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One-Way and two-way Interaction Gas-Particulate Flow through Porous Media

By

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Dedication

This thesis is respectfully dedicated to my beloved parents, my wife, my daughter, my beloved brothers and my sister for their help, support and encouragement.

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Abstract

Sets of partial differential equations describing the motion of a dusty fluid in porous media are developed in both cases of one–way and two–way interaction. The governing equations are derived using intrinsic volume averaging and are based on Saffman's dusty gas model. The effect of the porous microstructure on the flowing mixture is analyzed via the concept of representative unit cell (RUC) and distinction is made between flow in consolidated and granular porous media.

The equations governing the flow of a dusty fluid through different types of porous media, and the boundary conditions associated with each equation are investigated. Numerical simulation is carried out for the flow of a dusty fluid through naturally occurring porous media in a configuration of interest. Results indicate that the value of the Forchheimer drag coefficient C_d should be less than 0.75 .

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Chapter one

Introduction

1.1 Importance of Gas – particle flow through porous media

Gas–particle flow, dusty fluid flow and the flow of suspension through porous media have received considerable attention due to the importance of these types of flow in studies associated with the design of industrial filters, liquid – dust separators and water purification plants.

Applications of the above types of flow in the environment are exhibited in solid – waste and pollutant dispersal and storage in the ground layers.

Fluid flow through porous media has become a topic of increasing importance due to its direct applicability in many physical situations including the prediction of oil reservoir behavior, groundwater flow, irrigation problems and the biophysical sciences where the human lungs, for example, are modeled as porous layers.

The above and many other applications emphasize two fundamental aspects of the study of gas particulate flow in porous media:

i) Models describing gas–particulate flow through porous media must accurately take into account the effect of the porous media quantities on the flow constitute on each other. This necessitates developing models that take into account the porous microstructure and its effects on the flowing phases.

ii) Solution of boundary and initial value flow problems that should accurately predict the flow patterns and the nature of the flow fields of the constituents involved.

1.2 Problem formulation and derivation of field equations

In this section a general derivation of the dusty fluid flow equations is presented, this developed general model which accounts for inertial effects and two - way interaction between the phases present is cast in different forms that take into account different porous media structures.

The model's equations that developed are based on the following physical assumptions on the porous medium and on the dusty fluid.

1. It is assumed that the dusty fluid flows through a porous matrix consisting of a sparse distribution of particles fixed in space.
2. It is assumed that porosity change of the medium due to the retention of dust particles on possible retention site are negligible and that clogging of the pores does not occur and thus straining action is neglected.
3. The permeability to the fluid is assumed to be constant and the reduction in permeability is therefor insignificant.
4. The distribution of the solid grains comprising the solid matrix is uniform.
5. The suspended dust particles in the viscous fluid are uniform in shape and size with particle diameter large enough so that diffusion by Brownian motion can be neglected and small enough so that clogging of the pores can also be neglected.
6. Flow redistribution in the porous medium is assumed to be unaltered by the presence of captured particles.
7. It is assumed that the main mechanism of particles capture is the direct interception on the surface of the grains of the porous matrix .The effect of settling is ignored in this study by neglecting gravitational forces.

8. It is assumed that the dust behaves as a continuum and that the porous structure and dimensions allow for this behavior and allow for the possibility of volume averaging over a certain control volume.
9. The dust particles are assumed to be non – interacting chemically or otherwise, and the dust concentration by volume is very small.

To this end we consider the fluid flow through a porous medium having a constant porosity and a constant permeability to the fluid. The fluid saturating the chosen medium is assumed to consist of two phases, the fluid – phase and the dust-phase. When the fluid – phase and the porous medium compressibility effects are neglected, the averaged Navier – Stokes equations averaged over a control volume of the porous medium, take the following macroscopic form in the absence of momentum transfer between the phases involved [5]

$$\gamma_j \vec{\rho}_j \left[\frac{\partial \hat{\vec{u}}_j}{\partial t} + (\hat{\vec{u}}_j . \vec{\nabla}) \hat{\vec{u}}_j \right] = - \gamma_j \vec{\nabla} \vec{p}_j + \gamma_j \mu_j \nabla^2 \hat{\vec{u}}_j + \vec{f}_j \quad . \tag{1.1}$$

The macroscopic equation of continuity in the absence of sources and sinks is given by

$$\frac{\partial \gamma_j \vec{\rho}_j}{\partial t} + \vec{\nabla} . \gamma_j \vec{\rho}_j \hat{\vec{u}}_j = 0 \; . \tag{1.2}$$

Where γ_j is the volume fraction of the j^{th} phase in the system, $\vec{\rho}_j$ is the intrinsic volume – averaged density of phase *j*, $\vec{p}_j$ is the intrinsic volume – averaged pressure of phase *j*, $\hat{u}_j$ is the volume – averaged velocity vector of phase *j*, μ_j is the viscosity of phase *j* and $\vec{f}_j$ is the sum of external forces exerted on phase *j*.

The volume fraction γ_j satisfies the relation

$$\sum_j \gamma_j = 1 \tag{1.3}$$

with γ_j is defined as

$$\gamma_j = V_j / V. \tag{1.4}$$

Where V denotes the control volume and V_j is the volume occupied by phase j.

Since it is assumed that the dust–phase has a very small concentration by volume, it is reasonable to take the volume fraction of each of the phases to be constant so that the intrinsic volume - averaged quantities in equation (1.1) are related to the volume – averaged quantities by

$$\hat{\Gamma}_j = \gamma_j \vec{\Gamma}_j \quad . \tag{1.5}$$

Where $\hat{\Gamma}_j$ refers to a volume – averaged quantity and $\vec{\Gamma}_j$ refers to an intrinsic volume-averaged quantity.

In light of (1.5) equation (1.1) and (1.2) take the following forms respectively,

$$\hat{\rho}_j \left[\frac{\partial \hat{\vec{u}}_j}{\partial t} + (\hat{\vec{u}}_j \,.\, \vec{\nabla}) \hat{\vec{u}}_j \right] = - \vec{\nabla} \hat{p}_j + \mu_j \nabla^2 \hat{\vec{u}}_j + \vec{F}_j \tag{1.6}$$

$$\frac{\partial \vec{\rho}_j}{\partial t} + \nabla . \hat{\rho}_j \hat{u}_j = 0 \quad . \tag{1.7}$$

Where

$$\vec{F}_j = \vec{f}_j / \gamma_j \quad . \tag{1.8}$$

Dropping "^" from equations (1.6) and (1.7)and expressing them for the first and second phases we obtain

i) For the fluid – phase

The continuity equation is

$$\frac{\partial \rho_1}{\partial t} + \vec{\nabla} \,.\, \rho_1 \overrightarrow{u_1} = 0 \quad . \tag{1.9}$$

The momentum equation is

$$\rho_1 \left[\frac{\partial \overrightarrow{u_1}}{\partial t} + \left(\overrightarrow{u_1} \,.\, \overrightarrow{\nabla} \right) \overrightarrow{u_1} \right] = -\vec{\nabla} p_1 + \mu_1 \nabla^2 \overrightarrow{u_1} + \vec{F}_1 \quad . \tag{1.10}$$

ii) For the dust - phase

The continuity equation is

$$\frac{\partial \rho_2}{\partial t} + \vec{\nabla} \,.\, \rho_2 \vec{u}_2 = 0 \quad . \tag{1.11}$$

The momentum equation is

$$\rho_2 \left[\frac{\partial \vec{u}_2}{\partial t} + \left(\vec{u}_2 \,.\, \vec{\nabla} \right) \vec{u}_2 \right] = - \vec{\nabla} p_2 + \mu_2 \nabla^2 \vec{u}_2 + \vec{F}_2 \quad . \tag{1.12}$$

Where $\vec{u}_1$ is the fluid – phase velocity vector, $\vec{u}_2$ is the dust – phase velocity vector, p_1 is the fluid – phase partial pressure, p_2 is the dust – phase partial pressure, μ_1 is the fluid – phase viscosity, μ_2 is the dust – phase viscosity, $\vec{F}_1$ is the sum of external forces exerted on a unit volume of the fluid – phase and $\vec{F}_2$ represents the sum of external forces exerted on a unit volume of dust – phase .Neglecting the dust–phase partial pressure and the dust –phase viscosity, and expressing the dust–phase density ρ_2 in terms of the macroscopic particle number density *N*, and the mass of a single dust particle *m*, then equation (1.12) takes the form

$$mN \left[\frac{\partial \vec{u}_2}{\partial t} + \left(\vec{u}_2 \,.\, \vec{\nabla} \right) \vec{u}_2 \right] = \vec{F}_2 \quad . \tag{1.13}$$

The dust – phase continuity equation (1.11) can then be written in the form

$$\frac{\partial N}{\partial t} + \vec{\nabla} .\, N \overrightarrow{u_2} = 0 \tag{1.14}$$

where *N* is the particle number density.

1.2.1Nature of forces acting on the fluid-phase and dust-phase

i) Forces acting on the fluid-phase:

When the flow considered is that of a dusty fluid, then there are two forces acting on the fluid–phase, one of these is a frictional force due to the solid matrix of the medium, and the other is due to the influence of dust on the clean fluid.

