept was introduced to distinguish between a proton and a neutron inside a nucleus. This is because in strong interactions neutrons and protons might as well be the same particles.

8. It is invariant under time reversal operator, since $\vec{L}.\vec{P}_{12}$ and $\vec{S}$ change sign under time reversal, while $\vec{r}_{12}$ does not.

We shall use these properties to introduce and develop each term of the NN potential in deformed nuclides. In the following section, we shall introduce the various terms that form the nuclear potential of deformed nuclides.

2.3.2 Detailed form of the NN potential of deformed nuclides

2.3.2.1 The central part of the potential:

The earliest effort to figure out the interaction between nucleons was made possible by Yukawa. In 1935, Yukawa suggested that an interaction of finite range could be generated if particles (mesons) of finite mass were exchanged between nucleons as shown in Figure 2.2. A typical form of Yukawa NN potential can be written as (Yukawa, 1935):

$$V(r) = \left(\frac{g^2}{4\pi}\right)\frac{e^{-\lambda r}}{r} \qquad (2.10)$$

With $r = \left|\vec{r}_i - \vec{r}_j\right|$, and g is referred to as a coupling constant since it measures the strength of the coupling of the meson to the nucleon (Krane, 1987). The estimated value of $\frac{g^2}{4\pi} \cong 10^{-26}$ J.m for a potential depth of ~30 MeV (Krane, 1987). The parameter λ is a measure of the range of the

interaction between the nucleons $\left(\lambda \approx \frac{m_\pi c}{\hbar} \approx 0.71\,\text{fm}^{-1}\right)$, where m_π is the pion mass, due to meson exchange (Cohen, 1971).

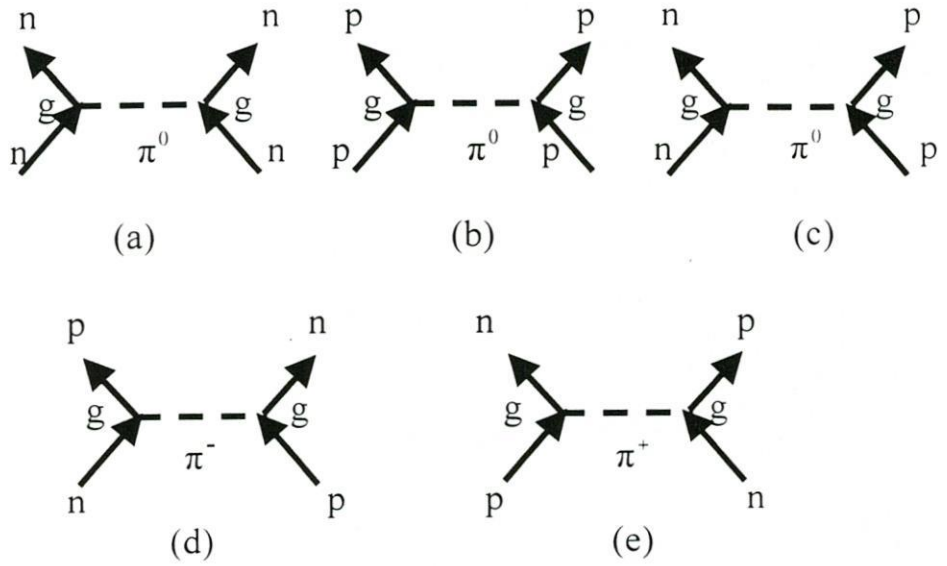


Figure 2.2 Illustration of pion exchange between two nucleons system. (a), (b), and (c) involve π^0 whilst (d) and (e) involve $\pi^{\pm}$ exchange leading to exchange scattering.

In order to develop a NN interaction in deformed nuclides, we shall consider a system consists of two nucleons within the deformed nuclear surface. If the coordinates of one nucleon are denoted $(r_1, \theta_1, \varphi_1)$ and the coordinates of the other are denoted $(r_2, \theta_2, \varphi_2)$ and $\vec{r}_{12}$ is the distance between them as shown in Figure 2.3, then the total NN potential of the ith nucleon inside the nucleus, according to Yukawa theory, can be written as:

$$V(r_i) = \left(\frac{g^2}{4\pi}\right)\sum_j \frac{e^{-\lambda\left|\vec{r}_i - \vec{r}_j\right|}}{\left|\vec{r}_i - \vec{r}_j\right|} = \left(\frac{g^2}{4\pi}\right)\sum_j \frac{e^{-\lambda\left|\vec{r}_{ij}\right|}}{\left|\vec{r}_{ij}\right|} \qquad (2.11)$$

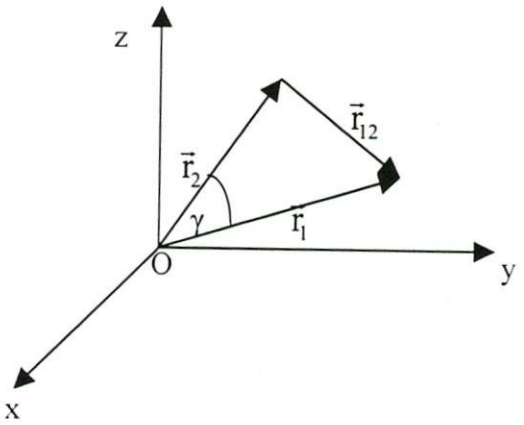


Figure 2.3 Coordinate representation of two nucleons system.

where

$$r_{12} = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\gamma} \qquad (2.12)$$

is the distance between nucleons 1 and 2. The central interaction term according to equation (2.11) decreases with increasing r_{12}. Therefore, for $r_1 > r_2$, this distance can be expanded using the addition theorem of spherical harmonics as (Arfken, 1997):

$$\frac{1}{\left|\vec{r}_{12}\right|} = \frac{1}{r_1}\sum_{\ell=0}^{\infty} P_\ell(\cos\gamma)\left(\frac{r_2}{r_1}\right)^\ell \qquad (2.13)$$

where

$$P_{\ell}(\cos\gamma) = \left(\frac{4\pi}{2\ell+1}\right)\sum_{m=-\ell}^{\ell}(-1)Y_{\ell,m}(\theta_1,\varphi_1)Y^{*}{}_{\ell,-m}(\theta_2,\varphi_2) \qquad (2.14)$$

Or equivalently

$$P_{\ell}(\cos\gamma) = \left(\frac{4\pi}{2\ell+1}\right)\sum_{m=-\ell}^{\ell}Y_{\ell,m}(\theta_1,\varphi_1)Y^{*}_{\ell,m}(\theta_2,\varphi_2) \qquad (2.15)$$

In equation (2.14) (θ_1,φ_1) and (θ_2,φ_2) denote the different directions for the nucleons in spherical coordinate systems, separated by an angle γ Figure 2.3. These angles satisfy the trigonometric identity (Arfken, 1997).

$$\cos\gamma = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2) \qquad (2.16)$$

Substituting equations (2.12), (2.13), (2.14) and (2.16) into equation (2.11), we get the Yukawa-type NN interaction for deformed nuclides. Thus,

$$V = \left(\frac{g^2}{2\ell+1}\right)\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}(-1)^{\ell}Y_{\ell,m}(\theta_1,\varphi_1)Y_{\ell,-m}(\theta_2,\varphi_2)\left(\frac{r_2}{r_1}\right)^{\ell}\times$$

$$e^{-\lambda\left\{r_1^2 + r_2^2 - 2r_1r_2[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2)\right\}^{\frac{1}{2}}} \qquad (2.17)$$

Equation (2.17) can be reduced to the well-known Yukawa potential form when θ_1 and $\theta_2 = 0$ and $\varphi_1 - \varphi_2 = \frac{\pi}{2}$.

2.3.2.2 The spin-spin residual interaction

One of the conclusions that can be drawn from the general properties of the nuclear forces is that the NN potential must be spin-dependent as mentioned in section 2.2. This conclusion is originated from the spin orientation of protons and neutrons in the nucleus, when analyzing the ground state of the deuteron. According to quantum theory, neutrons and protons possesses an intrinsic angular momentum, or spin, equal to $\frac{\hbar}{2}$ (Messiah, 1962; Gasiorowicz, 1996). Knowing that $\vec{s}_1.\vec{s}_2$ having eigenvalues of $\frac{\hbar^2}{4}$ and $-\frac{3\hbar^2}{4}$ for $S=1$ and $S=0$, respectively, different strengths for singlet and triplet states are expected. This will results in different energies for different spin-state symmetry. Clearly, there is a difference in strength and range of the triplet and singlet potentials (Flügge, 1974). This means that any mathematical expression for the interaction potential must also involve the spin operators $\vec{s}_1$ and $\vec{s}_2$ (or $\vec{\sigma}_1$ and $\vec{\sigma}_2$; where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli matrices of nucleon 1 and 2, respectively) for the two nucleons. Besides, it may be resulted in a repulsive potential correction in states with all spins parallel (triplet state) and corresponds to the observed and doublet states. The spin-spin residual interaction of the two-body interaction may be written as (Brown and Jackson, 1975):

$$V_{res}(|\vec{r}_i - \vec{r}_j|) = V_{res}(r) = \sum_{i \neq j} F_s(r)\vec{s}_i.\vec{s}_j \qquad (2.18)$$

where F_s is a function representing the form and strength of the spin dependent part of the potential which differs according to the spin state that imposed the weakness or strength of the spin part potential. Equation (2.18) can be written in another form as follows (Cohen, 1971):

$$V_{res} = \sum_{i \neq j} g(\vec{r}_i - \vec{r}_j)\vec{\sigma}_i.\vec{\sigma}_j \qquad (2.19)$$

where g is the coupling strength containing constant and radial part strength integral and $\vec{\sigma}_i$ and $\vec{\sigma}_j$ are the Pauli matrices of nucleon i and j, respectively. A constant value of g is of the order 25 MeV (Heyde, 1999).

Assuming that the residual interaction is composed of triplet state potential of parallel spins, $V_t(\vec{r})$, and a singlet state potential of antiparallel spins, $V_s(\vec{r})$, the spin-spin residual interaction can be expressed as (Flügge, 1974):

$$V_{res} = \frac{1}{4}[(3V_t + V_s) + (V_t - V_s)(\vec{\sigma}_n.\vec{\sigma}_p)] \qquad (2.20)$$

where $\vec{\sigma}_n$ and $\vec{\sigma}_P$ are the neutron and the proton spin vectors (Pauli matrices).

2.3.2.3 The tensor potential

It is well known that the nuclear force does not conserve orbital and angular momenta separately, but only their sum (Arima and Iachello,

1984). Besides, the angular momentum can be changed by only a torque. Changing the orbital angular momentum implies that the potential V is a function of θ and not merely a function of $\vec{r}$ (Bohr and Jackson, 1975). The operator that commutes with the total angular momentum $\vec{J}$, is known as the tensor operator. In its simplest form, the tensor operator can be written as (Cohen, 1987; Flügge, 1974):

$$S_{12} = 12[\frac{(\vec{s}_1.\vec{r})(\vec{s}_2.\vec{r})}{r^2}] - 4\vec{s}_1.\vec{s}_2 = \frac{6}{r^2}(\vec{S}.\vec{r})^2 - 2S^2 \qquad (2.21)$$

The operator S_{12} is invariant under spin exchange. In general, an interaction proportional to S_{12} results from the exchange of mesons between nucleons and is called tensor potential. The nuclear potential can be written as the sum of a central and a tensor potential. When done in this way, the tensor potential is some function of r times S_{12}. Thus,

$$V = F_T(r)S_{12} \qquad (2.22)$$

Using equation (2.21) for S_{12}, we get (Brown and Jackson, 1975):

$$V_T = F_T(r)[\frac{3(\vec{S}.\vec{r})^2}{r^2} - \vec{S}.\vec{S}] \qquad (2.23)$$

Here $\vec{S}$ is the spin vector and $F_T(r)$ is a function giving the strength and radial dependence of the potential and V_T is the tensor potential. Equation (2. 23) can be rewritten in a more widely used form in terms of Pauli matrices as (Flügge, 1974):

$$V_T = F_T(r)[(\frac{3}{r^2})(\vec{\sigma}_i.\vec{r})(\vec{\sigma}_j.\vec{r}) - (\vec{\sigma}_i.\vec{\sigma}_j)] \quad (2.24)$$

The operator S_{12} is invariant under spin exchange. Using the addition theorem of spherical harmonics to equation (2.24), the scalar form of equation (2. 22) can be expressed as:

$$V_T = F_T(r)\lfloor\sigma_i\sigma_j(2\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\varphi)\rfloor \quad (2.25)$$

Where θ_1 and θ_2 are the angles of $\vec{s}_1$ and $\vec{s}_2$ (or $\vec{\sigma}_1$ and σ_2) relative to $\vec{r}$, while φ is the azimuth of $\vec{s}_2$ relative to $\vec{s}_1 - \vec{r}$ plane as in Figure 2.4.

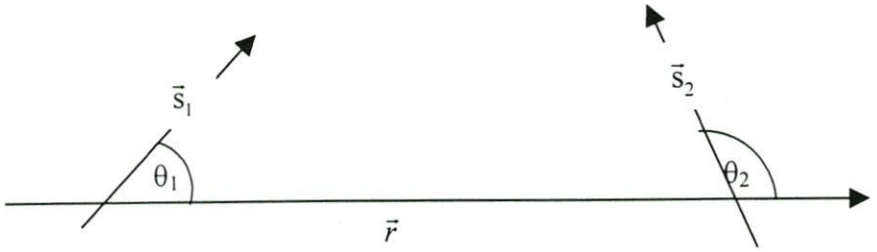


Figure 2.4. Typical representation of general spin orientation for two nucleons.