Let $\vec{F}_{11}$ be the frictional force per unit volume on the fluid – phase. This friction force has to balance Darcy's pressure gradient in the medium. In the absence of body forces, Darcy's law is expressed in terms of the seepage type and takes the form

$$\vec{u}_1 - \vec{u}_2 = -\frac{k}{\mu_1}\vec{\nabla}\hat{p} \quad . \tag{1.15}$$

The expression for $\vec{F}_{11}$ can thus be obtained [7]

$$\vec{F}_{11} = -\frac{\mu_1}{k}(\vec{u}_1 - \vec{u}_2) \quad . \tag{1.16}$$

Let $\vec{F}_{12}$ denotes the force due to the influence of dust on the clean fluid. The assumption of a small concentration of dust by volume leads to the following expression for the effect of dust on the clean fluid

$$\vec{F}_{12} = C_r N(\overrightarrow{u_2} - \overrightarrow{u_1}) \quad . \tag{1.17}$$

Where C_r is the coefficient of resistance in the porous medium which is considered here to be constant under the assumption of uniform size and distribution of the dust particles and N is the macroscopic number density.

By substituting the contributions to $\vec{F}_{11}$ given by equation (1.16) and (1.17) into Equ. (1.10), the fluid – phase momentum equation takes the form

$$\rho_1\left[\frac{\partial \overrightarrow{u_1}}{\partial t} + \left(\overrightarrow{u_1}.\vec{\nabla}\right)\overrightarrow{u_1}\right] = -\vec{\nabla} p_1 + \mu_1 \nabla^2 \overrightarrow{u_1} + C_r N(\vec{u}_2 - \overrightarrow{u_1}) + \frac{\mu_1}{k}(\vec{u}_2 - \overrightarrow{u_1}) . \tag{1.18}$$

The fluid–phase continuity equation (1.9) is then expressed in the form

$$\vec{\nabla} . \vec{u}_1 = 0 \; . \tag{1.19}$$

ii) Forces acting on the dust–phase

The term $\vec{F}_2$ in equation (1.13) represents the sum of external forces exerted on a unit volume of the dust–phase. Although it might be possible to consider that $\vec{F}_2$ is composed of two forces $\vec{F}_{21}$ and $\vec{F}_{22}$, where the first force represents the effect of the fluid–phase on the dust–phase and the other due to the solid matrix which is smaller than the first and therefore $\vec{F}_{22}$ is negligible.

Hence the only contribution to the force $\vec{F}_2$ is due to the fluid–phase influence on the dust, thus $\vec{F}_2$ is given by

$$\vec{F}_2 = \vec{F}_{21} = C_r N(\vec{u}_1 - \vec{u}_2) \quad . \tag{1.20}$$

Substituting equation (1.20) into (1.13), the dust–phase momentum equation will always assume the form

$$mN\left[\frac{\partial \vec{u}_2}{\partial t}+(\vec{u}_2.\vec{\nabla})\,\vec{u}_2\right]=C_r N\,(\vec{u}_1-\vec{u}_2) \quad . \tag{1.21}$$

While the dust–phase continuity equation is given by (1.14) .

The general equations governing the flow of an incompressible dusty fluid in a homogeneous porous medium are given by equations (1.18) and (1.19) while the fluid–phase coupled with the dust–phase equations are given in equations (1.14) and (1.21) respectively.

1.3 Overview of previous work

A set of general averaged transport equations for a multiphase system consisting of an arbitrary number of phases, interfaces and contact lines is established. A structure for the system is proposed and hydrodynamic interaction between the phases, interfaces and contact lines is also structured [18].

The common features of dispersed two–phase flows from a continuum–mechanical approaches are examined [5].

In [21] currently used averaging theorems are extended to allow for averaging volumes, which vary in space and time.

A model describing the flow of a dusty gas in porous media was developed [12] and is based on the differential equations approach. It incorporates the factors affecting the gas–particulate mixture in the type of porous media where Brinkman's equation is applicable and thus inertial effects were ignored under the assumption of creeping motion in a high–porosity medium.

Dusty fluid flow through porous media with applications to deep filtration has been widely studied [14] via the empirical and semi–empirical approach, and takes into account the optimal design of filters, liquid–dust separation and clogging mechanisms of the pores. In the case of flow of suspensions through a deep porous bed [14] gave a review of the a available literature and outlined the mechanisms of deposition and the possible capturing processes which include sedimentation, inertial impacting, direct interception, hydrodynamic effects and diffusion by Brownian motion. In cases where the particle size is greater than one micro-meter the particle diffusion is negligible [18] while the effect of inertial impacting is negligible if the fluid–phase is liquid [21]. In addition, if the particles are spherical in shape then the hydrodynamic effects can be neglected [14]. This leaves the interception capture mechanism to be dominant and the particles are captured mainly on the surfaces of the media grains. Settling of particles by sedimentation is also possible due to the high density associated with the dust particles.

Equations governing flow of a dusty fluid between tow porous flat plates with suction and injection are developed and closed – form solutions for the velocity profiles, displacement thickness and skin friction coefficients for both phases are obtained. Graphical results of the exact solutions are presented and discussed [3].

The problem of gas–particulate flow through a two – dimensional porous channel bounded by curved boundaries is considered [13].

Entry conditions to a porous channel compatible with the equations governing the flow of a dusty fluid in porous media are derived [10].

The equations governing the flow of a dusty fluid through isotropic, granular porous media are developed [1]. A set of fluid–phase momentum equations, and the boundary conditions associated with each equation are investigated [10].

A derivation of time–dependent field equations that are postulated to govern the flow of a two-phase fluid, with one of the phases being an oil phase, through porous sediment are presented [22].

1.4 Scope of the current work.

This work probes the recesses of single-phase and two – phase flow models describing the motion of the flow through porous media. Different forms of fluid phase momentum equations are derived and applied in a variety of settings and in different types of porous media. Averaging processes are applied so as to obtain equations that do not contain the details of the flow. The analysis to the governing equations are based on averaging the volume of these equations over a representative elementary volume (REV), and the microscopic inertial effects which are accounted for via the representative unit cell (RUC), in an isotropic porous media.

An averaging rules were used to average Saffman's dusty gas equations model so as to develop a set of partial differential equations which describing the motion of a dusty fluid in porous media taking into account the cases of one-way and two – way interactions between the fluid–phase and the dust–phase. Analysis of the deviation terms, analysis of the surface integral and distinction between flow in consolidated and granular porous media were considered in the current work.

In this work, we offer analysis of Darcy – Lapwood – Brinkman model that was recently derived [8]. Numerical solution to this model is obtained in a configuration of interest in an attempt to illustrate the determinate characters of the flow variables.

1.5 Organization of the thesis

In this chapter, a general introduction has been given to provide an overview of the scope of the current thesis. Chapter two deals with the basic concepts of single and two–phase flow models through porous media. In chapter three, the averaging rules are used to average a dusty gas flow equations through porous media so as to derive a set of partial differential equations. Chapter four tackles Darcy–Lapwood–Brinkman (DLB)–model in naturally–occurring porous media, boundary conditions and numerical simulation. While the conclusion will summarize the accomplishments of this work and some recommendations for future work.

Chapter two

Single and Two–Phase Flow Models through Porous Media

Introduction

The developed general model, which accounts for inertial effects and two–way interaction between the phases present is cast in different forms that take into account different porous media structure, rendering models that are valid for different flow situations in different types of porous media.

2.1 Definition of the porous medium

The porous matrix may be viewed as consisting of a number of dust collectors fixed in an ordered or disordered manner in space. This definition of the porous medium excludes compacted porous Darcy's Law is valid. It also excludes the naturally occurring porous media where the Forchheimer's equation is applicable.

2.2 Governing equations

We now consider the fluid flow through a porous medium having a constant porosity and a constant permeability to the fluid. The fluid saturating the chosen medium is assumed to consist of two phases: the fluid–phase and the dust–phase.

The general equations governing the flow of incompressible dusty fluid in a homogeneous porous medium are given by the following equations

i) For the fluid–phase

The continuity equation is

$$\nabla \, . \, u = 0 \tag{2.1}$$

and the linear momentum equation is

$$\rho\left[\frac{\partial u}{\partial t}+(u_1 . \nabla)\, u_1\right]=-\nabla P+\mu \nabla^2 u_1+C_r\, N(u_2-u_1)+\frac{\mu}{\eta}(u_2-u_1)\ . \tag{2.2}$$

ii) For the dust–phase

The continuity equation is

$$\frac{\partial N}{\partial t} + \nabla . N u_2 = 0 \qquad (2.3)$$

and the linear momentum equation is

$$m N \left[\frac{\partial u_2}{\partial t} + (u_2 . \nabla) u_2 \right] = C_r N (u_1 - u_2) . \qquad (2.4)$$

Where u_1 and u_2 are the fluid and dust macroscopic velocity vectors respectively, N is the dust particle number density, C_r is the coefficient of resistance in the porous medium, p is the pressure, ρ is the density, μ is the viscosity coefficient and m is the mass of a single dust particle.

2.3 Single–phase flow models

In the case of a single–phase fluid–flow through porous media the governing equations take different forms depending on the type of flow and the type of porous medium considered. It is assumed that the dust–phase momentum equation and the continuity equations of both phases remain the same for the different regimes of interest and that only the fluid–phase momentum equation changes. Of particular interest to the current work are the following models describing the flow of a single–phase incompressible fluid through porous media [16] in the absence of body forces.