2.3.2.4 The spin-orbit interaction term

There is nothing to prevent the target and the projectile nucleons from exchanging angular momentum, or even trading spin for orbital angular momentum $\vec{L}$, since the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ has to be conserved. If the orbital motion is supposed to occur in S-state, where

$\vec{L}=0$ and $\vec{J}=\vec{S}$, the momenta S_z and S^2 will be conserved and the forces must be independent of J_z.

Accordingly, the nuclear force does not conserve orbital and spin angular momenta separately, but only their sums. Hence, there must be a component of the nuclear interaction which does not commute with L^2 and S^2 separately, but it does commute with J^2. Nucleons polarized experiments indicate the presence of a spin-orbit potential that has the following form (Krane, 1987)

$$V_{LS}=V_{SO}=F_{LS}(r)\vec{L}.(\vec{s}_1+\vec{s}_2) \tag{2.26}$$

The action of the operator $\vec{L}.\vec{S}$ on the angular dependence in $P(\cos\theta)$ may be found from $\vec{L}=-\vec{P}\times\vec{r}$ expression in momentum representation. A good description of the scattering of nucleons requires spin-dependent forces. The main spin-orbit effect can be obtained by adding a spin-orbit potential term in the Hamiltonian of the form (Jelley, 1990)

$$V_{LS}=\vec{L}.\vec{S}V_{LS}(\vec{r})=c\vec{L}.\vec{S} \tag{2.27}$$

The spin-orbit potential $c\vec{L}.\vec{S}$ does not change the spatial motion of the nucleons in the spherically symmetric oscillator potential. It merely couples the orbital and spin motion to obtain eigenstates of the total angular momentum. In its most general form the spin-orbit potential may be written as (Brown and Jackson, 1975):

$$V_{LS} = G\ \lambda\ [Y(\lambda r)^2[1 + AY(\lambda r)] \qquad (2.28)$$

where $Y(x) = \frac{e^{-x}}{x}$.

2.3.2.5 The NN isospin interaction

While the nuclear forces have strong spin dependence, they have also show a remarkable independence of the charge of the nucleons (Eisenberg and Greiner, 1970). Indeed, it is the spin dependence of the forces that is responsible for the lack of a dineutron bound state, since the only two nucleons bound system (deuteron) is restricted to a certain direction of the two nucleons spins. This interesting phenomenon in the field of nuclear physics, which is generally referred to as exchange, is observed in high energy scattering experiment of high energy neutrons by protons. It is found in such a scattering experiment, that the proton is just as likely to be scattered in the neutron's direction with high energy in a way that can be understood as though the neutron picks up the proton's charge, converting itself into a proton and leaving the original proton behind as a neutron. This phenomenon must also be accounted for in any representation of the interaction potential.

For strong interactions, neutrons and protons might as well be the same particle. The significant difference in the neutron-proton (np) and proton-proton (pp) systems arise from the fermions antisymmetry. A convenient formulation, which exploits the charge independence of the

nuclear force, consists of introducing the concept of isospin, an abstract, inner degree of freedom, somewhat analogous to spin, which each nucleon possesses. The nucleons are in a subspace of this new degree of freedom, a Hilbert space with two basis states analogous to "up" and "down" states of spin $\frac{1}{2}$. We say that the nucleon has isospin $\tau = \frac{1}{2}$. Since the isospin of one nucleon τ has magnitude $\frac{1}{2}$, the total isospin of a nucleus is then

$$T = \sum_{k=1}^{A} \tau_k \qquad (2.29)$$

The third component τ_z, which is the sum of individual contributions of $-\frac{1}{2}$ for neutrons and $+\frac{1}{2}$ for protons is

$$\tau_z = -\frac{1}{2}(N - Z) \qquad (2.30)$$

If the nuclear potential depends on spin coordinates, $\vec{\sigma}_1$ and $\vec{\sigma}_2$, it may also depends as well on the isospin coordinates $\vec{\tau}_1$ and $\vec{\tau}_2$. It is important to remember that $\vec{\tau}$ and $\vec{\sigma}$ are only mathematically similar, and not physically so. Thus, the spin operator $\vec{\sigma}$ has eigenvalues corresponding to up and down in real space, but the space in which the up means proton and the down means neutron, i.e, the isospin space, is a purely mathematical device and in no way related to the space in which we measure distances and spin directions. For that reason, products

between isospin vectors and spin or space vectors, such as $\vec{\sigma}.\vec{\tau}$ and $\vec{r}.\vec{\tau}$ are meaningless. Hence, the NN potential has no such terms. Moreover, since nuclear force between nucleons is charge-independent and therefore independent of the isospin projection T_z but depends on the total isospin T. It might be thought now that the only possible isospin dependent term must be of the form $\vec{\tau}.\vec{\tau}$ (Cohen, 1971). Other possible forms $\tau_{z1}, \tau_{z2}, \tau_{z1} + \tau_{z2}$, where $\vec{T} = \frac{1}{2}(\vec{\tau}_1 + \vec{\tau}_2)$ are due to charge conservations. Generally speaking, all energy terms commute with charge operator, $\frac{1}{2}(1 + \tau_{z1} + 1 + \tau_{z2})$.

The isospin NN interaction in nucleus must have a term J_t, which is proportional to the scalar product of the two-nucleons isospin. Thus,

$$J_t(\vec{r}_{12}, \vec{\tau}_1, \vec{\tau}_2) = \vec{\tau}_1.\vec{\tau}_2 J_t(\vec{r}_{12}) \qquad (2.31)$$

This is a consequence of the requirement that the interaction must be a scalar in isospin space. However, for higher energies the NN charge-exchange effective interaction is predominantly the spin two types (Fesbach, 1995)

$$V_{st}(\vec{r}_{12}, \vec{\tau}_1, \vec{\tau}_2, \vec{s}_1, \vec{s}_2) = (\vec{s}_1.\vec{s}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{st}(\vec{r}_{12}) \qquad (2.32)$$

Where $F(\vec{r}_{12})$ is a function giving the strength and radial dependence of the potential, the above interaction is known as the Gamow-Teller interaction (Brown and Jackson, 1975).

2.4 The general form of nucleon-nucleon interaction

The general form of the two-nucleon NN potential is dictated by general invariance properties (invariance under exchange nucleon coordinates, translational invariance, Galilean invariance, space reflection invariance, time reversal invariance, -------etc.). A general form could be (de Shalit and Feshbach, 1974; Brown and Jackson, 1975)

$$V = V_c(r) + V_\sigma(r)\vec{\sigma}_1.\vec{\sigma}_2 + V_T(r)(\vec{\tau}_1.\vec{\tau}_2) + V_{\sigma T}(r)(\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)$$
$$+ V_{tensor}(r)S_{12} + V_{tensor}(r)S_{12}(\vec{\tau}_1.\vec{\tau}_2) + V_{LS}(r)\vec{L}.\vec{S} + V_Q(r)Q_{12} \qquad (2.33)$$

Where $Q_{12} = (\vec{\sigma}_1.\vec{L})(\vec{\sigma}_2.\vec{L})$ is the quadratic spin-orbit operator. General expressions for central $V_c(r)$ and Q potentials $V_Q(r)$ are derived elsewhere (Hammada and Johnston, 1962). The results can be summarized as:

$$V(r) = G\,\lambda\, Y(\lambda r)(1 + AY(\lambda r) + B(Y(\lambda r))^2) \qquad (2.34)$$

Where $Y(x) = \frac{e^{-x}}{x}$. The tensor potentials are given by

$$V_{tensor}(r) = G\,\lambda Y(\lambda r)[1 + \frac{3}{\lambda r} + \frac{3}{(\lambda r)^2}][1 + AY(\lambda r) + B(Y(\lambda r))^2] \qquad (2.35)$$

And spin-orbit potential by

$$V_{LS}(r) = G\,\lambda(Y(\lambda r))^2(1 + AY(\lambda r)) \qquad (2.36)$$

All of these potential values are obtained for r > D; while for r < D they are all have infinite values. Here D is the hard core radius, A, B and G are parameters whose numerical values are determined from the experimental values of NN potential (Hammada and Johnston, 1962).

Substituting equations (2.10), (2.19), (2.24), (2.26), (2.31) and (2.32) and into equation (2.33), the general form of the NN potential for deformed nuclides can be written as:

$$\begin{aligned} V = &\left(\frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}\right) + g(\vec{r}_1 - \vec{r}_2)\vec{\sigma}_1.\vec{\sigma}_2 + \tau_1.\tau_2 J_t(\vec{r}_{12}) + (\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r}) \\ &+ V_{tensor}(\vec{r})F_T\left[\left(\frac{3}{r^2}\right)(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r}) - (\vec{\sigma}_1.\vec{\sigma}_2)\right] + c\vec{L}.\vec{S} + V_Q(\vec{r})(\vec{\sigma}_1.\vec{L})(\vec{\sigma}_2.\vec{L}) \\ &+ V_{tensor}(\vec{r})F_T\left[\left(\frac{3}{r^2}\right)(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r}) - (\vec{\sigma}_1.\vec{\sigma}_2)](\vec{\tau}_1.\vec{\tau}_2)\right] \end{aligned} \qquad (2.37)$$

The form factors and potential strengths appeared in equation (2.37) are calculated somewhere else (Mar'i, 1987). Several attempts and investigations were conducted to simplify the nuclear potential in equation (2.37), among these the Nilsson one (Cohen, 1971) and Ben Day (Day, 1981).

Chapter Three

Effective non-core Hamiltonian for two nucleons systems

3.1 Introduction

It has been shown that the scattering of nucleons of velocities of about 0.3 c (c is the speed of light) can be, for a given spin state, accurately described by two parameters, the scattering length and the effective range. The description of the nucleons interaction by a potential V(r), which determines the scattering by means of Schrödinger equation, seems to be a rather circuitous procedure. Obviously, it is much more efficient to parametrize the phase shifts in terms of the relative momentum. On the other hand, it is convenient to formulate results of the scattering experiments in terms of a quantum mechanical operator, so that we can use the language and results of quantum mechanics to relate the scattering information to properties of more complex nuclear systems. The phase shifts are not a special suitable choice, since they are not easily expressed in terms of operators in Hilbert space.

A convenient way to describe the measurements of scattering amplitude is by the effective interaction. This is an operator $\mathfrak{I}$ in the Hilbert space of two nucleons which has the property of giving in the

Born approximation, the same scattering as the true interaction V gives when the Schrödinger equation with true interaction is solved exactly (Messiah, 1962). There are two different approaches to accomplish this goal. In the first approach, one assumes the existence of an effective interaction and determines the NN interaction with relatively few parameters. This can be accomplished by fitting the specific few-body matrix elements or a simple form containing a few parameters (strength, range, etc). For the second approach, the many-body perturbation theory can be used. The second approach will be used in this study and a general outline will be given in the following sections for such approach.

3.2 The wave functions of the two nucleons system

In the present section, we shall discuss how to introduce the two-nucleon wave functions and we shall use such wave functions to investigate the effective interaction of two nucleons. In general the total wave function of two nucleons system can be written in terms of space part ψ_{space} , spin part ψ_{spin} , and the isospin part $\psi_{isospin}$ as:

$$\Psi_{\text{two-nucleon}} = \psi_{space}(\vec{r})\psi_{spin}(\chi_{sm_z})\psi_{isospin}(\chi_{T\tau_z}) \tag{3.1}$$

the total wave function should be antisymmetric since nucleons are fermions. The radial part of the wave function is the one needed to calculate energy. It is now possible to establish the Schrödinger equation

for the radial part of the wave function of the two-nucleon system. This can be achieved by substituting equation (3.1) into the general form of Schrödinger equation and separate variables. The general result can be written as:

$$\left\{-\frac{\hbar^2}{2\mu}\nabla^2 + E - V\right\}\Psi_{\text{two-nucleon}} = 0 \qquad (3.2)$$

where μ is the two-nucleon reduced mass, E is the energy and V is the NN potential that is given by equation (2.37) in its general form. Equation (3.2) is not easy to solve, it can be solved numerically. Only small perturbations are considered, in order that space, spin and isospin wave functions are not allowed to couple.