1) When the inertial effects and viscous shearing effects are neglected, then Darcy's Law takes the form describes the motion of a single–phase fluid flow through porous media. In this case we obtain

$$u_1 = -k p / \mu \quad . \qquad (2.5)$$

2) Brinkman's equation: when the inertial effects are negligible as compared to the dominate viscous shear effects, the motion of single–phase fluid flow through porous media is described by Brinkman's equation which takes the form

$$-\nabla p + \mu \nabla^2 u - \frac{\mu}{k} u_1 = 0 \quad . \qquad (2.6)$$

Equation (2.6) is postulated to be valid in situations where the porosity of the medium is close to unity.

3) Darcy–Lapwood (DL) and Darcy–Forchheimer (DL) – models

The (DL) and (DF) models that describe the dust flow when the porous medium possesses a sparse structure take the following forms respectively describe the flow

$$\rho\left[\frac{\partial u_1}{\partial t}+(u_1\,.\,\nabla\,)\,u_1\right]=-\nabla\,p-\frac{\mu}{k}\,u_1 \qquad (2.7)$$

and

$$\rho\left[\frac{\partial u_1}{\partial t}+C_d\,u_1\,|u_1|\,/\,\sqrt{k}\right]=-\nabla\,p-\frac{\mu}{k}\,u_1\ . \qquad (2.8)$$

Where C_d is the form drag coefficient. In these equations the inertial effects are taken into account but the viscous shearing effects are neglected and replaced by the viscous dumping term $-\frac{\mu}{k}u_1$.

4) Darcy–Lapwood–Brinkman (DLB), Darcy–Forchheimer–Brinman (DFB) and Darcy–Lapwood–Forchheimer–Brinkman (DLFB) – models

In certain type of porous media where the speed of the flow is not small and the viscous shearing action is important, the following models describe the flow

i) DLB – model

$$\rho\left[\frac{\partial u_1}{\partial t}+(u_1\,.\nabla)\,u_1\right]=-\nabla\,p+\mu_{eff}\,\nabla^2\,u_1-\frac{\mu}{k}\,u_1 \qquad (2.9)$$

where μ_{eff} is the effective viscosity of the fluid in the medium .

ii) DFB-model

$$\rho\left[\frac{\partial u}{\partial t}+C_d\,u_1\,|u_1|\,/\,\sqrt{k}\right]=-\nabla\,p-\frac{\mu}{k}\,u_1+\mu_{eff}\nabla^2\,u_1\ . \qquad (2.10)$$

iii) DLFB–model

$$\rho\left[\frac{\partial u_1}{\partial t}+(u_1\,.\nabla)\,u_1\right]=-\nabla\,p+\mu_{eff}\,\nabla^2\,u_1-\frac{\mu}{k}\,u_1+\rho\left[C_d\,u\,|\,u\,|\,/\sqrt{k}\right]\ . \qquad (2.11)$$

2.4 Two-phase flow models

In parallel subclassification to that of single-phase flow models, the dusty fluid flow models may be cast in different forms depending on the type of the medium and the type of flow described. It is assumed that the dust-phase momentum equation and the continuity equations of both phases remain the same for the different regimes of interest and that only the fluid-phase momentum equation changes. The following subclassifications of equation (2.2) are considered.

1) Hamdan-Barron-Brinkman (HBB)-model

Ignoring the fluid-phase inertial effects while retaining the viscous shear effect, then equation (2.2) reduces to

$$-\nabla P + \mu\, \nabla^2 u_1 + C_r\, N(u_2 - u_1) + \frac{\mu}{k}(u_2 - u_1) = 0\ . \tag{2.12}$$

The HBB-model is thus composed of equations (2.1), (2.12), (2.3) and (2.4). This model is postulated to describe the flow of a dusty fluid in a medium with porosity close to unity. In the absence of dust effects, equation (2.12) reduces to Brinkman's equation.

2) Hamdan – Barron – Darcy – Lapwood (HBDL) – model

Ignoring the viscous shear effects while retaining inertial effects, equation (2.2) reduces to

$$\rho\left[\frac{\partial u_1}{\partial t} + (u_1\,.\nabla)\,u_1\right] = -\nabla p + C_r\, N(\,u_2 - u_1\,) + \frac{\mu}{k}(u_2 - u_1)\ . \tag{2.13}$$

Hence equations (2.1), (2.13), (2.3) and (2.4) constitute the HBDL – model, which describe the flow of a dusty fluid in a porous medium that is of a sparse structure where the (DL) – model is valid.

Furthermore, in the absence of dust effect equation (2.13) reduces to DL – equation.

3) Hamdan – Barron – Darcy – Lapwood – Brinkman (HB – DLB) – model

Equations (2.1) – (2.4) constitute (HB-DLB) model, it should be noted that when the dust effects are absent then equation (2.2) reduces to (DLB)–equation. Furthermore in the absence of the porous matrix effects the (HB)–(DLB) model reduces to the usual dusty fluid flow equations in free space as given by [16].

2.5 Basic concepts of volume averaging

Averaging processes are applied so that to obtain equations that do not contain the details of the flow .The advantages of averaging are less obvious. First, the various terms appearing in the macroscopic equations are shown to arise from appropriate microscopic considerations. Second, the resulting macroscopic variables are related to microscopic variables.

Averaging of the microscopic scale conservation equations in order to obtain macroscopic scale conservation equations provides a powerful method of analyzing flow in porous media.

2.5.1 Representative Elementary Volume (REV)

A Representative Elementary Volume (REV) is defined as an arbitrarily shaped control volume over which averages will be calculated [2]. This control volume is denoted by V and it will contain fluid and porous matrix in the same proportion as the whole porous medium. In other words , it is a control volume whose porosity is the same as that of the whole porous medium .The pore volume V_Φ will comprise the volume of the fluid–phase V_f and the volume containing the particle dust–phase V_p clearly

$$V_\Phi = V_f + V_p \quad . \tag{2.14}$$

Where porosity Φ is defined as the ratio of the pore volume to the bulk volume of the medium. In terms of the REV the porosity is defined as

$$\Phi = \frac{V_\Phi}{V} \quad . \tag{2.15}$$

When averaging point equations, it is necessary to use an averaging volume of a size large enough that the averaged quantities obtained are meaningful and defined in equation (2.16) that represents the average distance between pores. Accordingly, the overlapping void volume available for flow is given by $Æ_p \, \ell$. The (RUC) was represented for consolidated media by three short square duct sections [6] oriented mutually perpendicular as shown in Figure 2.

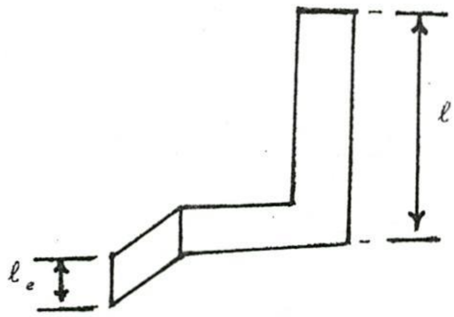


Fig .2. A typical representative unit cell

The following results based on the concept of the RUC, which are listed here for the sake of clarity [6] .

1. The total wetted surface within the RUC is given by

$$S = 3(1-\tau)\,(3\tau - 1)\,\ell^2 / \tau^2 \quad . \tag{2.18}$$

2. The total tortuous path length within the RUC is defined by

$$\ell_e = V_\Phi \,/\, Æ_p \, . \tag{2.19}$$

3. The pore area, $Æ_p$, within the RUC is defined by

$$Æ_p = \left[1 - (1-\Phi)^{2/3}\right]\ell^2 \quad . \tag{2.20}$$

4. The porosity Φ is defined by

$$\Phi = (3\tau - 1)^2 / \left(4\tau^3\right) \quad . \tag{2.21}$$

suitable for the flow under consideration. However, the averaging volume should not be so large that macroscopic inhomogeneities affect values. For example, in terms of microscopic and macroscopic length scales ℓ and L respectively, the REV is chosen such that

$$\ell^3 << V << L^3 . \tag{2.16}$$

Figure 1 represents a sketch of a typical (REV) consisting of a solid volume and a pore space.

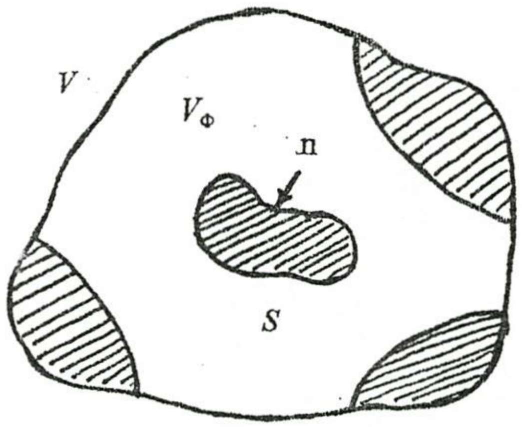


Fig .1. A typical representative elementary volume

It is within V_Φ that the effect of the phases on each other takes place and on the surface area S bounding V_Φ the porous matrix interacts with the flowing phases.