3.2.1 The space part of the two-nucleon wave functions

We shall assume that the full Schrödinger wave equation for space motion can be written in the same form as equation (3.2). Assuming that the Schrödinger wave equation is a separable one (small deformation), the space wave function can be written as:

$$\psi_{\text{nlm}_\ell} \cong \frac{\Phi_{\text{nl}}(r)}{r} Y_{\ell, m_\ell}(\theta, \varphi) \qquad (3.3)$$

where n, ℓ, and m_ℓ are, respectively, the principle (radial), the orbital and the magnetic (projection) quantum numbers. We shall take the unperturbed wave function $\Phi_{n\ell}$ to be a spherical harmonic oscillator wave function, such that (Messiah, 1962)

$$\Phi_{n\ell} \sim \exp\left(-\frac{m\omega r^2}{2\hbar}\right) r^{\ell+1} L^{\ell+\frac{1}{2}}_{n+\ell-\frac{1}{2}}\left(\sqrt{\frac{m\omega}{\hbar}}r\right)$$

$$= \left[\frac{2^{\ell-n+2}(2\ell+2n+1)!!}{\sqrt{\pi}n!(2\ell+1)!!}\right]^{\frac{1}{2}} \exp\left(-\frac{m\omega r^2}{2\hbar}\right) r^{\ell+1} \times$$

$$\sum_{k=0}^{n} \frac{(-1)^k 2^k n!(2\ell+1)!!}{k!(n-k)!(2\ell+2k+1)!!}\left(\frac{m\omega r^2}{2\hbar}\right)^k \qquad (3.4)$$

The $\Phi_{n\ell}$ describes a state with energy $\left(2n+\ell+\frac{3}{2}\right)\hbar\omega$ and $L^{\ell+\frac{1}{2}}_{n+\ell-\frac{1}{2}}$ are Laguerre polynomials of order ℓ. The motivation of using the harmonic oscillator basis follows from the dependence of the two nucleons interaction on the relative coordinates. By using the harmonic oscillator wave functions, we can readily separate the relative motion from the center-of-mass motion of nucleons. Because of this separability we may write the spatial part of the two nucleons wave functions as:

$$\psi_{LM}(n_1\ell_1(1); n_2\ell_2(2)) = \left[\Phi_{n_1\ell_1}(\vec{r}_1)\Phi_{n_2\ell_2}(\vec{r}_2)\right]_{LM}$$

$$= \sum_{n\ell N} M_L(n\ell NL, n_1\ell_1 n_2\ell_2)\left[\Phi_{n\ell}(\vec{r})\Phi_{NL}\right]_{LM} \qquad (3.5)$$

Here the notation $n_1\ell_1(1), n_2\ell_2(2)$ means nucleon number 1in the state $n_1\ell_1$ and nucleon number 2 in the state $n_2\ell_2$ orbit, while $n\ell$ and NL denote the relative and center-of-mass (CM) quantum numbers. The coefficient M_L is the Mishinsky coefficient (Lawson, 1980) and its squared value gives the probability that the two nucleons system will be

found in a state of relative motion characterized by the oscillator wave function introduced in equation (3.3).

3.2.2 The two nucleons spin operators and wave functions

Generally speaking, it is not sufficient to specify their wave functions only by radial dependence (Skyrme, 1956; Detraz, 1995), the wave functions also have to be specified in the space of their intrinsic spins. We shall denote the spin wave functions (spinor) by either up spinor α or down spinor β, depending on whether the nucleon alignment is parallel or antiparallel to an arbitrary chosen axis. Therefor, the single nucleon wave function $\chi_{sm_s}(s_z)$ has two terms that can be written as:

$$\left.\begin{aligned} \chi\left(\frac{1}{2}\right) = \alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \chi\left(-\frac{1}{2}\right) = \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}\right\} \tag{3.6}$$

With

$$s_z\alpha = \frac{\hbar}{2}\alpha\,; \quad s^2\alpha = \frac{3}{4}\hbar^2\alpha; \quad s_z\beta = -\frac{\hbar}{2}\beta\,; \qquad s^2\beta = \frac{3}{4}\hbar^2\beta \tag{3.7}$$

The general spin wave function can be written as a linear combination of up and down spinors as:

$$\chi = a\,\alpha + b\beta = \begin{pmatrix} a \\ b \end{pmatrix} \tag{3.8}$$

where $|a|^2$ is the probability for spin up and $|b|^2$ is the probability for spin down.

The spin wave function for a system of two identical particles should be symmetrized. Using the χ_{sm_s} notations to represent the spin wave function, the singlet state (S=0, S_z=0) wave function for a two-nucleon system is antisymmetric and can be written in terms of individual spinors as (Merzbacher, 1996):

$$\chi_{00} = \frac{\alpha_1 \beta_1 - \alpha_2 \beta_2}{\sqrt{2}}; \quad \text{antisymmetric} \tag{3.9}$$

The antisymmetry of the wave function for two like fermions means that, for two neutrons (or two protons) to have an even-parity orbital wave function (e.g. L=0), their spin wave function must be odd under the interchange of the particles spins, implying S=0 (Lawson, 1980). The S=0, even-L phase shifts are practically the same (within 1%) for neutron-neutron and proton-proton scattering (Schiff, 1968).

The triplet state (S=1, S_z=1, 0, -1) wave function of the two nucleons system is symmetric and the different wave functions are related to individual spinors in the following way

$$\left.\begin{aligned}\chi_{11} &= \alpha_1\,\alpha_2 \\ \chi_{10} &= \frac{\alpha_1\,\beta_2+\alpha_2\,\beta_1}{\sqrt{2}} \\ \chi_{1\text{-}1} &= \beta_1\beta_2\end{aligned}\right\};\quad \text{symmetric} \tag{3.10}$$

Let $\left|(s_1s_2)\,S\,m_S\right\rangle$ stands for $\chi_{S\,m_S}(1,2)$, the two nucleons spin wavefunction. This wave function can be expressed in terms of Clebsh-Gordan form as:

$$\chi_{sm_s}(1,2)=\left|(s_1s_2)Sm_s\right\rangle=\sum_{m_{s1}.m_{s2}} C^{s_1s_2S}_{m_{s_1}m_{s_2}m_s}\chi_{m_{s_1}}(1)\chi_{m_{s_2}}(2) \tag{3.11}$$

In either case, there are four spin basis vectors. The square of the coefficients $C^{s_1s_2S}_{m_{s_1}m_{s_2}m_s}(1,2)$ gives the probability for finding nucleon 1 with spin up and nucleon 2 with spin down and vise versa. Using the Clebsh-Gordan coefficients properties, we can write

$$C^{s_2s_1S}_{m_{s2}m_{s1}m_s}=(-1)^{s_1+s_2-S}C^{s_1s_2S}_{m_{s_1}m_{s_2}m_s} \tag{3.12}$$

Alternately, the spinor wave functions may be specified in the basis of eigenfunctions of the total spin $\vec{S}=\vec{s}_1+\vec{s}_2$, or more precisely its square and its z-component

$$S^2\left|\chi_{Sm_S}\right\rangle=\hbar^2S(S+1)\left|\chi_{Sm_S}\right\rangle \tag{3.13}$$

$$S_z\left|\chi_{Sm_s}\right\rangle=\hbar S_z\left|\chi_{Sm_s}\right\rangle \tag{3.14}$$

In general, the basis $\left|\chi_{Sm_S}\right\rangle$ is convenient for $\vec{L}=0$ and S_z. Applying the first term in equation (2.24) on the two-nucleon spinors (triplet state) we get (Flügge, 1974)

$$\frac{(\vec{\sigma}_1.\vec{r}(\vec{\sigma}_2.\vec{r})}{r^2}\begin{bmatrix}\chi_{1,1}\\ \chi_{1,0}\\ \chi_{1,-1}\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta\chi_{1,1}+\sqrt{2}\sin\theta\cos\theta e^{i\varphi}\chi_{1,0}+\sin^2\theta e^{2i\varphi}\chi_{1,-1}\\ \sqrt{2}\sin\theta\cos\theta e^{-i\varphi}\chi_{1,1}+(\sin^2\theta-\cos^2\theta)\chi_{1,0}-\sqrt{2}\sin\theta\cos\theta e^{i\varphi}\chi_{1,-1}\\ \sin^2\theta e^{-2i\varphi}\chi_{1,1}-\sqrt{2}\sin\theta\cos\theta e^{-i\varphi}\chi_{1,0}+\cos^2\theta\chi_{1,-1}\end{bmatrix} \tag{3.15}$$

The second term in equation (2.24) gives

$$(\vec{\sigma}_1.\vec{\sigma}_2)\begin{bmatrix}\chi_{1,m_s}\\ \chi_{0,0}\end{bmatrix}=\begin{bmatrix}\chi_{1,m_s}\\ -3\chi_{0,0}\end{bmatrix} \tag{3.16}$$

And for the singlet state we get

$$\frac{(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})}{r^2}\chi_{0,0}=-\chi_{0,0} \tag{3.17}$$

using equations (3.16) and (3.17) we have

$$S_{12}\chi_{0,0}=0 \tag{3.18}$$

The effect of the tensor operator on the triplet spin states can be expressed in terms of Y_{lm} in the following way (Flügge, 1974)

$$S_{12}\chi_{1,1} = \frac{2}{3}\sqrt{\frac{4\pi}{5}}\{Y_{2,0}\chi_{1,1} + \sqrt{3}Y_{2,1}\chi_{1,0} + \sqrt{6}Y_{2,2}\chi_{1,-1}\} \tag{3.19}$$

$$S_{12}\chi_{1,0} = \frac{2}{3}\sqrt{\frac{4\pi}{5}}\{-\sqrt{3}Y_{2,-1}\chi_{1,1} - 2Y_{2,0}\chi_{1,0} - \sqrt{3}Y_{2,1}\chi_{1,-1}\} \tag{3.20}$$

$$S_{12}\chi_{1,-1} = \frac{2}{3}\sqrt{\frac{4\pi}{5}}\left(\sqrt{6}Y_{2,-2}\chi_{1,1} + \sqrt{3}Y_{2,-1}\chi_{1,0} + Y_{2,0}\chi_{1,-1}\right) \tag{3.21}$$

Accordingly, the scattering matrix must be diagonal and independent of S_z, it may, however, depends on $\vec{S}$.

3.2.3 The two nucleons isospin operators and wave functions

The proton and neutron are treated as two states of the nucleon differ only in charge. We shall introduce the charge operator $\vec{\tau}$ to be identical to that of the spin operator $\vec{\sigma}$. The isospin wave functions can be introduced as follows

$$\left.\begin{aligned} \chi(\text{proton}) &= \chi(p) = \chi\left(\frac{1}{2}\right) = \gamma = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \chi(\text{neutron}) &= \chi(n) = \chi\left(-\frac{1}{2}\right) = \delta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}\right\} \tag{3.22}$$

The neutron and proton are eigenstates of the operator τ_z analogous to S_z. Thus,

$$\left.\begin{aligned}\tau_z\gamma &= \frac{\hbar}{2}\gamma \\ \tau_z\delta &= -\frac{\hbar}{2}\delta\end{aligned}\right\} \qquad (2.23)$$

The isospin two nucleons wave function $\chi_{T\tau_z}$ also can be either a symmetric or an antisymmetric one depending on the total isospin of the two nucleons. The isospin raising and lowering operators, $\tau_\pm = \tau_x \pm i\tau_y$ change neutrons into protons and vice versa. Two nucleon states having isospin $T=1$ or $T=0$, are belonging to isospin symmetric and antisymmetric wave functions, respectively. The generalized antisymmetry for all multi fermions states including the isospin part must be antisymmetric under interchange two fermions coordinates (Krane, 1987).

The antisymmetric wave function corresponding to the singlet state of the two nucleons has a total isospin T=0 and an isospin projection $T_z = 0$. Similarly to spin case, introducing χ_{TT_z} notations, the antisymmetric singlet isospin wave function can be written as

$$\chi_{00} = \frac{\gamma_1\,\delta_1 - \gamma_2\,\delta_2}{\sqrt{2}}; \quad \text{antisymmetric} \qquad (3.24)$$

Moreover, a two-proton or two-neutron state has $T_z = \tau_z(1) + \tau_z(2) = \pm 1$ which implies $T=1$, a neutron-proton state has $T_z = 0$ and this may be either $T=0$ or 1. The two-nucleon isospin triplet

state (T=1, T_z=1, 0, -1) wave functions is symmetric and the different wave functions are related to individual isospin spinors by

$$\left.\begin{aligned}\chi_{11} &= \gamma_1\,\gamma_2 \\ \chi_{10} &= \frac{\gamma_1\,\delta_2+\gamma_2\,\delta_1}{\sqrt{2}} \\ \chi_{1\text{-}1} &= \delta_1\delta_2\end{aligned}\right\};\quad \text{symmetric} \tag{3.25}$$

This is similar to the spin wave functions.