2.5.2 Representative Unit Cell (RUC)

The concept of an (RUC) was introduced by [6], and is defined as the minimal (REV) in which the average properties of the porous medium are embedded such that

$$V_\Phi = \Phi \ \ell^3 . \tag{2.17}$$

With the mean pore cross – sectional area $Æ_p$ being the same in all directions by virtue of isotropy of the medium. The length ℓ is the microscopic length

5.The Tortuosity τ is given by

$$\tau = \frac{\ell}{\ell_e} \quad . \tag{2.22}$$

For granular porous media the RUC is represented by a cube of linear dimension ℓ aligned centrally within the solid element that is also of linear dimension.

6.The total wetted surface within the RUC is given by

$$S = 6\left[(1-\Phi)^{2/3}\right]\ell^2 \quad . \tag{2.23}$$

7.The tortuosity τ is defined by

$$\tau = \left[1-(1-\Phi)^{2/3}\right]/\Phi \, . \tag{2.24}$$

2.5.3 The Averaging Rules

Let F be a volumetrically additive quantity of the fluid–particle mixture within the REV. The phase average F is defined as

$$<F> = (1/V)\int_V F \, dV \quad . \tag{2.25}$$

While the intrinsic phase average is expressed in the form

$$<F>_\Phi = \frac{1}{V_\Phi} \int_{V_\Phi} F \, dV \quad . \tag{2.26}$$

It follows from equations (2.25) and (2.26) that the two averages are related by the expression

$$<F> = \Phi <F>_\Phi \tag{2.27}$$

The relationship between the true quantity F and its intrinsic average is given by $F = \langle F \rangle_\Phi + F^\circ$ where F° is the deviation from F.

Letting F and G be two volumetrically additive scalar quantities and F a vector quantity, the following set of rules have been established [9],

(i) $\langle F \rangle = \Phi \langle F \rangle_\Phi = \Phi F_\Phi$

(ii) $\langle F \pm G \rangle_\Phi = \langle F \rangle_\Phi \pm \langle G \rangle_\Phi$

(iii) $\langle F G \rangle_\Phi = \langle F \rangle_\Phi \langle G \rangle_\Phi + \langle F^\circ G^\circ \rangle_\Phi$

(iv) $\langle \nabla F \rangle = \nabla \langle F \rangle + (1/V)\int\limits_{S} n F\, ds$, where n is a unit normal directed into the solid matrix .

(v) $\langle \nabla F \rangle = \Phi\, \nabla\, (\langle F \rangle_{\Phi}) + (1/V)\int\limits_{S} n F^{\circ}\, ds$

(vi) $\langle \nabla . F \rangle = \nabla . \langle F \rangle + (1/V)\int\limits_{S} n . F\, ds$

(vii) The surface integral over the fluid matrix interface vanishes if the integrand contains the fluid–phase and dust-phase velocities explicitly.

(viii) If c is a constant, $\langle c F \rangle = c \langle F \rangle$.

Chapter Three

Dusty gas flow through porous media

Introduction

In the current work an attempt is made to derive differential equations governing the gas–particulate flow through porous media of variable porosity. The models are intended to parallel the existing single–phase models of flow in porous media that take into account the effect of the porous microstructure. The governing equations are derived using intrinsic volume averaging and are based on Saffman's dusty gas model and take into account the cases of one-way and two–way interaction between the phases present. The effect of the porous microstructure on the flowing mixture is analyzed via the concept of representative unit cell (RUC), the analysis to flow is based on volume averaging the governing equations over a representative elementary volume (REV).

A distinction was made in these models between consolidated and isotropic porous media through the concept of representative unit cell (RUC) which was introduced [6]. In the developed models an attempt is made to analyze the dispersion terms.

3.1 Preliminaries

Consider the flow of a fluid in an isotropic porous media and assume that the fluid contains a small bulk volume of dust particles. The same flow in free – space is governed by Saffman's dusty gas equations [6], which are considered here for steady incompressible fluid flow in the following form

i) For the dust–phase

The continuity equation is

$$\nabla \cdot (N \mathrm{v}) = 0 \tag{3.1}$$

ii) For the fluid–phase

The continuity equation is

$$\nabla . \mathrm{u} = 0 \tag{3.7}$$

The linear momentum equation is

$$\rho\nabla .(\mathrm{u\,u}) = -\nabla p + \mu\nabla^2 \mathrm{u} \ . \tag{3.8}$$

3.2.1 Averaging the dusty gas equations

In the Saffman's dusty fluid model the effects of the flowing phases on each other are represented by a drag term proportional to the relative velocity of the phases involved. When the same dusty fluid flows in a porous medium the other interactions that occur are consequences of the introduction of the porous matrix in the flow domain. We will therefore assume at the onset of the analysis that when a dusty fluid flows in a porous medium the effects of the phases on each other are represented by drag terms proportional to the relative velocity. This assumption eliminates consideration of the interfacial surface area between the fluid and dust phases.

In order to accomplish the intrinsic averaging procedure, we derive the required set of equations by a doping the intrinsic phase averaging procedure over the REV and applying the averaging rules (i) – (viii) in chapter two to the governing equations (3.5) – (3.8) as follows

i) Averaging the dust–phase equations

The continuity equation is

$$\nabla . (\Phi \langle N_\Phi \ \mathrm{v}_\Phi) + \nabla .(\Phi \langle N^\circ \ \mathrm{v}^\circ \rangle_\Phi) = 0 \tag{3.9}$$

The momentum equation is

$$\nabla . (\Phi \, \mathrm{v}_\Phi \ \mathrm{v}_\Phi) + \nabla . (\Phi \langle \, \mathrm{v}^\circ \ \mathrm{v}^\circ \rangle_\Phi) = k \, \Phi / m \, (\mathrm{u}_\Phi - \mathrm{v}_\Phi) \ . \tag{3.10}$$

ii) Averaging the fluid–phase equations

The continuity equation is

$$\nabla . (\Phi \, \mathrm{u}_\Phi) = 0 \tag{3.11}$$

The momentum equation is

$$\rho\nabla\,.\,(\Phi\ \mathrm{u}_\Phi \mathrm{u}_\Phi)\ +\rho\ \nabla\,.(\Phi\,\langle \mathrm{u}^\circ \mathrm{u}^\circ\rangle_\Phi)=-\ \Phi\ \nabla p\ \ +\mu\nabla^2(\Phi \mathrm{u}_\Phi)$$

$$+\ (1/V)\int_S(-np^\circ+\mu n\ .\ \nabla \mathrm{u})\,\mathrm{ds}\ . \tag{3.12}$$

(See appendix 1)

Equation (3.9) – (3.12) represents the intrinsic averaged equations governing the flow of a dusty fluid in a porous medium in the case of one–way interaction. The deviation terms in these equations and the surface integral in equation (3.12) contain the necessary information on the interactions between the phases present and the porous medium. In particular, the surface integral represents the effect of the porous matrix on the flowing mixture.

3.2.2 Analysis of the deviation terms

The terms $\nabla.\ (\Phi\,\langle \mathrm{v}^\circ \mathrm{v}^\circ\rangle_\Phi)$ and $\nabla.\ (\Phi\,\langle \mathrm{u}^\circ \mathrm{u}^\circ\rangle_\Phi)$ in equations (3.10) and (3.12) respectively are related to the hydrodynamic dispersion of the average dust–phase and fluid–phase velocities .

Suppose that high velocity and porosity gradients are absence, then we obtain

$$\nabla\,.\,(\Phi\,\langle \mathrm{v}^\circ \mathrm{v}^\circ\rangle_\Phi)=0 \tag{3.19}$$

and

$$\nabla\,.\,(\Phi\,\langle \mathrm{u}^\circ \mathrm{u}^\circ\rangle_\Phi)\ =\ 0 \tag{3.20}$$

Substituting equation (3.19) and (3.20) into equation (3.10) and (3.18) respectively, then we obtain the following forms

$$\nabla.(\Phi \mathrm{v}_\Phi \mathrm{v}_\Phi)=(\Phi\ \mathrm{k/m})\,(\mathrm{u}_\Phi-\mathrm{v}_\Phi) \tag{3.21}$$

$$\rho\ \nabla.(\Phi \mathrm{u}_\Phi \mathrm{u}_\Phi)=-\ \Phi\ \nabla p_\Phi+\mu\nabla^2(\Phi \mathrm{u}_\Phi)+(1/\,V\,)\int_S(-\ n\ p^\circ+\mu\ n\ .\nabla u)\ ds\,. \tag{3.22}$$

The term $\langle N^\circ \mathrm{v}^\circ\rangle_\Phi$ appearing in equation (3.9) is dispersion vector of the average number density due to the influence of average phase velocity vectors. If the particle distribution is uniform and N is constant then

$$\langle N^\circ \mathrm{v}^\circ\rangle_\Phi=0\,. \tag{3.23}$$

However, if N is not constant then this term can be sufficiently quantified by modeling the dispersion vectors as a diffusion mechanism only.

The term $\nabla.(\Phi \langle N^{\circ} \mathrm{v}^{\circ} \rangle_{\Phi})$ appearing in the dust–phase continuity equation (3.9) represents the mass transfer Γ between the dust–phase and the porous medium in the form of settling of the particle on the solid matrix boundaries and the sedimentation and suspension of particles of the solid porous matrix in the flow field [9]. It should be noted that there is no mass transfer between the fluid–phase and the dust–phase.

To quantify Γ we let ε be the mass transfer coefficient then in light of channeling concept of the porous medium it may be argued that the mass transfer between the dust–phase and the solid matrix is dependent on the wetted surface area of the REV and thus the mass transfer Γ is taken as

$$\Gamma = \nabla.(\Phi \langle N^{\circ} \mathrm{v}^{\circ} \rangle_{\Phi}) = \varepsilon S \ . \tag{3.24}$$

And thus the dust–phase continuity equation (3.9) takes the form

$$\nabla.(\Phi N_{\Phi} \mathrm{v}_{\Phi}) = -\varepsilon S \tag{3.25}$$

It should be emphasized at this point that determination of the mass transfer coefficient ε that is postulated here to be a function of some quantities including the porosity rigidity of the medium should be experimental. When the particle distribution is uniform, i.e., dust particle number density N is taken to be constant through or when the dispersion of the dust phase is not taken into account, then $\Gamma = 0$ and the dust–phase continuity equation reduces to the form

$$\nabla.(\Phi N_{\Phi} \mathrm{v}_{\Phi}) = 0 \ . \tag{3.26}$$

The averaged equations of the flow at hand are

i) For the dust–phase

The continuity equation is

$$\nabla.(\Phi N_{\Phi} \mathrm{v}_{\Phi}) = -\varepsilon S \tag{3.27}$$

The linear momentum equation is

$$\nabla \,.\, (\Phi \mathrm{v}_{\Phi} \mathrm{v}_{\Phi}) = (\Phi k / m)(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) \ . \tag{3.28}$$

ii) For the fluid–phase

The continuity equation is

$$\nabla \,.\, (\Phi \mathrm{u}_{\Phi}) = 0 \ . \tag{3.29}$$

The linear momentum equation is

$$\rho\, \nabla.(\Phi \mathrm{u}_{\Phi}\, \mathrm{u}_{\Phi}) = -\Phi \nabla p_{\Phi} + \mu \nabla^2 (\Phi \mathrm{u}_{\Phi}) + (1/V) \int_S (-n\, p^{\circ} + \mu\, n\,.\, \nabla u)\, ds\,. \qquad (3.30)$$

3.2.3 Analysis of the surface integral

The interactions that take place within V_{Φ} are the effects of the flowing phases on each other and the effect of the porous matrix on the flowing phases. The former occurs through the interfacial area between the fluid and dust phases is represented at the onset by a drag force proportional to the relative velocity of the flowing phases. The effects of the porous matrix on the flowing phases occur through the portion S of the surface area of V_{Φ} that is in contact with the dusty fluid at hand .The surface integral appearing in equation (3.27) contains the necessary information on the effect of the porous matrix on the flowing phases .Its accurate evaluation depends on the knowledge of the porous matrix structure and its geometric description. However, in the absence of a dust–phase partial pressure and a dust–phase viscosity, the effect of the porous matrix on the flowing mixture arises in the form of the fluid–phase quantities only.

In order to evaluate the surface integral $\int_S (-n p^{\circ} + \mu n. \nabla u) ds$, it can be written in the terms of the directional derivative along the unit normal vector as

$$(1/V) \int_S (-n\, p^{\circ} + \mu\, \frac{\partial u}{\partial n})\ \mathrm{ds}. \qquad (3.31)$$

This integral surface is imperative to provide an accurate description of the porous medium. Evaluation of a similar integral was addressed in [7] for the case of single–phase flow in porous media, where they distinguish between consolidated and granular isotropic porous media. To take into account the effect of the porous microstructure on the flow they introduced the concept of an RUC, which is defined as the minimal REV in which the average properties of the porous medium are embedded.

The surface integral was evaluated in terms of the average wall shear stress over the stream wise flow length [6], and expressed it in terms of the product of the Reynolds number and the apparent friction factor for:

1) Fully developed flow in a square straight duct (for consolidated media)

2) Fully developed flow between parallel plates (for granular media).

Defining u_p to be the area average fluid–phase velocity at pore level in a stream wise oriented pore and letting in the current analysis $\Re$ into be the product of the Reynolds number and the friction factor associated with the flow of a dusty fluid in a square duct (for consolidated media) or between parallel plates (for granular media) then the surface integral in (3.27) takes the form

$$(1/V)\int_S (-np^\circ + \mu n.\nabla u)\, ds = -\mu S \Re\, u_p / \tau\,. \tag{3.32}$$

The volumetric flow rate of the fluid–phase is defined by: $q_1 = \Phi \tau u_p$ or $q_1 = \Phi u_\Phi$. Thus, $u_p = u_\Phi / \tau$. Substituting for u_p in the expression (3.29), yield

$$(1/V)\int_S (-np^\circ + \mu n.\nabla u)\, ds = -\mu\, S\, u_\Phi\, \Re / \tau\, V\,. \tag{3.33}$$

If equation (2.15) is substituted into (2.17), we get

$$V = V_\Phi / \Phi = \frac{\Phi \ell^3}{\Phi} = \ell^3\,. \tag{3.34}$$

Upon substituting (2.18) and (3.34) into (3.33), then we obtain

$$(1/V)\int_S (-np^\circ + \mu n.\nabla u)\, ds = -\left[3\Re(1-\tau)(3\tau-1)/\ell\tau^3\right]\mu\, u_\Phi\,. \tag{3.35}$$

By substituting (3.35) into (3.31) we obtain the following fluid–phase momentum equation that is valid in consolidated porous media

$$\rho\, \nabla.(\Phi u_\Phi u_\Phi) = -\Phi \nabla p_\Phi + \mu \nabla^2(\Phi u_\Phi) - \left[3\Re(1-\tau)(3\tau-1)/\ell\tau^3\right]\mu\, u_\Phi\,. \tag{3.36}$$

And substituting (2.25) and (3.34) into (3.32) we obtain the following fluid–phase momentum equation that is valid in granular media

$$\rho\, \nabla.(\Phi u_\Phi u_\Phi) = -\Phi \nabla p_\Phi + \mu \nabla^2(\Phi u_\Phi) - \left[6\Re(1-\Phi)^{2/3}/\ell\tau\right]\mu u_\Phi\,. \tag{3.37}$$

The dust–phase continuity equation (3.26) can be written with the help of (2.18) and (2.23) in the form

For consolidated porous media (3.26) takes the form

$$\nabla.(\Phi N_{\Phi}\,\mathrm{v}_{\Phi}) = -3\varepsilon(1-\tau)(3\tau-1)\,\ell^2/\tau^2 \; . \tag{3.38}$$

While for granular porous media equation (3.26) becomes

$$\nabla.(\Phi N_{\Phi}\,\mathrm{v}_{\Phi}) = -6\varepsilon(1-\Phi)^{2/3}\,\ell^2 . \tag{3.39}$$

To this end, we summarize the governing equations:

For consolidated media

i) Dust–phase

The continuity equation is

$$\nabla.(\Phi N_{\Phi}\,\mathrm{v}_{\Phi}) = -3\varepsilon(1-\tau)(3\tau-1)\,\ell^2/\tau^2 \tag{3.40}$$

The linear momentum equation is

$$\nabla.(\Phi N_{\Phi}\,\mathrm{v}_{\Phi}) = (\Phi k/m)(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) \quad . \tag{3.41}$$

ii) Fluid–phase

The continuity equation is

$$\nabla\,.\,(\Phi\,\mathrm{u}_{\Phi}) = 0 \tag{3.42}$$

The linear momentum equation is

$$\rho\,\nabla.(\Phi\,\mathrm{u}_{\Phi}\,\mathrm{u}_{\Phi}) = -\Phi\nabla p_{\Phi} + \mu\nabla^2(\Phi\,\mathrm{u}_{\Phi}) - \left[3\Re(1-\tau)(3\tau-1)/\ell\tau^3\right]\mu\,\mathrm{u}_{\Phi} \quad . \tag{3.43}$$

For granular media

i) Dust–phases

The continuity equation is

$$\nabla\,.\,(\Phi N_{\Phi}\,\mathrm{v}_{\Phi}) = -6\varepsilon(1-\Phi)^{2/3}\,\ell^2 \tag{3.44}$$

The linear momentum equation is

$$\nabla.\,(\Phi N_{\Phi}\,\mathrm{v}_{\Phi}) = (\Phi\,k/m)(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) . \tag{3.45}$$

ii) Fluid–phase

The continuity equation is

$$\nabla\,.\,(\Phi\,\mathrm{u}_{\Phi}) = 0 \tag{3.46}$$

The linear momentum equation is

$$\rho\,\nabla\,.\,(\Phi\,\mathrm{u}_{\Phi}\,\mathrm{u}_{\Phi}) = -\Phi\nabla p_{\Phi} + \mu\nabla^2(\Phi\,\mathrm{u}_{\Phi}) - \left[6\Re(1-\Phi)^{2/3}/\ell\tau\right]\mu\,\mathrm{u}_{\Phi} \quad . \tag{3.47}$$