3.2.4 The two nucleons total wave functions

The two-particle angular momentum coupled wave functions can be constructed as $\Psi(j_1(1)j_2(2);JM)$, with $j_1=\{n_1,l_1,j_1\}$ and 1,2,----- is a notation for all coordinates $\vec{r}_1,\vec{\sigma}_1,.....:\vec{r}_2,\vec{\sigma}_2,.....$ The eigenstates of the two-nucleon state may be written as $|Nljm\rangle$, where N is the principle quantum number of the orbital motion. The above wave function is constructed via angular momentum coupling

$$\Psi(j_1(1)j_2(2);JM)=\sum_{m_1.m_2}\langle j_1m_1,j_2m_2|JM\rangle\Psi_{j_1m_1}(1)\Psi_{j_2m_2}(2) \tag{3.26}$$

the coefficients $\langle j_1m_1,j_2m_2|JM\rangle$ (Clebsch-Gordon coefficients) vanish unless $M=m_1+m_2$ (Jelley, 1990; Rotenberg, Bivins, Metropolis and Wooten, 1959). In the case of identical particles (p-p, n-n) the wave function should be antisymmetric under the interchange of all coordinates. We explicitly construct them

(a) $j_1 \neq j_2$

$$\Psi_{as.}(j_1 j_2; JM) = N \sum_{m_1 . m_2} \langle j_1 m_1, j_2 m_2 | JM \rangle [\Psi_{j_1 m_1}(1)\Psi_{j_2 m_2}(2) - \Psi_{j_1 m_1}(2)\Psi_{j_2 m_2}(1)] \qquad (3.27)$$

Rewriting in the convention of angular momentum coupling particle 1 and 2, in that order, and using the symmetry properties of Clebsch-Gordan coefficients, we obtain (Merzbacher, 1996)

$$\Psi_{ans.}(j_1 j_2; JM) = \frac{1}{\sqrt{2}}[\Psi(j_1 j_2; JM) - (-1)^{j_1 + j_2 - I}\Psi(j_1 j_2; JM)] \qquad (3.28)$$

Here the subscript "ans" means that the wave function is normalized and antisymmetric. The nucleon coordinates need not be written any more since we use the convention of always in the order (1,2)

(b) $j_1 = j_2$

From equation (3.28) in which we placed the restriction $j_1 = j_2 = j$, we obtain the wave function

$$\Psi_{ans.}(j^2; JM) = N'(1 + (-1)^J) \sum_{m_1, m_2} \langle j m_1 j m_2 | JM \rangle \Psi_{j m_1}(1)\Psi_{j m_2}(2) \rangle \qquad (3.29)$$

So, one gets N′ =½ and the restriction J=even. This leads to two-nucleon systems with spin J=0, 2, 4, -----,2j-1 ;in general. The normalized wave function for the two nucleons system can be expressed in terms of Clebsh-Gordan coefficients as (Mar'i, 1987):

$$\Psi_{ans.}(j^2;JM) = \left|n_1\ell_1 s_1 j_1, n_2\ell_2 s_2 j_2; JM(\tau_1,\tau_2)T\right\rangle$$

$$= \frac{1}{\sqrt{2(1+\delta)}} \sum_{LS} \left[\hat{j}_1\hat{j}_2\hat{L}\hat{S}\right]^{\frac{1}{2}} \begin{Bmatrix} \ell_1 s_1 j_1 \\ \ell_2 s_2 j_2 \\ L\ S\ J \end{Bmatrix} \times$$

$$\left| \begin{matrix} \left[n_1\ell_1(1)j_1, n_2\ell_2(2)Lm_\ell(s_1s_2)Sm_s; JM(\tau_1,\tau_2)T\right] \\ +(-1)^T n_1\ell_1(1)j_1, n_2\ell_2(2)Lm_\ell(s_1s_2)Sm_s; JM(\tau_1,\tau_2)T \end{matrix} \right\rangle \quad (3.30)$$

where $\delta = \delta_{n_1n_2}\delta_{\ell_1\ell_2}\delta_{j_1j_2}\delta_{s_1s_2}\delta_{\tau_1\tau_2}$.

The calculation of the two body matrix $\langle j_1, j_2; JM|V_{eff}(1,2)|j_1 j_2; JM\rangle_{ans}$ is the one required in this study. This will be done for some special cases in the following chapter.

3.3 The effective interaction operators

In order to obtain the difference between the free NN interaction V and the effective model interaction V_{eff}, we consider a system described by the Hamiltonian H

$$H = H_0 + V \quad (3.31)$$

Here, H_o denotes the unperturbed Hamiltonian with a set of unperturbed wave functions Ψ_i° and V is the perturbed term. Thus

$$H_0\Psi_i^0 = E_i^0\Psi_i^0 \quad (3.32)$$

For a particular state, the true wave function Ψ obeys the full Schrödinger equation (Messiah, 1962):

$$(H_0 + V)\Psi = E\Psi \quad (3.33)$$

Where, E is the total energy of the many-body system.

The standard perturbation expression expresses the true wave function Ψ in terms of Ψ_i° as (Gasiorowicz, 1996):

$$\Psi = \sum_{i=1} a_i \Psi_i^0 \tag{3.34}$$

For a limited, small model space, only a small number of basis states Ψ_i° are used and one obtains a wave function

$$\Psi' = \sum a_i \Psi_i^0 \tag{3.35}$$

We now impose the condition that the effective Hamiltonian reproduces the true energy E for the corresponding normalized wave function Ψ' thus

$$\left\langle \Psi' \middle| H_{eff} \middle| \Psi' \right\rangle = E \tag{3.36}$$

We divide the Hilbert space between the model space M and the remaining part. The remaining part is given by the projection operators such that (Merzbacher, 1996)

$$\hat{P} = \sum_{i \in M} \left| \Psi_i^0 \right\rangle \left\langle \Psi_i^0 \right| \tag{3.37}$$

And $\hat{Q}$

$$\hat{Q} = \sum_{i \notin M} \left| \Psi_i^0 \right\rangle \left\langle \Psi_i^0 \right| \tag{3.38}$$

Operators $\hat{P}$ and $\hat{Q}$ have the following properties (Villars, 1966):

$$\left.\begin{aligned} &\hat{P}+\hat{Q}=\hat{I},\\ &\hat{P}'=P,\\ &\hat{Q}'=Q,\\ &[\hat{P},\hat{Q}]=0. \end{aligned}\right\} \tag{3.39}$$

where $\hat{I}$ is the identity operator. Introducing the space wave function as $\Psi' = \hat{P}\Psi$, the true wave function can be written as follows:

$$\Psi = (\hat{P}+\hat{Q})\Psi = \Psi' + \hat{Q}\Psi \tag{3.40}$$

When operators $\hat{Q}$ and $\hat{P}$ are applied to equation (3.33), Schrödinger equation can be written as (Brüeckner *et al.*, 1961):

$$(H_0 - E + \hat{Q}V\hat{Q})\hat{Q}\Psi = -\hat{Q}V(\hat{P}\Psi) \tag{3.41}$$

And

$$(H_0 - E - \hat{P}V\hat{P})\hat{P}\Psi = -\hat{P}V(\hat{Q}\Psi) \tag{3.42}$$

Solving equation (3.41) for $\hat{Q}\Psi$ we get

$$\hat{Q}\Psi = -\hat{Q}(H_0 - E + \hat{Q}V\hat{Q})^{-1}\hat{Q}V\hat{P}(\hat{P}\Psi) \tag{3.43}$$

Substituting equation (3.43) into equation (3.42) to get

$$(H_0 - E + \hat{P}V\hat{P} - \hat{P}V\hat{Q}(H_0 - E + \hat{Q}V\hat{Q})^{-1}\hat{Q}V\hat{P})\hat{P}\Psi = 0 \tag{3.44}$$

Or

$$\hat{P}(H_0 - E + V - V\hat{Q}(H_0 - E + \hat{Q}V\hat{Q})^{-1}\hat{Q}V)\hat{P}\Psi = 0 \tag{3.45}$$

In terms of the model wave function Ψ', equation (3.45) can be written as

$$\hat{P}(H_0 - E + V_{eff})\Psi' = 0 \qquad (3.46)$$

Equation (3.46) represents an eigenvalue problem in the model space for the true energy eigenvalue E at the expense of using an effective model interaction $V_{eff}(E)$. The effective interaction can be introduced as (Brüeckner *et al.*, 1961; Bethe, 1971; Köhler, 1965)

$$V_{eff} = V + V\hat{Q}(E - H_0 - \hat{Q}V\hat{Q})^{-1}\hat{Q}V \qquad (3.47)$$

Let us introduce the identity operator as follows (Messiah, 1962):

$$\hat{1} = \frac{1}{\hat{A}}.(\hat{A} - \hat{B}) + \frac{1}{\hat{A}}.\hat{B} \qquad (3.48)$$

Or

$$\frac{1}{(\hat{A} - \hat{B})} = \frac{1}{\hat{A}} + (\frac{\hat{B}}{\hat{A}})(\frac{1}{\hat{A} - \hat{B}}) \qquad (3.49)$$

The effective interaction can be expressed in terms of the identity operator as in the following equation (Bethe, 1971):

$$V_{eff} = V + V(\frac{\hat{Q}}{E - H_0})V \qquad (3.50)$$

For an arbitrary effective interaction, the latter expression in equation (3.50) should be expanded to include all nucleons eigenstates. We can iterate the effective interaction into the series (Brüeckner *et al.*, 1961)

$$V_{eff} = V + V(\frac{\hat{Q}}{E - H_0})V + V(\frac{\hat{Q}}{E - H_0})V(\frac{\hat{Q}}{E - H_0})V + ... \qquad (3.51)$$

Similarly, we can obtain an expression of the true wave function Ψ in terms of the model wave function as

$$\Psi = \Psi' + (\frac{\hat{Q}}{E - H_0})V\Psi \qquad (3.52)$$

Equation (3.52) can also be iterated in the same way. The result is

$$\Psi = \Psi' + (\frac{\hat{Q}}{E - H_0})V\Psi' + (\frac{\hat{Q}}{E - H_0})V(\frac{\hat{Q}}{E - H_0})V\Psi' + ... \qquad (3.53)$$

Since $\hat{Q}|\Psi'\rangle = 0$, the true wave functions can be normalized according to the following relation

$$\langle\Psi|\Psi'\rangle = \langle\Psi'|\Psi'\rangle = 1 \qquad (3.54)$$

Starting form equation (3.51) and equation (3.53) the following relation can easily verified

$$V_{eff}\Psi' = V\Psi \qquad (3.55)$$

Accordingly, the action of the effective interaction on the model wave function gives the same result as the action of the realistic interaction on the true wave function (Brown and Jackson, 1975).

3.4 The Effective interaction and the $\Im$ matrix

In order to relate the effective interaction to $\Im$, let us consider an integral equation to be satisfied by the wave function $\Psi_{\vec{k}}$ as

$$\Psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}.\vec{r}} + \int G_E^{(+)}(\vec{r},\vec{r}')V(\vec{r}')\Psi_{\vec{k}}(\vec{r})d^3\vec{r} \qquad (3.56)$$

Where $G_E^{(+)}(\vec{r},\vec{r}')$ is Green's function satisfies the relation

$$\delta(\vec{r}-\vec{r}') = (E - H_0)G_E^{(+)}(\vec{r},\vec{r}') = (\frac{\hbar^2}{2\mu}\nabla_r^2 + E)G_E^{(+)}(\vec{r},\vec{r}') \qquad (3.57)$$

In a more explicit way, Green's function can be written in the following form:

$$G_E^{(+)}(\vec{r},\vec{r}') = -\frac{\mu}{2\hbar^2}\frac{e^{i\sqrt{\frac{2\mu}{\hbar^2}}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \qquad (3.58)$$

Where, by convention, the (+) denotes the outgoing boundary condition. It is easy to verify that equation (3.56) guarantees that $\Psi_{\vec{k}}$ solves the following Schrödinger equation (Eisenberg and Greiner, 1970)

$$(-\frac{\hbar^2}{2\mu}\nabla^2 + V(\vec{r}))\Psi_{\vec{k}}(\vec{r}) = E\Psi_{\vec{k}}(\vec{r}) \qquad (3.59)$$

Where μ is the reduced mass of the two-nucleons system. At all positions, $\Psi_{\vec{k}}$ satisfies the Schrödinger equation (3.59). Equation (3. 56) can be used to evaluate the matrix elements of equation $\langle\phi_{\vec{k}'}|V|\Psi_{\vec{k}}\rangle$, for $|\vec{k}'|$ and $|\vec{k}|$. Thus,

$$\begin{aligned}\langle\phi_{\vec{k}}|V|\phi_{\vec{k}'}\rangle &= \langle\phi_{\vec{k}'}|V|\phi_{\vec{k}}\rangle + \int d^3\vec{r}\int d^3\vec{r}'\phi_{\vec{k}'}V(\vec{r})G_E^{(+)}(\vec{r},\vec{r}')V(\vec{r}')\Psi_{\vec{k}}(\vec{r}) \\ &= \langle\phi_{\vec{k}'}|V|\phi_{\vec{k}}\rangle + \int d^3\vec{r}\int d^3\vec{r}'\langle\phi_{\vec{k}'}|V|\vec{r}\rangle\langle\vec{r}|G_E^{(+)}|\vec{r}'\rangle\langle\vec{r}'|V|\Psi_{\vec{k}}\rangle \qquad (3.60)\end{aligned}$$

By introducing the complete set of basis states $|\varphi_{\vec{k}''}\rangle$ and $|\varphi_{\vec{k}'''}\rangle$ instead of $|\vec{r}\rangle$ and $|\vec{r}'\rangle$ states, equation (3. 60) can be written in another form using Fourier transform as:

$$\langle\phi_{\vec{k}'}|V|\Psi_{\vec{k}}\rangle=\langle\phi_{\vec{k}'}|V|\phi_{\vec{k}}\rangle+\frac{1}{(4\pi\hbar)^6}\int d^3\vec{k}''\int d^3\vec{k}'''\langle\phi_{\vec{k}'}|V|\phi_{\vec{k}''}\rangle\langle\phi_{\vec{k}'}|G_E^{(+)}|\phi_{\vec{k}'''}\rangle\times \langle\phi_{\vec{k}''}|V|\Psi_{\vec{k}}\rangle \qquad (3.61)$$

The matrix elements of $G_E^{(+)}$ between plane waves are easily evaluated using equation (3. 57) or directly from the definition equation (3. 58), we may write

$$\langle\phi_{\vec{k}'}|G_E^{(+)}|\phi_{\vec{k}''}\rangle=\lim_{\varepsilon\to 0}\langle\phi_{\vec{k}''}|\frac{1}{E-H_o+i\varepsilon}|\phi_{\vec{k}'''}\rangle \qquad (3.62)$$

The matrix in equation (3. 62) is diagonal in the plane wave basis. This can be verified using the following completeness relation

$$(\frac{1}{2\pi\hbar})^6\int d^3\vec{k}|\phi_{\vec{k}}\rangle\langle\phi_{\vec{k}}|=1 \qquad (3.63)$$