3.2.4 The final form of the governing equations

For computational purposes the above equations may be cast in terms of the volumetric flow rates of the phases involved by the substituting: $q_1 = \Phi\,\mathrm{u}_\Phi$ and $q_2 = \Phi\,\mathrm{v}_\Phi$ and the macroscopic number densities $n = \Phi\,\mathrm{N}_\Phi, n_d = \Phi\,\mathrm{N}_{d\Phi}$. The following equations are thus obtained

For consolidated media:

i) Dust–phase: Equations (3.41) and (3.42) yield

$$\nabla\,.\,(n q_2 / \Phi) = -3\varepsilon\,(1-\tau)(3\tau-1)\,\ell^2/\tau^2 \ . \tag{3.48}$$

$$\nabla\,.\,(q_2\, q_2 / \Phi) = (k/m)\,(q_1 - q_2) \quad . \tag{3.49}$$

ii) Fluid–phase: Equations (3.45) and (3.46) become

$$\nabla\,.\,(q_1) = 0 \tag{3.50}$$

$$\nabla\,.\,(q_1\, q_1 / \Phi) = -\Phi\nabla p_\Phi + \mu\nabla^2(q_1) - 12\Re\mu\,\mathrm{q}_1[(1-\tau)/\,\ell\,(3\tau-1)]\,. \tag{3.51}$$

For granular media:

i) Dust–phase: Equations (3.45) and (3.46) yield

$$\nabla\,.\,(n q_2 / \Phi) = -6\varepsilon\,(1-\Phi)^{2/3}\ell^2 \tag{3.52}$$

$$\nabla\,.\,(q_2\, q_2 / \Phi) = (k/m)\,(q_1 - q_2)\ . \tag{3.53}$$

ii) Fluid–phase: Equations (3.47) and (3.48) take the following form respectively

$$\nabla\,.\,(q_1) = 0 \tag{3.54}$$

$$\rho\nabla\,.\,(q_1\, q_1 / \Phi) = -\Phi\nabla p_\Phi + \mu\nabla^2(q_1) - 6\,\Re\,\mu\,\mathrm{q}_1[(1-\Phi\tau)/(\ell\tau\Phi)]. \tag{3.55}$$

The hydrodynamic permeability of the flow may be defined in terms of the Tortuosity that appearing in equation (3.48) and (3.51) which can easily be expressed in terms of the linear dimensions of the RUC depending on the type of flow and the type of porous media.

3.3 Two–way interaction

In the following analysis we consider the case of two–way interaction approximations that in the case of dusty flow through porous media take into account the influence of the dust on the clean fluid.

Assuming that the fluid–phase affects the dust–phase and the dust–phase effect on the fluid–phase is not negligible, then the fluid–phase momentum equation is given by equation (3.4).

Consider the flow of a fluid in an isotropic porous medium and assume that the fluid contains a small bulk volume of dust particles. This same flow in free–space is governed by Saffman's gas equations [17] which are considered here for steady incompressible fluid flow in the following form

i) For the dust–phase:

The continuity equation is

$$\nabla . (N \mathrm{v}) = 0 \tag{3.56}$$

The linear momentum equation is

$$\nabla . (\mathrm{v}\,\mathrm{v}) = (k/m) (\mathrm{u} - \mathrm{v}) \quad . \tag{3.57}$$

ii) For the fluid–phase:

The continuity equation is

$$\nabla . \mathrm{u} = 0 \tag{3.58}$$

The linear momentum equation is

$$\rho \nabla . (\mathrm{u}\,\mathrm{u}) = - \nabla p + \mu \nabla^2 \mathrm{u} + kN (\mathrm{v} - \mathrm{u}) \quad . \tag{3.59}$$

3.3.1 Averaging equations

The required set of equations are derived by adopting the intrinsic phase averaging procedure over the (REV) and applying the averaging rules (i) – (viii) which were given in section (2.5.3) to the governing equations (3.56)–(3.59). Consequently, the intrinsic averaged equations governing the flow of a dusty fluid in a porous medium in the case of two–way interaction are given by

i) For the dust–phase

The continuity equation is

$$\nabla . (\Phi \langle N_\Phi \, v_\Phi \rangle + \nabla . (\Phi \langle N^\circ \, v^\circ \rangle_\Phi) = 0 \quad . \tag{3.60}$$

The momentum equation is

$$\nabla . (\Phi \, v_\Phi \, v_\Phi) + \nabla . (\Phi \langle \, v^\circ \, v^\circ \rangle_\Phi) = \quad k \, \Phi / m \, (u_\Phi - \, v_\Phi) \quad . \tag{3.61}$$

ii) For the fluid–phase:

The continuity equation is

$$\nabla . (\Phi \, u_\Phi) = 0 \tag{3.62}$$

The momentum equation is

$$\begin{aligned} \rho \nabla . (\Phi \, u_\Phi u_\Phi) + \rho \, \nabla . (\Phi \langle u^\circ u^\circ \rangle_\Phi) = & - \Phi \, \nabla p \, + \mu \nabla^2 (\Phi u_\Phi) \\ & + k \Phi \, N_\Phi (v_\Phi - \, u_\Phi) + k \Phi (\langle N^\circ v^\circ - \, N^\circ u^\circ \rangle_\Phi) \\ & + \, (1/V) \int_S (-n p^\circ + \mu n \, . \nabla u) \, ds \quad . \end{aligned} \tag{3.63}$$

Equations (3.60) – (3.63) represent the intrinsic averaged equations governing the flow of a dusty fluid in a porous medium. The deviation terms in these equations and the surface integral in equation (3.63) contain the necessary information on the interactions between the phases present and the porous medium. In particular, the surface integral represents the effect of the porous matrix on the flowing mixture.

3.3.2 Analysis of the deviation terms

The deviation terms appearing in equations (3.60),(3.61) and (3.63) are analyzed in section (3.3.2) while the deviation term $(\langle N^\circ v^\circ - N^\circ u^\circ \rangle_\Phi)$ which appears in equation (3.62) is the difference between two dispersion vectors and arises from averaging the relative velocity vector $v - u$.

In order to analyze the term $(\langle N^\circ v^\circ - N^\circ u^\circ \rangle_\Phi)$ we apply an averaging rule (ii) and obtain

$$\langle N^\circ v^\circ - N^\circ u^\circ \rangle_\Phi = \langle N^\circ v^\circ \rangle_\Phi - \langle N^\circ u^\circ \rangle_\Phi \tag{3.64}$$

Using rule (iii) to the right hand side, we have

$$\langle N^\circ \mathrm{v}^\circ\rangle_\Phi = \langle N^\circ \rangle_\Phi \langle \mathrm{v}^\circ\rangle_\Phi + \langle (N^\circ \mathrm{v}^\circ)\rangle^\circ_\Phi \tag{3.65}$$

and

$$\langle N^\circ \mathrm{u}^\circ \rangle_\Phi = \langle N^\circ \rangle_\Phi \langle \mathrm{u}^\circ\rangle_\Phi + \langle (N^\circ \mathrm{u}^\circ)\rangle^\circ_\Phi . \tag{3.66}$$

Equation (3.64) becomes

$$\langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi = \langle N^\circ \rangle_\Phi \langle \mathrm{v}^\circ\rangle_\Phi + \langle (N^\circ \mathrm{v}^\circ)\rangle^\circ_\Phi - \langle N^\circ \rangle_\Phi \langle \mathrm{u}^\circ\rangle_\Phi - \langle (N^\circ \mathrm{u}^\circ)\rangle^\circ_\Phi . \tag{3.67}$$

The terms $\langle (N^\circ \mathrm{v}^\circ)\rangle^\circ_\Phi$ and $\langle (N^\circ \mathrm{u}^\circ)\rangle^\circ_\Phi$ represent fluctuations of the average deviations of the averaged products over the REV. Because of the smoothing action of the averaging procedure, then these terms are individually negligibly small. Alternatively, they may be taken to be of the same order and hence their net effect $\langle (N^\circ \mathrm{v}^\circ)\rangle^\circ_\Phi - \langle (N^\circ \mathrm{u}^\circ)\rangle^\circ_\Phi$ is zero. In either case with the help of averaging rule (ii), equation (3.68) takes the form

$$\langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi = \langle N^\circ \rangle_\Phi (\langle \mathrm{v}^\circ \rangle_\Phi - \langle \mathrm{u}^\circ \rangle_\Phi) . \tag{3.68}$$

To analyze the right–hand side of equation (3.68) we consider the following cases:

1. If the dust particle distribution is uniform and hence N is constant then $\langle N^\circ \rangle_\Phi = 0$ and the term $\langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi$ vanishes. Furthermore, if dispersion is negligible then clearly the dispersion vectors $\langle N^\circ \mathrm{v}^\circ\rangle_\Phi$ and $\langle N^\circ \mathrm{u}^\circ \rangle_\Phi$ both vanish.

2. If there are no abrupt changes in the averaged relative velocity vector in the porous medium, and $\langle \mathrm{v}\rangle_\Phi$ and $\langle \mathrm{u}\rangle_\Phi$ are sufficiently well behaved, then the difference $\langle \mathrm{v}^\circ \rangle_\Phi - \langle \mathrm{u}^\circ \rangle_\Phi$ is sufficiently small and hence negligible.