Accordingly, equation (3.61) satisfies the following equation (Heyde,1999)

$$\langle\phi_{\vec{k}''}|\Im|\phi_{\vec{k}}\rangle=\langle\phi_{\vec{k}'}|V|\phi_{\vec{k}}\rangle+\langle\phi_{\vec{k}}|VG_E^{(+)}\Im|\phi_{\vec{k}}\rangle \qquad (3.64)$$

Or, simply,

$$\Im_E=V+VG_E^{(+)}\Im_E=V+\lim_{\varepsilon\to 0}V\frac{1}{E-H_o+i\varepsilon}\Im_E \qquad (3.65)$$

The relation (3.65) between $\Im$ and V involves the energy E as parameters, because E appears in the Green's function $G_E^{(+)}$. Thus, equation (3.65) actually defines a whole family of operators $\Im_E$. Thus we are led to identify the matrix element of the effective interaction $\Im$ (the transition or $\Im$ matrix) by the following approximation:

$$\left\langle \phi_{\vec{k}'} \middle| \Im \middle| \phi_{\vec{k}} \right\rangle = \left\langle \phi_{\vec{k}'} \middle| V \middle| \Psi_{\vec{k}} \right\rangle \tag{3.66}$$

The solution of this equation (3.66) can be by using the general expression of NN interaction in equation (2.37) and by using the following total wave function of the two nucleons system

$$\Psi_{tot.} = \Phi(r)\chi_{s.m_s}\chi_{T.\tau_z} \equiv \left| n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2, \sigma_1, \sigma_2, \tau_1, \tau_2 \right\rangle \tag{3.67}$$

3.5 Approximation of the effective interaction $\Im_E$ matrix elements by the differential cross sections

Sometimes it is difficult to evaluate the exact matrix element of NN potential and approximate methods are recommended. This section is aimed to relate the $\Im$ matrix to the differential cross section $\frac{d\sigma}{d\Omega}$, in a step to simplify the complex expression of the effective interaction. The differential cross section can be expressed in terms of the scattering amplitude $f_k(\theta)$ as follows (Schiff, 1968):

$$\frac{d\sigma}{d\Omega} = \left| f_k(\theta) \right|^2 \tag{3.68}$$

Where θ is the scattering angle, and $f_k(\theta)$ describes the asymptotic term of the scattering wave function for the relative motion

$$\Psi_{\vec{k}}(\vec{r}) \to e^{i\vec{k}\cdot\vec{r}} + f_k(\theta)\frac{e^{ikr}}{r} \qquad \text{for} \qquad kr \to \infty \qquad (3.69)$$

Where $\hbar\vec{k}$ is the relative momentum before the scattering.

Let us now figure out the relationship between the $\Im$ matrix and $f_k(\theta)$. It is well known that the Born approximation consists of approximating $\Psi_{\vec{k}}$ by an unperturbed plane wave $\Phi_{\vec{k}}$. By comparison, a plane wave $\Phi_{\vec{k}'}(\vec{r})$ of the same energy but directed along another arbitrary direction $\vec{k}'$ satisfies also Schrödinger equation (Merzbacher, 1996)

$$-\frac{\hbar^2}{2\mu}\nabla^2\Phi_{\vec{k}'}(\vec{r}) = E\Phi_{\vec{k}'}(\vec{r}) \qquad (3.70)$$

The wave function $\Phi_{\vec{k}'}(\vec{r})$ appears in equation (3.70) is the same as equation (3.69) for large $\vec{r}$, but differs inside the range of the force. Multiplying equation (3.69) by $\Phi_{\vec{k}'}(\vec{r})$, subtracting $\Psi_{\vec{k}}(\vec{r})$ times the complex conjugate of equation (3.70) and integrating over a sphere of radius $R \to \infty$, to obtain

$$\left\langle \Phi_{\vec{k}'} \middle| V \middle| \Psi_{\vec{k}} \right\rangle = \lim_{R\to\infty} \int_{|\vec{r}|<R} d^3\vec{r}\,\Phi_{\vec{k}'} V(\vec{r})\Psi_{\vec{k}}(\vec{r}) = \lim_{R\to\infty} \int_{|\vec{r}|<R} d^3\vec{r}\,\frac{\hbar^2}{2\mu} \times$$
$$[\Phi_{\vec{k}'}(\vec{r})\nabla^2\Psi_{\vec{k}}(\vec{r}) - \Psi_{\vec{k}}(\vec{r})\nabla^2\Phi_{\vec{K}'}(\vec{r})] \qquad (3.71)$$

The right hand side of equation (3.71) may be evaluated using Green theorem

$$\int_{\text{volume}} d^3\vec{r}(f\nabla^2 g - g\nabla^2 f) = \int_{\text{suface}} R^2 d\Omega(f\vec{\nabla}g - g\vec{\nabla}f).\hat{n} \qquad (3.72)$$

Where $\hat{n}$ is a unit vector normal to the surface. Accordingly, by inserting equation (3.69) in equation (3.71), we get

$$\langle \phi_{\vec{k}}(\vec{r})|V|\Psi_{\vec{k}}\rangle = \frac{\hbar^2}{2\mu}\lim_{R\to\infty} R^2 \int d\Omega[e^{-i\vec{K}'.\vec{r}}\frac{\partial}{\partial r}(e^{i\vec{k}.\vec{r}} + \frac{f_{k(\theta)}}{r}e^{ikr}) - e^{i\vec{k}.\vec{r}} + \frac{f_k(\theta)}{r}e^{ikr})$$
$$\frac{\partial}{\partial r}e^{i\vec{K}'.\vec{r}}] = -4\pi\frac{\hbar^2}{2\mu}f_k(\theta') \qquad (3.73)$$

Where θ' is the angle corresponding to $\vec{k}'$. According to equation (3. 66) and equation (3.68), the right-hand side result of equation (3.73) is related to the scattering amplitude through the following relation

$$\langle \phi_{\vec{k}'}|\Im|\phi_{\vec{k}}\rangle = -4\pi\frac{\hbar^2}{2\mu}f_k(\theta) \qquad (3.74)$$

Generally, f_k depends only on the magnitude of k and on the angel between $\vec{k}$ and $\vec{k}'$.

The requirement that the effective interaction produces the correct scattering amplitude in the Born approximation is not sufficient to specify completely the operator $\Im$. However, it gives the same matrix elements between states of the same energy as the $\Im$ operator does. The obvious generalization is obtained using equation (3.66) to define the matrix

elements of $\Im$ for all $\vec{k}$ and $\vec{k}'$. In order to have relations between the effective interaction and $\Im$, the effective interaction can be replaced by the true interaction V. This approximation enable us in finding an equation equivalent to equation (3.74), but having an explicit dependence on the wave vector $\vec{k}$.

The relation to the scattering amplitude f_k, equation (3.74), only holds for the effective interaction $\Im_E$ whose energy E is equal to the experimental energy $E = \frac{\hbar^2 k^2}{2\mu}$. Equation (3.74) can also be used as a definition of the $\Im$-matrix, from which its other properties follow. This definition is not restricted to local potentials, or to non-relativistic quantum mechanics.

3.6 In medium NN cross section

In order to use the approximated expression in equation (3.74) for evaluating the matrix elements of the NN potential, the in medium actual cross section, σ_{NN}^{in}, which takes into account Pauli blocking and the finite nuclear matter density must be known.

The in medium cross section σ_{NN}^{in} can be calculated by solving many-body Bethe-Goldston equation (Faessler *et al.*, 1984). In this case, the calculated values are limited because values of nuclear density and incident energy values are restricted to include small ranges. The semi-

empirical formula used for calculating σ_{NN}^{in} within energy range between 50-500 MeV and matter density up to $2\rho_0$ suggests that $\sigma^{in} = 0.80\sigma^{free}$. A general formula that will serve wider energy range extended from 10 MeV to 1 GeV can be written as (Xiangzhou *et al.*, 1997):

$$\sigma_{nn}^{in} = (13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{-4})\frac{1 + 7.772E_{lab}^{0.06}\rho^{1.48}}{1 + 18.01\rho^{1.46}} \quad (3.75)$$

$$\sigma_{nn}^{in} = (-70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta)\frac{1 + 20.88E_{lab}^{0.04}\rho^{2.02}}{1 + 35.86\rho^{1.9}} \quad (3.76)$$

where

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad (3.77)$$

is the ratio of projectile velocity to light velocity

$$\gamma = \frac{E_{lab}}{931.5} + 1 \quad (3.78)$$

ρ is the nuclear matter density E_{lab} is the incident energy in lab frame

In this study, we shall adopt the following approximation:

$$\frac{d\sigma}{d\Omega} \cong \sigma_{NN}^{in} = \frac{\sigma_{nn}^{in} + \sigma_{np}^{in}}{2} \quad (3.79)$$

Chapter Four

Results and discussion

4. 1 Introduction

In chapter 3 we have introduced the two nucleons matrix elements of the effective NN interaction. Our philosophy is to adjust the matrix element of V_{eff} in order to achieve agreement with the measured ground state properties in the central spherical potential well approximations, before proceeding to include the deformed potentials in the present model.

The deformed types are classified according to their L-values. Typical deformations to be investigated in this study include ℓ=2, 3, 4, and 5. The dipole deformation of nuclear shape having ℓ=1 has no physical meaning since it corresponds to the movement of center of mass the nucleus. The ℓ=2 corresponds to ellipsoidal shape for the deformed nucleus and is called the quadrupole deformation. The ℓ=3 is referred to as an octupole deformation. The ℓ=4, 5 are called hexadecapole and fifth pole deformation. Therefore, we adjust the deformation parameters simultaneously to achieve the desired binding energy for a given nucleus for each choice of the model. The deformation parameters are usually less than unity. This is because as the deformation parameters become large, the effective interaction approaches the bare interaction whose matrix

elements are large and positive. Thus, our whole procedure will eventually breakdown with increasing deformation parameter values. The results we present are obtained neglecting the Coulomb interaction, since the Coulomb interaction is a relatively weak interaction this should be a safe approximation for the nuclei we consider.

In this chapter we shall present our results for ^{16}O and ^{40}Ca nuclides. We have evaluated the ground state properties of these nuclides and have calculated effective interaction total energies. We outline this chapter as follows: In section (4.2) we describe the technique we have used for efficient computation. In section (4.3) we test the model against the delta potential and the Nilsson one. In section (4.4) we present our results for ^{16}O and in section (4.5) we present our results for ^{40}Ca.

4.2 Numerical methods

Our calculations include up to 6 major shells. For such calculations, considerable computer time and space for storage are needed. To overcome such difficulties we omit some nucleons and some orbits that are not expected to play significant roles. We have tested the validity of this procedure in the deuteron nucleus (Figure 4.1), we have found that the results obtained are in a good agreement with the experimental results (Cohen, 1971). We have used these orbits to determine which of the two-body matrix elements of the effective Hamiltonian to be evaluated. The matrix element of the effective

interaction $\mathfrak{I}$ (the transition or $\mathfrak{I}$ matrix) was introduced in chapter 3 as follows:

$$\langle \phi_{\vec{k}'} | \mathfrak{I} | \phi_{\vec{k}} \rangle = \langle \phi_{\vec{k}'} | V | \Psi_{\vec{k}} \rangle \tag{4.1}$$

The solution of this equation can be as follows: Consider the total wave function of the two particles can be written as

$$\Psi_{tot.} = \phi(\vec{r})\chi_{s.m_s}\chi_{T.\tau_z} \equiv |n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2, \sigma_1, \sigma_2, \tau_1, \tau_2\rangle \tag{4.2}$$

Using equation (3.51), equation (4.1) becomes

$$\langle \Psi_{tot.} | \mathfrak{I} | \Psi_{tot.} \rangle = \langle n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2, \sigma_1, \sigma_2, \tau_1, \tau_2 | V + V(\frac{\hat{Q}}{E - H_0}) \times$$
$$V + V(\frac{\hat{Q}}{E - H_0}) V (\frac{\hat{Q}}{E - H_0}) V + - - - - -$$
$$| n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2, \sigma_1, \sigma_2, \tau_1, \tau_2 \rangle \tag{4.3}$$

Moreover, by classifying each part of the potential with its own part of the wave function we get: The first term:

$$\langle n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2 | (\frac{g^2}{4\pi} e^{-\mu r} (\frac{1}{r_1} \sum_{l=0}^{\infty} P_l(\cos\theta)(\frac{r_{12}}{r_1})^l) + (\frac{g^2}{4\pi} e^{-\mu r} (\frac{1}{r_1} \sum_{l=0}^{\infty} P_l(\cos\theta)(\frac{r_{12}}{r_1})^l)$$
$$(\frac{\hat{Q}}{E - H_0})(\frac{g^2}{4\pi} e^{-\mu r} (\frac{1}{r_1} \sum_{l=0}^{\infty} P_l(\cos\theta)(\frac{r_{12}}{r_1})^l) + (\frac{g^2}{4\pi} e^{-\mu r} (\frac{1}{r_1} \sum_{l=0}^{\infty} P_l(\cos\theta)(\frac{r_{12}}{r_1})^l)(\frac{\hat{Q}}{E - H_0})$$
$$(\frac{g^2}{4\pi} e^{-\mu r} (\frac{1}{r_1} \sum_{l=0}^{\infty} P_l(\cos\theta)(\frac{r_{12}}{r_1})^l)(\frac{\hat{Q}}{E - H_0})(\frac{g^2}{4\pi} e^{-\mu r} (\frac{1}{r_1} \sum_{l=0}^{\infty} P_l(\cos\theta)(\frac{r_{12}}{r_1})^l))$$
$$| n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2 \rangle \tag{4.4}$$

The second term:

$$\langle \sigma_1, \sigma_2 | g\,\vec{\sigma}_1.\vec{\sigma}_2 + g\,\vec{\sigma}_1.\vec{\sigma}_2 (\frac{\hat{Q}}{E - H_0}) g\,\vec{\sigma}_1.\vec{\sigma}_2 + g\vec{\sigma}_1.\vec{\sigma}_2 (\frac{\hat{Q}}{E - H_0}) g(\vec{r}_1 - \vec{r}_2)\vec{\sigma}_1.\vec{\sigma}_2 \times$$
$$(\frac{\hat{Q}}{E - H_0}) g(\vec{r}_1 - \vec{r}_2)\vec{\sigma}_1.\vec{\sigma}_2 | \sigma_1, \sigma_2 \rangle \tag{4.5}$$

The third term:

$$\langle \tau_1, \tau_2 | \tau_1.\tau_2 J_t(\vec{r}_{12}) + \tau_1.\tau_2 J_t(\vec{r}_{12})(\frac{\hat{Q}}{E - H_0})\tau_1.\tau_2 J_t(\vec{r}_{12}) + \tau_1.\tau_2 J_t(\vec{r}_{12})(\frac{\hat{Q}}{E - H_0}) \times$$

$$\tau_1.\tau_2 J_t(\vec{r}_{12})(\frac{\hat{Q}}{E - H_0})\tau_1.\tau_2 J_t(\vec{r}_{12}) | \tau_1, \tau_2 \rangle \qquad (4.6)$$

The fourth term:

$$\langle \sigma_1, \sigma_2, \tau_1, \tau_2 | (\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r}) + (\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r})(\frac{\hat{Q}}{E - H_0}) \times$$

$$(\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r}) + (\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r})(\frac{\hat{Q}}{E - H_0})(\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r}) \times$$

$$(\frac{\hat{Q}}{E - H_0})(\vec{\sigma}_1.\vec{\sigma}_2)(\vec{\tau}_1.\vec{\tau}_2)F_{\sigma T}(\vec{r}) | \sigma_1, \sigma_2, \tau_1, \tau_2 \rangle \qquad (4.7)$$

The fifth term:

$$\langle \sigma_1, \sigma_2 | V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.r)(\vec{\sigma}_2.r) - (\vec{\sigma}_1.\vec{\sigma}_2)] + V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.r)(\vec{\sigma}_2.r) \times$$

$$- (\vec{\sigma}_1.\vec{\sigma}_2)](\frac{\hat{Q}}{E - H_0}) V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.r)(\vec{\sigma}_2.r) - (\vec{\sigma}_1.\vec{\sigma}_2)] + V_{tensor}(\vec{r})F_T[(\frac{3}{r^2}) \times$$

$$(\vec{\sigma}_1.r)(\vec{\sigma}_2.r) - (\vec{\sigma}_1.\vec{\sigma}_2)](\frac{\hat{Q}}{E - H_0})V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.r)(\vec{\sigma}_2.r) - (\vec{\sigma}_1.\vec{\sigma}_2)] \times$$

$$(\frac{\hat{Q}}{E - H_0})V_{tensor}(\vec{r}) \times F_T[(\frac{3}{r^2})(\vec{\sigma}_1.r)(\vec{\sigma}_2.r) - (\vec{\sigma}_1.\vec{\sigma}_2)] | \sigma_1, \sigma_2 \rangle \qquad (4.8)$$

The sixth term:

$$\langle \sigma_1,\sigma_2,\tau_1,\tau_2 | V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})-(\vec{\sigma}_1.\vec{\sigma}_2)](\vec{\tau}_1.\vec{\tau}_2)+V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})\times$$

$$(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})-(\vec{\sigma}_1.\vec{\sigma}_2)](\vec{\tau}_1.\vec{\tau}_2)(\frac{\hat{Q}}{E-H_0})\,V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})-(\vec{\sigma}_1.\vec{\sigma}_2)]\times$$

$$(\vec{\tau}_1.\vec{\tau}_2)+V_{tensor}(\vec{r})\times F_T[(\frac{3}{r^2})(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})-(\vec{\sigma}_1.\vec{\sigma}_2)](\vec{\tau}_1.\vec{\tau}_2)(\frac{\hat{Q}}{E-H_0})V_{tensor}(\vec{r})\times$$

$$F_T[(\frac{3}{r^2})(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})-(\vec{\sigma}_1.\vec{\sigma}_2)](\vec{\tau}_1.\vec{\tau}_2)(\frac{\hat{Q}}{E-H_0})\;\; V_{tensor}(\vec{r})F_T[(\frac{3}{r^2})(\vec{\sigma}_1.\vec{r})(\vec{\sigma}_2.\vec{r})\times$$

$$-(\vec{\sigma}_1.\vec{\sigma}_2)](\vec{\tau}_1.\vec{\tau}_2)|\sigma_1,\sigma_2,\tau_1,\tau_2\rangle \qquad (4.9)$$

The seventh term:

$$\langle n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2, \sigma_1,\sigma_2 | c\vec{L}.\vec{S}+c\vec{L}.\vec{S}(\frac{\hat{Q}}{E-H_0})c\vec{L}.\vec{S}+c\vec{L}.\vec{S}(\frac{\hat{Q}}{E-H_0})c\vec{L}.\vec{S}\times$$

$$(\frac{\hat{Q}}{E-H_0})\,c\vec{L}.\vec{S}|n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2,\sigma_1,\sigma_2\rangle \qquad (4.10)$$

The final term:

$$\langle n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2, \sigma_1,\sigma_2 | V_Q(\vec{r})(\vec{\sigma}_1.\vec{I})(\vec{\sigma}_2.\vec{I})+V_Q(\vec{r})(\vec{\sigma}_1.\vec{I})(\vec{\sigma}_2.\vec{I})(\frac{\hat{Q}}{E-H_0})\times$$

$$V_Q(\vec{r})(\vec{\sigma}_1.\vec{I})(\vec{\sigma}_2.\vec{I})+V_Q(\vec{r})(\vec{\sigma}_1.\vec{I})(\vec{\sigma}_2.\vec{I})(\frac{\hat{Q}}{E-H_0})V_Q(\vec{r})(\vec{\sigma}_1.\vec{I})(\vec{\sigma}_2.\vec{I})\,(\frac{\hat{Q}}{E-H_0})\times$$

$$V_Q(\vec{r})(\vec{\sigma}_1.\vec{I})(\vec{\sigma}_2.\vec{I})\;|n_1 j_1 l_1 m_1, n_2 j_2 l_2 m_2,\sigma_1,\sigma_2\rangle \qquad (4.11)$$

Evaluating the above equations requires a supercomputer to accomplish the job. The computer program has been developed according to equations (4.4-4.11) and is restricted to work within the basis space defined by these orbits despite the complication of numerical analysis. In

developing the program several simplifications and approximations have been made and will be discussed separately.

The central part of the potential decreases with increasing the separation distance between nucleons and the matrix element can be separated into two parts making use of the addition theorem of spherical harmonics. If the deformation effects are allowed, the deformation surface can be described by equation (2.1). We have considered axially symmetric deformations only with z-axis used as symmetric axis. In this case, all $\alpha_{\ell m}$ values vanish except for m=0. The central part of the potential is calculated using the expression in equation (2.17). The matrix element can be separated into two parts making use of the expansion in equation (2.15)

The spin contribution to the internucleon effective interaction is represented by an expression involving both nucleon spins (Cohen, 1971). The simplest term involving both spins is $\vec{s}_1.\vec{s}_2$. The expectation values of $\vec{s}_1.\vec{s}_2$ is calculated for both singlet and triplet states of nucleons. In doing these calculations we are making use of the squared value of the total spin $\vec{S} = \vec{s}_1 + \vec{s}_2$. Thus

$$\vec{s}_1.\vec{s}_2 = \frac{1}{2}\left(S^2 - s_1^2 - s_2^2\right) \qquad (4.12)$$

Knowing from quantum mechanics that the eigenvalue of an angular momentum S^2 can be expressed as $\hbar^2 S(S+1)$, the expectation value of $\vec{s}_1.\vec{s}_2$ can be written as

$$\langle \vec{s}_1.\vec{s}_2 \rangle = \frac{1}{2}\left(S(S+1) - s_1(s_1+1) - s_2(s_2+1)\right)\hbar^2 \qquad (4.13)$$

With nucleon spins $\vec{s}_1$ and $\vec{s}_2$ of $\frac{1}{2}$, the value of $\vec{s}_1.\vec{s}_2$ is, for triplet (S=1) states $\frac{1}{4}\hbar^2$ and for singlet states (S=0) is $-\frac{3}{4}\hbar^2$. Thus, a spin dependent expression of the form $V_s(\vec{r})\,\vec{s}_1.\vec{s}_2$ can be calculated and added to the internucleon potential. The effect of such potential will resulted in having different cross sections for the singlet and the triplet states. The magnitude of V_s can be adjusted to give the correct differences between the singlet and triplet cross sections and the radial dependence can be adjusted to give the proper dependence on energy. In this study, the value of V_s that fulfils this requirement was found to be ~30 MeV. The general expression of V_s that includes both the triplet as well as the singlet contributions to the internucleon potential is given by equation (2.20).

The tensor contribution to give the internucleon potential was taken to be of the form $V_T(\vec{r})\,S_{12}$ as given by equation (2.25). The $V_T(\vec{r})$ function gives the force and the proper radial dependence and magnitude is included in equation (2.24). The expression in equation

(2.23) gives the force proper tensor character that also averages to zero over all angles.

The spin-orbit level scheme contains the major effects relating to the internucleon potential deformation and can be evaluted via

$$\left\langle \vec{L}.\vec{S} \right\rangle = \frac{1}{2}\left\langle J^2 - L^2 - S^2 \right\rangle = \begin{cases} \frac{\hbar^2}{2}\ell, & \text{for } j = \ell - \frac{1}{2} \\ -\frac{\hbar^2}{2}(\ell+1), & \text{for } j = \ell + \frac{1}{2} \end{cases} \qquad (4.14)$$

Where we have used $s = \frac{1}{2}$ for the spin of a nucleon.

The isospin is mathematically like the spin as been introduced. Integration over spin-isospin spaces, these equations can be simplified by using

$$\left\langle SS_z \left| s_1.s_2 \right| SS_z \right\rangle = [2S(S+1) - 3]/4 \qquad (4.15)$$

and

$$\left\langle T\tau_z \left| \tau_1.\tau_2 \right| T\tau_z \right\rangle = [2T(T+1) - 3]/4 \qquad (4.16)$$

Using equation (2.31), the isospin contribution to the NN potential can be written as:

$$U^t(\vec{r},\vec{\tau}) = \vec{\tau}.\sum_{i=1}^{A} \left\langle i \left| \vec{\tau}_i J_t \ (\vec{r}_i - \vec{r}) \right| i \right\rangle \qquad (4.17)$$

Where $\vec{\tau}$ is the isospin of the nucleon, and $|i\rangle$ is the single-nucleon state. The isospin potential can be approximated by (Seimens and Jensen, 1987):

$$U^{t} = -\tau_z \frac{N-2}{A} 48 f(r) \tag{4.18}$$

The interaction energy of the two-body potential in equation (2.31) can be written as (Seimens and Jensen, 1987):

$$J_t^{(2)} = \frac{48\,\text{MeV}}{A} \sum_{i \neq k} \vec{\tau}_i . \vec{\tau}_k \tag{4.19}$$

For a state of given isospin T and third component $T_z = \frac{1}{2}(Z-N)$, this gives immediately

$$\begin{aligned} \langle T\tau_z | J_t^{(2)} | T\tau_z \rangle &= \frac{48}{A} \sum_i \langle T\tau_z | \vec{\tau}_i . (\sum_k \vec{\tau}_k - \vec{\tau}_i) | T\tau_z \rangle \\ &= \frac{48}{A} [T(T+1) - \frac{1}{2}(\frac{1}{2}+1) \sum_{i=1}^{A} I] \\ &\approx \frac{48}{A} T(T+1) - 36, \text{MeV} \end{aligned} \tag{4.20}$$

Thus the nuclear force corresponds to the lowest energies should be symmetric in the space-spin state. This is because the lowest energy is obtained with the lowest possible values of T, i.e., $T = |T_z| = \left|\frac{1}{2}(N-Z)\right|$ (Jelley, 1990).

Now, let us outline the main processes in one iteration of the calculations in sequence. The following iteration is read and stored, the

number of occupied states and their 2j-values, the 2j, L and n values of the orbits.

In the basis $n\ell jm_j$, the first quantities to be calculated and stored are the $n\ell jm_j \, V \, n'\ell'j'm'_j$, the matrix elements according to the expressions in equations (4.4-4.11). The appropriate two-body matrix elements of the effective interaction according to equation (1.12) can be obtained by summing all contributions from all nucleons. The effective interaction is calculated according to equation (4.3) using the matrix elements from the second, the third, the fourth and the fifth multipole deformation parameters using equations (4.4)-(4.11). This set of matrix elements is used to calculate the corresponding total energy of the nucleus. This ends the first iteration. The second iteration proceeds as in the first, in order to calculate the effective interaction and total energy for other states. This procedure process will continue until convergent solution is achieved.