3. Since hydrodynamic dispersion (or the mixing of the phases in each other) is negligible in dusty gases, it is possible to drop out the relative dispersion vector under the premise that the fluid and dust phases only exert a drag force on each other.

4. If none of the above conditions is satisfied, then we treat dispersion as a diffusion process, expressed in terms of the product of a diffusion coefficient δ and a number density driving differential $(\langle N\rangle_\Phi - \langle N_d \rangle_\Phi)$ and thus we obtain

$$\langle N^{\circ} \mathrm{v}^{\circ} - N^{\circ} \mathrm{u}^{\circ} \rangle_{\Phi} = \delta(\langle N \rangle_{\Phi} - \langle N_d \rangle_{\Phi}) \ . \tag{3.69}$$

Upon substituting equation (3.67) into (3.61) we get

$$\rho \nabla.(\Phi \mathrm{u}_{\Phi} \mathrm{u}_{\Phi}) = -\Phi \nabla p_{\Phi} + \mu \nabla^2 (\Phi \mathrm{u}_{\Phi}) + k\Phi[N_{\Phi}(\mathrm{v}_{\Phi} - \mathrm{u}_{\Phi}) + \delta(N_{\Phi} - N_{d\Phi})] + (1/V)\int_S (-n p^{\circ} + \mu n.\nabla \mathrm{u})\, ds \ . \tag{3.70}$$

The averaged equations of the flow at hand are:

i) Fore the dust–phase

The continuity equation is

$$\nabla.(\Phi N_{\Phi} \mathrm{v}_{\Phi}) = -\varepsilon \, \ell \, . \tag{3.71}$$

The linear momentum equation is

$$\nabla.(\Phi \mathrm{v}_{\Phi} \mathrm{v}_{\Phi}) = (\Phi k/m)(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) \ . \tag{3.72}$$

ii) For the fluid–phase

The continuity equation is

$$\nabla.(\Phi \mathrm{u}_{\Phi}) = 0 \ . \tag{3.73}$$

The linear momentum equation is

$$\rho \nabla.(\Phi \mathrm{u}_{\Phi} \mathrm{u}_{\Phi}) = -\Phi \nabla p_{\Phi} + \mu \nabla^2 (\Phi \mathrm{u}_{\Phi}) + k\Phi[N_{\Phi}(\mathrm{v}_{\Phi} - \mathrm{u}_{\Phi}) + \delta(N_{\Phi} - N_{d\Phi})] + (1/V)\int_S (-n p^{\circ} + \mu n.\nabla \mathrm{u})\, ds \ . \tag{3.74}$$

3.3.3 Analysis of the surface integral

The surface integral appearing in equation (3.74) was analyzed in section (3.3.3) and it represents the effect of the porous matrix on the flowing phases. Its accurate evaluation depends on the knowledge of the porous matrix structure and its geometric description and it takes the form

$$(1/V)\int_S (-n p^{\circ} + \mu n.\nabla \mathrm{u})\, ds = -[6\Re(1-\Phi)^{2/3}/l\tau]\mu \, \mathrm{u}_{\Phi} \ . \tag{3.75}$$

Substituting (3.75) into (3.74), we obtain the following fluid–phase momentum equation that is valid in granular porous media

$$\rho \nabla.(\Phi \mathrm{u}_{\Phi} \mathrm{u}_{\Phi}) = -\Phi \nabla p_{\Phi} + \mu \nabla^2 (\Phi \mathrm{u}_{\Phi}) + k\Phi[N_{\Phi}(\mathrm{v}_{\Phi} - \mathrm{u}_{\Phi}) + \delta(N_{\Phi} - N_{d\Phi})] - [6\Re(1-\Phi)^{2/3}/l\tau]\mu \, \mathrm{u}_{\Phi} \ . \tag{3.76}$$

To this end, the governing equations are

For consolidated media:

i) Fore the dust–phase

The continuity equation is

$$\nabla \, . \, (\Phi \, N_{\Phi} \, \mathrm{v}_{\Phi}) = -3\varepsilon(1-\tau)(3\tau-1)l^2 / \tau^2 \ . \tag{3.77}$$

Linear momentum equation

$$\nabla \, . \, (\Phi \, N_{\Phi} \, \mathrm{v}_{\Phi}) = (\Phi k / m)(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) \ . \tag{3.78}$$

ii) Fore the fluid–phase

The continuity equation is

$$\nabla \, . \, (\Phi \, \mathrm{u}_{\Phi}) = 0 \ . \tag{3.79}$$

The linear momentum equation is

$$\rho \, \nabla . (\Phi \, \mathrm{u}_{\Phi} \, \mathrm{u}_{\Phi}) = -\Phi \nabla p_{\Phi} + \mu \, \nabla^2 (\Phi u_{\Phi}) + k\Phi[N_{\Phi} (\mathrm{v}_{\Phi} - \mathrm{u}_{\Phi}) + \delta (N_{\Phi} - N_{d\Phi})] - \left[3\Re (1-\tau)(3\tau-1) / \ell \tau^3 \right] \mu \, \mathrm{u}_{\Phi} \ . \tag{3.80}$$

For granular media:

i) Fore the dust–phase

The continuity equation is

$$\nabla \, . \, (\Phi \, N_{\Phi} \, \mathrm{v}_{\Phi}) = -6\varepsilon(1-\Phi)^{2/3} \ell^2 \ . \tag{3.81}$$

The linear momentum equation is

$$\nabla \, . \, (\Phi \, \mathrm{v}_{\Phi} \, \mathrm{v}_{\Phi}) = (\Phi k / m)(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) \ . \tag{3.82}$$

ii) Fore the fluid–phase

The continuity equation is

$$\nabla . (\Phi \, \mathrm{u}_{\Phi}) = 0 \ . \tag{3.83}$$

The linear momentum equation is

$$\rho \, \nabla . (\Phi \, \mathrm{u}_{\Phi} \, \mathrm{u}_{\Phi}) = -\Phi \nabla p_{\Phi} + \mu \nabla^2 (\Phi \, \mathrm{u}_{\Phi}) + k\Phi[N_{\Phi} (\mathrm{v}_{\Phi} - \mathrm{u}_{\Phi}) + \delta (N_{\Phi} - N_{d\Phi})] - [6\Re (1-\Phi)^{2/3} / \ell \tau] \mu u_{\Phi} \ . \tag{3.84}$$

3.3.4 The final form of the governing equations

Following the same procedure in section (3.3.5) yield

For consolidated media:

i) Dust–phase: Equations (3.77) and (3.78) yield

$$\nabla . (nq_2 / \Phi) = -3\, \varepsilon(1-\tau)\, (3\tau - 1)\, \ell^2 / \tau^2 \tag{3.85}$$

$$\nabla . (q_2\, q_2 / \Phi) = (k/m)\, (q_1 - q_2) \quad . \tag{3.86}$$

ii) Fluid–phase: Equations (3.79) and (3.80) become

$$\nabla . (q_1) = 0 \quad . \tag{3.87}$$

$$\begin{aligned} \nabla . (q_1 q_1 / \Phi) \; = \; & -\Phi \nabla p_\Phi + \mu \nabla^2 (q_1) + (k / \Phi)[n(q_2 - q_1)] + \delta(n - n_d) \\ & -12 \Re \mu q_1 [(1-\tau)/\ell(3\tau - 1)] \quad . \end{aligned} \tag{3.88}$$

For granular media:

i) Dust–phase: Equations (3.82) and (3.83) yield

$$\nabla . (nq_2 \Phi) = -6\varepsilon(1-\Phi)^{2/3} \ell^2 \quad . \tag{3.89}$$

$$\nabla . (q_2\, q_2 / \Phi) = (k/m)(q_1 - q_2) \quad . \tag{3.90}$$

ii) Fluid–phase: Equations (3.84) and (3.85) become

$$\nabla . (q_1) = 0 \quad . \tag{3.91}$$

$$\begin{aligned} \rho \nabla . (q_1 q_1 / \Phi) = & -\Phi \nabla p_\Phi + \mu \nabla^2 (q_1) + (k / \Phi)[n(q_2 - q_1)] + \delta(n - n_d) \\ & - 6 \Re \mu q_1 [(1 - \Phi\tau) / (\ell \tau \Phi)] \quad . \end{aligned} \tag{3.92}$$

The hydrodynamic permeability of the flow may be defined in terms of the tortuosity that appearing in equation (3.85) and (3.88), which can easily be expressed in terms of the linear dimensions of the RUC depending on the type of flow and the type of porous media.

Chapter Four

Dusty gas model of flow through naturally occurring porous media

Introduction

The steady flow of an incompressible dusty fluid in a porous medium of constant permeability is given by the following coupled set of equations that take into account the absence of dust effects, that is, the fluid–phase momentum equation takes the well–known (DLB) equation, in this type of porous media the speed of the flow is not small and the viscous shearing action is important.

i) Fluid–phase equations:

The continuity equation is

$$\nabla . u = 0 \quad . \tag{4.1}$$

The momentum equation is

$$\rho \nabla . (u\, u) = -\nabla p + \mu \nabla^2 u - \frac{\mu}{k} u \quad . \tag{4.2}$$

ii) Dust–phase equations:

The continuity equation is

$$\nabla . N \mathrm{v} = 0 \; . \tag{4.3}$$

The momentum equation is

$$\nabla . (\mathrm{v}\,\mathrm{v}) = t/m \; (u - \mathrm{v}) \quad . \tag{4.4}$$

Where u and v are the fluid and dust macroscopic velocity vectors, p is the pressure, μ is the viscosity coefficient, ρ is the density, N is the particle number density, m is the mass of a single dust particle, k is the permeability and t is the drag coefficient on the dust particle in the porous medium.