4.3 The model testing

In this section we shall test the model against the well known results of the deuteron and the Nilsson results.

In the deuteron case, we have used a central radial interaction of the form $V(\vec{r}_1 - \vec{r}_2)$ to evaluate the two-body matrix elements. Expanding

the central interaction in the orthonormal set of Legendre polynomials, we can obtain the result

$$V(\vec{r}_1 - \vec{r}_2) = \sum_{k=0}^{\infty} V_k(r_1, r_2) P_k(\cos\theta_{12}) \qquad (4.21)$$

where the index k counts the various multipole components present in the expansion. For a short range interaction to be represented by delta-type, $\delta(\vec{r}_1 - \vec{r}_2)$, interaction, the expansion coefficients $V_k(r_1, r_2)$ can be written as (Heyde, 1999):

$$V_k(r_1, r_2) = \left(\frac{\delta(r_1 - r_2)}{r_1 r_2}\right)\left(\frac{2k+1}{4\pi}\right) \qquad (4.22)$$

When nucleons interact in the nuclear ground state, their relative momentum is not greater than twice the Fermi momentum, corresponding to scattering beam of 140 MeV nucleons. Thus it is reasonable to say that the effective interaction in nuclear matter may have a similar approximate expansion. Skyrme propose (Negéle, 1982) that the plane wave matrix elements of the local density dependent effective interaction in nuclear matter could be expanded in a power series in the relative momenta of the initial and final two-body states

$$\begin{aligned}\vec{k}_1\vec{k}_2\,\Im\,\vec{k}_1'\vec{k}_2' &= \left(\frac{1}{2\hbar}\right)^3 \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_1' - \vec{k}_2')\{\Im_0(1 + \frac{1}{8}\Im_1[(\vec{k}_1 - \vec{k}_2)^2 \\ &+ (\vec{k}_1' - \vec{k}_2')^2]/\hbar^2 + \frac{1}{4}\Im_2[(\vec{k}_1 - \vec{k}_2)(\vec{k}_1' - \vec{k}_2') + \ldots]/\hbar^2\} \qquad (4.23)\end{aligned}$$

where primes denote final states. Since nuclear forces depend strongly on spin, the power series coefficients $\mathfrak{I}_0, \mathfrak{I}_1$ and $\mathfrak{I}_2$ can be introduced as operators in the spin space. In this space, the effective interaction can be approximated by

$$\mathfrak{I}_k \rightarrow \mathfrak{I}_k(1+\chi_k P_S) \tag{4.24}$$

where $P_S = \frac{1}{2}(1+4\vec{s}_1.\vec{s}_2)$ (spin exchange operator) is -1 for the singlet and $+1$ for the spin-triplet states of relative motion (P_s values according to $\vec{s}_1.\vec{s}_2$ eigenvalues). The final result concerning the two-body matrix elements results in the expression

$$j_1, j_2; JM\ \delta(\vec{r}_1 - \vec{r}_2)\ j_1, j_2 : JM\ _{\text{ans.}} = F^0(2j_1+1)(2j_2+1)\begin{bmatrix} j_1 & j_2 & J \\ 1\ 2 & -1\ 2 & 0 \end{bmatrix}^2 \times \frac{1+(1)^{j_1+j_2+J}}{2} \tag{4.25}$$

where F° is a Slater integral expressing the strength of the interaction. The j-dependent however, only rests in the Wigner 3j-symbol and the phase factor.

The effective interaction of the two nucleons system in a deuteron nucleus is shown in Figure 4.1. The total effective NN potential is obtained by adding contribution from all terms. The calculated results are in agreement with the experimental results in which approximately 70 MeV for the depth of potential well is predicted. This figure can be used as an example to figure out the relation between L and V_{eff}. For L=0,

there is no repulsive centrifugal force and no spin-orbit interaction (Cohen, 1971). We note that as L increases the potential becomes narrower and shallower the justification is as follows (Cohen, 1971): the narrowing arise because the particle with high orbital angular momentum is strongly repelled from small radii by the centrifugal force. When the potential well becomes narrower, the wave length of a wave which fits a given number of half wave length into the width of the well becomes smaller, so a shorter wave length leads to higher value of V. Hence, the distance from the bottom of the equivalent potential well to any energy level increases with L. Moreover, the bottom of the well rises with increasing L so the energy levels are pushed up as shown in Figure 4.1.

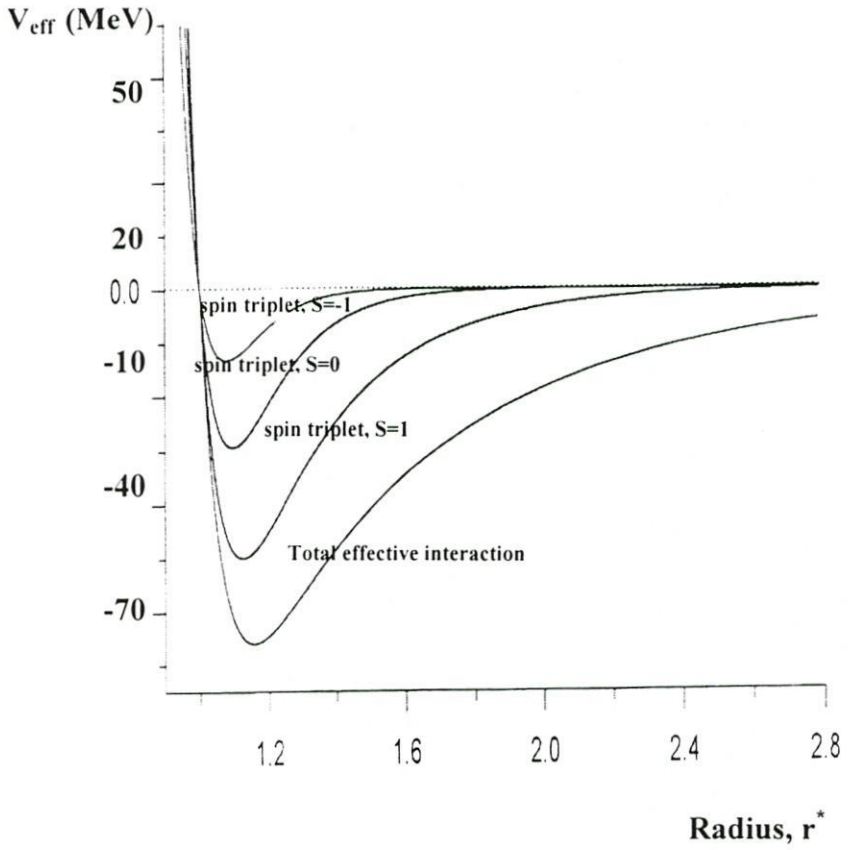


Figure 4.1. The various terms of the effective interaction for deuteron

Before proceeding in citing the deformation results, let us examine the Nilsson model by considering the spin-orbit interaction only and have a look at this deformation energy spectra illustrated in Figure 4.2. The results shown in Figure 4.2 obtained by assuming a deformed surface are quite similar to those obtained by Nilsson (Cohen, 1971; Jelley, 1990)

who assumed a deformed potential. Accordingly, we shall discuss other results belonging to different deformed nuclides.

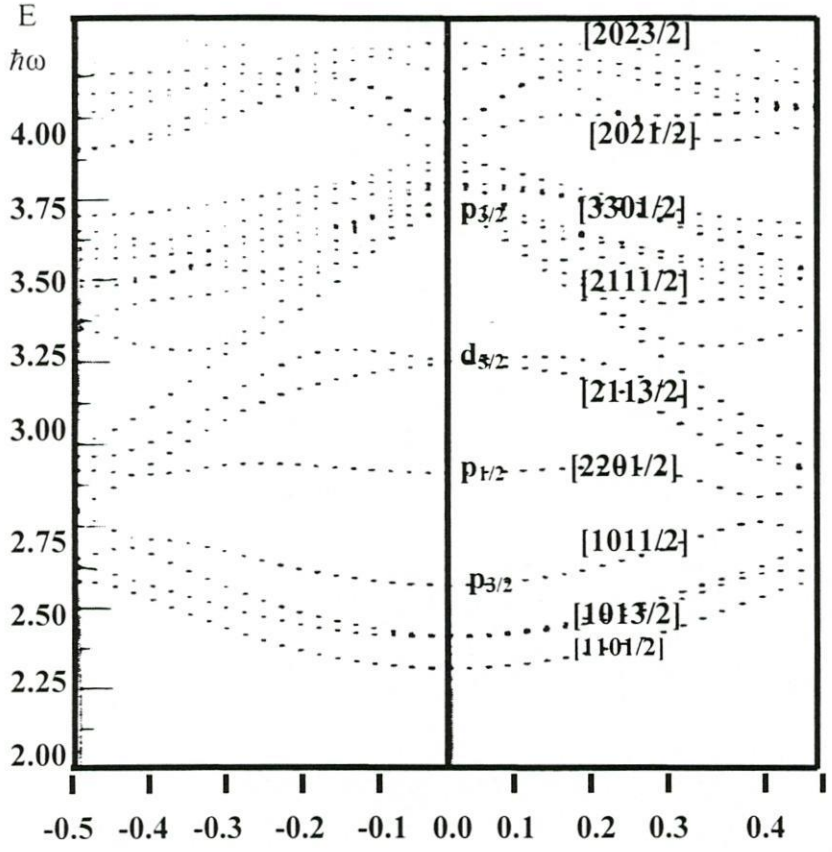


Figure 4.2. Orbit energies of ^{12}C deformed nuclide (A<20) as a function of deformation parameter.

One can characterize a deformed eigenstate with the quantum numbers $[Nn_z\Lambda\Omega]$ where N: principle quantum number, n_z : projection of N, $\Lambda = m_l$ and $\Omega = \Lambda + \Sigma$ where $\Sigma = \pm \frac{1}{2}$ the spin projection.

4. 4 Results for ^{16}O .

The $V_{eff}(^{16}O)$ calculated values for some nucleon states are displayed in Figure 4.3 and some deformed energy levels are shown in Figure 4.4. The occupied orbits we find for ^{16}O are in agreement with the standard shell model for A<20, where A is the mass number (Heyde, 1999). Besides, the effective interactions are in agreement with the results obtained for the ^{16}O nuclide (Faessler *et al.*, 1984)

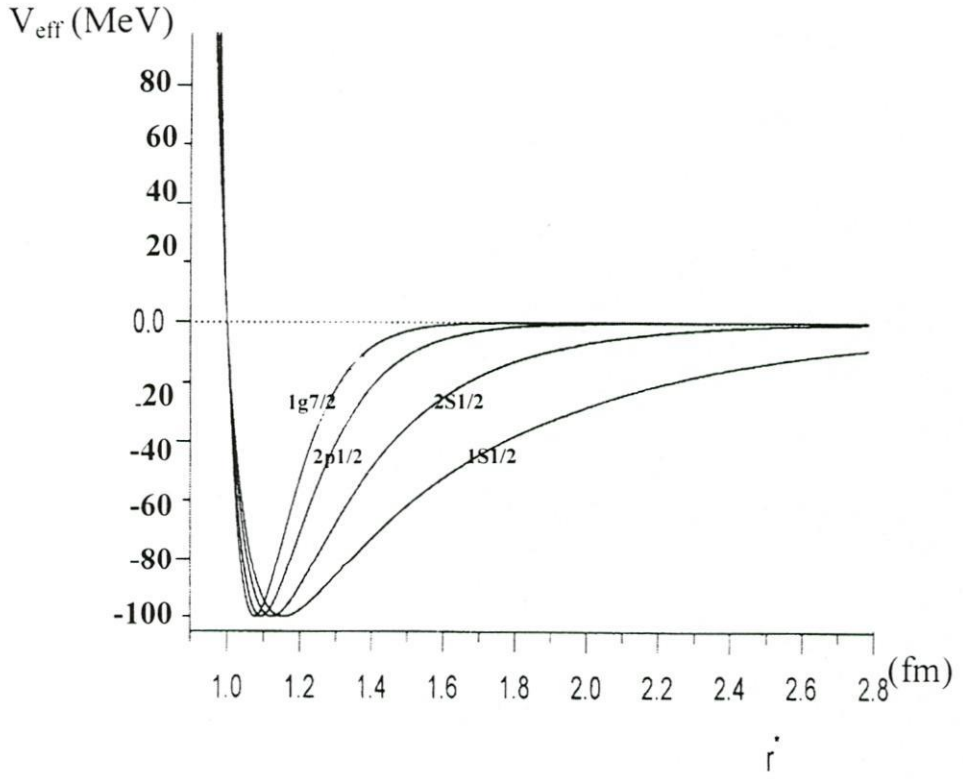


Figure 4.3. Typical effective interaction for nucleons in different states of ^{16}O deformed nuclide.

Figure 4.4 shows the ratio of the total orbit energy to an oscillator energy of $\hbar\omega$=14 A MeV for ^{16}O nucleus as a function of the deformation parameter α. Considerations of energy ratio was adopted for

comparing the obtained results with that of Nilsson results. For positive α we have prolate deformation and for negative α we have oblate deformation (spheroidal deformation), for non-zero α each level splits into (2j+1)/2 levels, each level is two-fold degenerate with eigenvalues ±mj (Jelley, 1990).

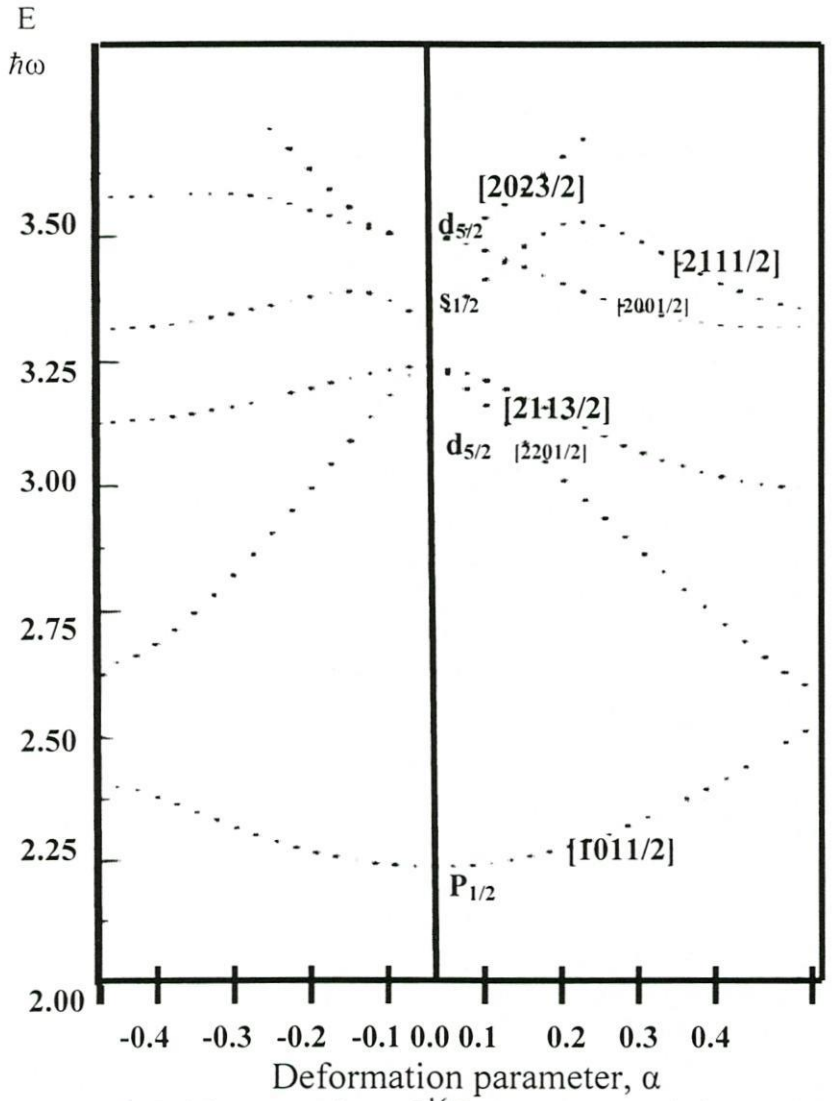


Figure 4.4. Orbits energies of ^{16}O (N, Z <20) deformed nuclide as a function of deformation parameter.

Based on Figure 4.4 results, the deformation effects can be explained as follow: For α=0 the energy levels are designated by the j-

quantum number (spherical nucleus) and all the states for m_j are degenerate, but for $\alpha \neq 0$ the potential becomes a function of θ (tensor force). Accordingly, the orbital angular momentum $\vec{L}$ is not conserved and consequently $\vec{L}$ is not good a quantum number. On the other hand, V is not a function of φ so we have no torque in the z-direction hence the z-component is conserved so m_j is a good quantum number. For this case ($\alpha \neq 0$) the energy states are designated only by m_j and the parity which is unchanged because of the deformation. For an oblate deformation we note that the energy increases with decreasing m_j. If the nucleus is prolate small m_j orbits lie lower in energy than the large m_j orbits (Cohen, 1971)

The model has numerous parameters: masses, coupling constant, and other parameters some are constrained by the coupled channel fits to the NN scattering data. The equilibrium root-mean square radius (rms) have been calculated and a value of 2.61 fm corresponding to total energy -142.7 MeV. The experimental rms radius for ^{16}O is 2.74 fm and the measured binding energy is -127.7 MeV. The difference between the experimental quantities and our calculated results may be attributed to neglecting Coulomb interaction which gives a repulsive energy of about 14 MeV for ^{16}O and including several deformation terms.

4.5 Results for ^{40}Ca

Here we restrict our calculations to a nucleus of mass number equals 40 and radius of 3.41 fm. Again the Coulomb effects are neglected. The calculations proceed in the same manner as the calculations for ^{16}O. Our results of $V_{eff}(^{40}Ca)$ are presented in Figure 4. 5

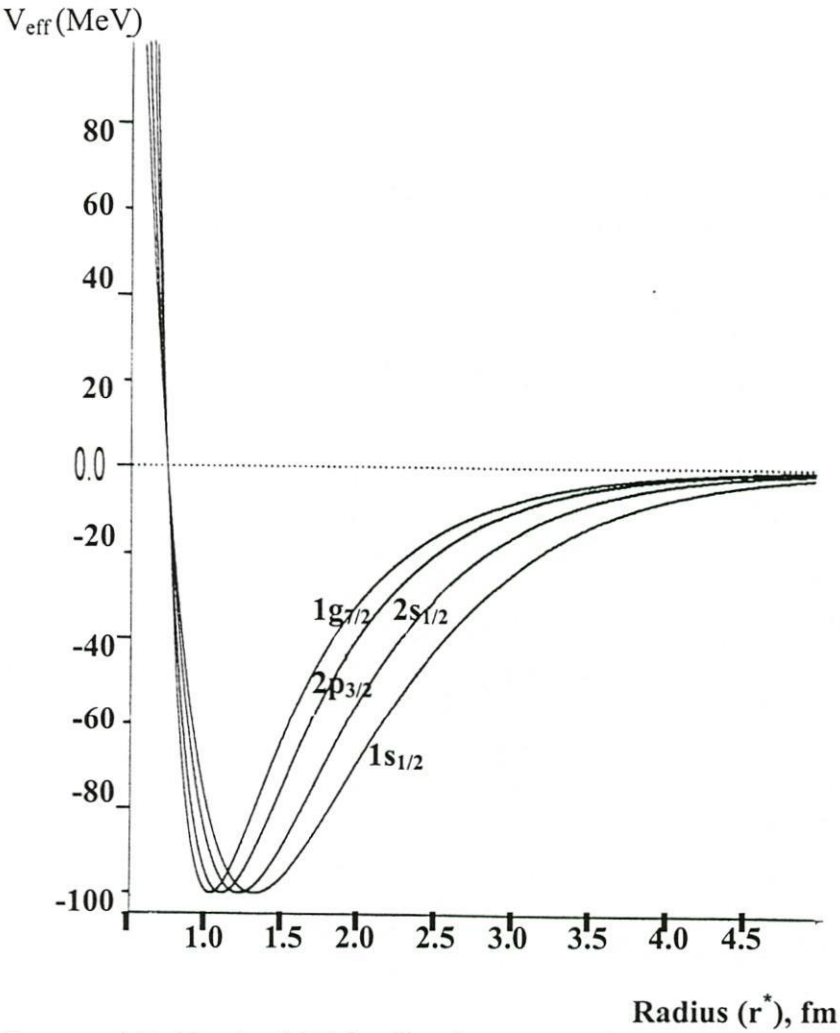


Figure 4.5. Typical NN effective potential curves for nucleons inside ^{40}Ca deformed nuclide.

and the deformation effect displayed in Figure 4.6 for ^{40}Ca. The ^{40}Ca results are similar to those results obtained for deformed nuclide ^{16}O. For the same L-values for the nuclides, we find the same depth of the potential well. This means the potential depends on L-value of the energy level. The ratio of the orbits energies with oscillator its energy given as $\hbar\omega$=14 A MeV for ^{40}Ca nucleus as a function of the deformation parameter α is plotted in Figure 4.6.

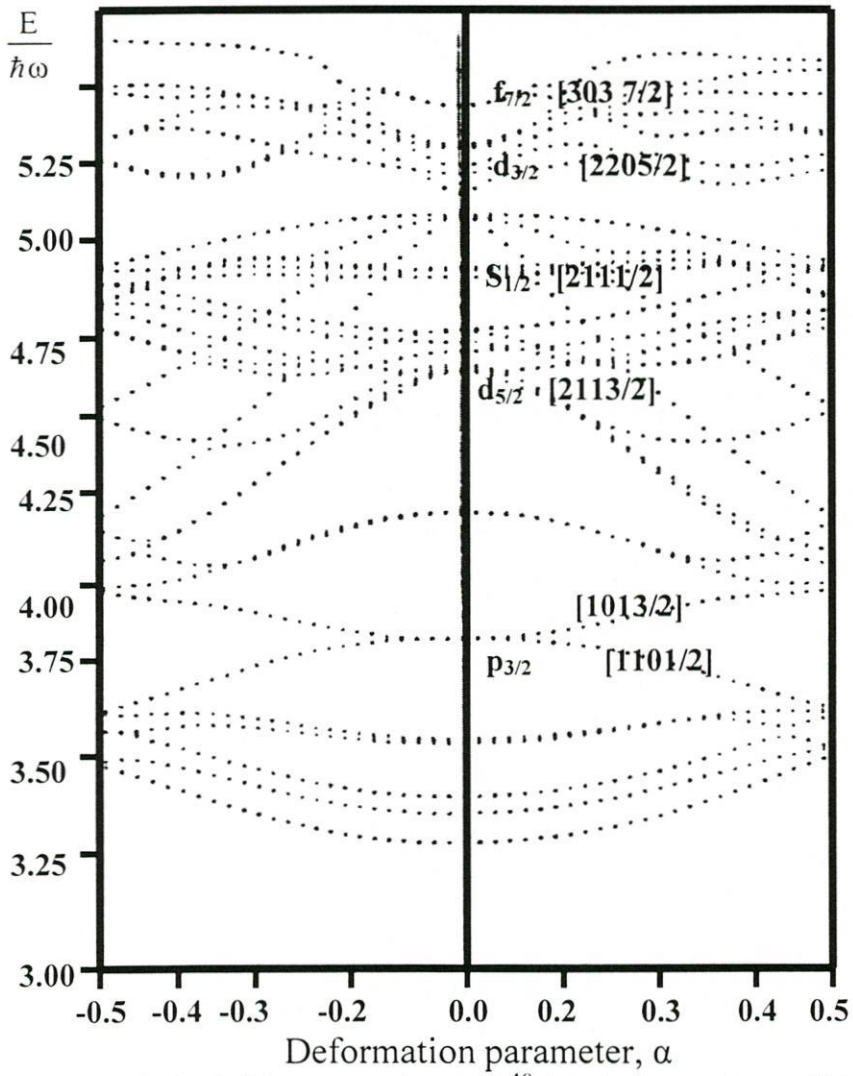


Figure 4.6. Orbits energies of ^{40}Ca deformed nuclide as a function of deformation parameter.

The equilibrium rms radius of ^{40}Ca is 3.38 fm and the corresponding total energy is -414.5 MeV. The experimental rms radius for ^{40}Ca is 3.48 fm and the binding energy is -342 MeV. The difference between our calculated results and the experimental results may be attributed to Coulomb effects which give a repulsive energy of about 70 MeV.

The results presented in this study for ^{16}O and ^{40}Ca are in agreement within <15% with other results obtained for deformed nuclides ($10<A<50$) (Nilsson *et al.*, 1955; Cohen, 1971; Heyde, 1999). Accordingly, we can conclude by stating the following conclusion: all nuclides have a certain deformation to a certain extent and the proposed spherical shape for nuclides is a special case from the general case, the deformed one. Therefore, we recommend to include deformation effects to have better results for nuclear structure.

Chapter Five

Conclusion and Future Work

5.1 Introduction

In this chapter we state the conclusion we believe and the expected extension in this scope.

5.2 Conclusion

In this study, the effective nuclear potential between nucleons is studied for deformed nuclei, in the hope that this knowledge will be useful in studying more complicated nuclear systems. Despite the good agreement between the results obtained using this model and other results obtained using other models such as HF, Nilsson, etc. there has been a space for uncertainties as in all such models. We remark here that one uncertainty that could be mentioned is the possibility of multiple parameter sets yielding equivalent fits to NN data. With this in mind, we sense some freedom in adjusting the over all strength of the transitions potentials. We have therefore increased the strength of the transition potentials in the effective interaction Hamiltonian by multiplying the matrix elements of transition potentials by a factor of 5 and 10 and repeated the calculations with the same strengths. We found that the number of other particles in the occupied orbits very small (of the order of 10^{-12}), and the effective interaction has not changed.

Generally speaking, from the obtained results in this model, we may draw the following conclusions. First, both V_{eff} versus rms radius curves manifest the general shape of a parabola as expected. Second, the energy is more sensitive to variations in the rms radius than other parameters. Third, it is possible to stretch the nucleus to larger radii.

Therefore, we recommend more extensive discussions and more calculations have to be conducted in order to assure the liability of the model.

5.3 Future work

This study can be extended to include:

1- The study of superdeformed (SD) nuclear shapes for more testing to nuclear shell structure. Since SD states of large deformations at high spin have been revealed in many nuclides (Bäck *et al.*, 1999).

2- The pion exchange force. The pions play a special role in the forces between nucleons, the longest-range part of this force arise from the nucleus, exchange of a single pion, just as the long-range Coulomb interaction between electrically charged particles is due to exchange of photons.

3- Pion-nucleon interaction should be considered for energies higher than 500 MeV.

4- Relativistic effects can be included.

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