4.1 Fluid – phase momentum equations:

Equations (4.1) – (4.4) is valid in a variety of settings and in different types of porous media. We will discuss the following cases that illustrate the variety of settings and the different types that these equations are valid in.

i) When the inertial effects are negligible or insignificant, that is, the flow is fully–developed with no macroscopic stream–wise fluid–phase velocity gradient, equ. (4.2) becomes

$$-\nabla p + \mu \nabla^2 u - \frac{\mu}{k} u = 0 \ . \qquad (4.5)$$

Equation (4.5) is Brinkman's equation that is postulated to be valid in situations where the porosity of the medium is close to unity.

ii) When the macroscopic boundary effects are not important, that is when the macroscopic cross–stream fluid–phase velocity gradients are absent, equ. (4.2) yields

$$\rho \nabla . (u u) = -\nabla p - \frac{\mu}{k} u \ . \qquad (4.6)$$

Equation (4.6) is (DL) model and it is valid when the porous medium possesses a sparse structure that is the flow is through a sparse distribution of particles fixed in space.

iii) When the inertial effects and viscous shearing effects are neglected, that is, both of the macroscopic stream–wise and cross–stream fluid–phase velocity gradients are absent, equ. (4.2) yields

$$-\nabla p - \frac{\mu}{k} u = 0 \ . \qquad (4.7)$$

Equation (4.7) is Darcy's law that is describing the motion of a single–phase fluid flow through porous media.

iv) When the porous medium is naturally occurring, the fluid–phase momentum equation reduces to a Forchheimer's equation that is postulated to govern the single–phase fluid flow.

The resistance to the fluid offered by the porous matrix is governed by an equation of the form:

$$-\nabla p = \alpha u + \beta u|u| \tag{4.8}$$

where $\alpha = \dfrac{\mu}{\rho k}$, $\beta = \dfrac{C_d}{\sqrt{k}}$ and C_d is the Forchheimer drag coefficient drag coefficient associated with single–phase flow, then the macroscopic inertial term may be expressed as a microscopic inertial term of the form

$$\nabla.(uu) = \alpha\, u + \beta\, u\,|\,u\,| \tag{4.9}$$

and β is referred to as the inertial parameter.

4.2 Dust –phase momentum equations:

The dust phase momentum equation which is represented by eq. (4.4) serves in all of the different types of porous media which take the above discussed cases so the macroscopic cross–stream velocity gradients are absent while the stream–wise velocity gradients are always present. In the presence of an impermeable boundary as a solid wall or otherwise, the dust particles settle on the macroscopic boundary and set into motion other particles that are already settled, while others reflect back into the flow field and assume a streamwise velocity gradients.

Consequently, the macroscopic convective term $\nabla.\mathrm{vv}$ always survives.

4.3 Flow through a Naturally Occurring Porous Channel

In the current work we consider the two–dimensional flow of an incompressible dusty fluid through a naturally occurring porous channel into a point–sink. The source–sink model that is a popular setting for irrigation problems, might be thought of here as a means for solid–waste disposal. The model at hand might thus shed some light on the behavior of the solid particles in the flowfield as they are dispersed in a porous layer. The determinate nature of the model equations is illustrated by numerically studying the flow at hand.

The two–dimensional steady flow of an incompressible dusty fluid is governed by the coupled set of equations (4.1), (4.3), (4.4) and (4.8).

We introduce the dimensionless macroscopic streamfunctions $\Psi_1(x,y)$, $\Psi_2(x,y)$ and macroscopic vorticities $\zeta_1(x,y)$ and $\zeta_2(x,y)$ in terms of the dimensionless horizontal and vertical components of velocity defined by

$$u_1 = \Psi_{1y} \quad , \quad v_1 = - \Psi_{1x}$$

$$u_2 = \Psi_{2y} \quad , \quad v_2 = - \Psi_{2x} \ . \tag{4.10}$$

And the dimensionless vorticities are defined by

$$\zeta_1 = v_{1x} - u_{1y} \tag{4.11}$$

$$\zeta_2 = v_{2x} - u_{2y} \ . \tag{4.12}$$

The stream function vorticity equations governing the flow at hand in each of the coupled set of equations (4.1) and (4.9) for the fluid–phase, equations (4.3) and (4.4) for the dust–phase are obtained by taking the curl of each of the momentum equations and the subsequent use of the equations of continuity are given by [11],

For the fluid–phase

$$\nabla^2 \Psi_1 = -\zeta_1 \tag{4.13}$$

and

$$\zeta_1 / kR_e \sqrt{\Psi_{1x}^2 + \Psi_{1y}^2} + \zeta_1 C_d / \sqrt{k} \left\{ \Psi_{1x}^2 + \Psi_{1y}^2 \right\} =$$

$$C_d / \sqrt{k} \left\{ \Psi_{1x}^2 \Psi_{1xx} + 2\Psi_{1x} \Psi_{1y} \Psi_{1xy} + \Psi_{1y}^2 \Psi_{1yy} \right\} \tag{4.14}$$

For the Dust–phase

$$\nabla^2 \Psi_2 = -\zeta_2 \tag{4.15}$$

and

$$\Psi_{2y} \zeta_{2x} - \Psi_{2x} \zeta_{2y} = (t/M) \, [\zeta_1 - \zeta_2] \ . \tag{4.16}$$

Where Ψ_1 is the fluid–phase stream function, Ψ_2 is the dust–phase streamfunction, ζ_1 is the fluid–phase vorticity and ζ_2 is the dust–phase vorticity, u_1 and v_1 are the fluid–phase velocity components, u_2 and v_2 are the dust–phase velocity components and M is the dimensionless mass of a dust particle.

The flow variables have been rendered dimensionless with respect to a characteristic velocity U_∞ and a characteristic length L using the following dimensionless equations

X = x/L, Y = y/L, $\vec{U} = \vec{u} / U_\infty$, $\zeta = \zeta^* L / U_\infty$, $T = t / L^2$, $\Psi = \Psi^* / LU_\infty$ and the Reynolds number is defined as $Re = \rho U_\infty L / \mu$.

4.4 Numerical example

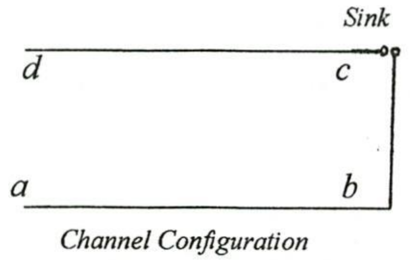
Consider the flow of a dusty fluid in a straight porous channel, bounded below and above by impermeable walls. Associated with this flow are boundary conditions and entry conditions to the channel. Any appropriate boundary conditions must be compatible with the type of medium, the governing model equations and the type of the impermeable bounding walls.

Suitable boundary conditions for the flow at hand, in the shown configuration, are as follows.

$\Psi_1 = \Psi_2 = 0$ along the walls ab and bc.

$\Psi_1 = \Psi_2 = 1$ along the upper wall dc.

$\Psi_1 = \Psi_2 = y$, $0 \le y \le 1$ along the inlet ad.



Channel Configuration

The fluid-phase and the dust–phase velocities are quantities to be determined on the impermeable walls. In addition no explicit boundary conditions are available for the vorticities of the phases involved. Thus, ζ_1 and ζ_2 are updated on the boundary in terms of the streamfunction values at internal grid points using standard first–order accurate schemes.

4.5 Numerical Results and Discussion:

Numerical integration is therefore accomplished by solving equations (4.13) and (4.14) iteratively using successive line over–relaxation procedure.

Results have been obtained for various values of permeability, Forchheimer drag coefficient and Reynolds number. The range of dimensionless permeability tested is k = 1 , 0.1 , 0.01 and 0.00001 for C_d = 0.5 , 0.55 , 0.75 and for the R_e = 0.5 , 0.65 , 0.85 and 1 .

Solutions were obtained using a grid size of 31× 31 which corresponds to step size $H = \Delta X = \Delta Y = 0.05$ and the computational domain defined by $0 \le X \le 1$ and $0 \le Y \le 1$. The numerical scheme for solving equ. (4.13) is

$$F^{(n+1)}_{(i,j)} = F^{(n)}_{(i,j)} + \frac{\omega}{4}\left(F^{(n+1)}_{(i-1,j)} + F^{(n)}_{(i+1,j)} + F^{(n+1)}_{(i,j-1)} + F^{(n)}_{(i,j+1)} - 4\,F^{(n)}_{(i,j)} - H^2 \zeta_{(i,j)}\right). \quad (4.17)$$

in which ω is a constant to be determined. The iterated result converges for $1 \le \omega < 2$, and it converges most rapidly where ω is assigned the optimum value

$$\omega = \frac{4}{2 + \sqrt{4 - \left[\cos\left(\frac{\Pi}{m}\right) + \cos\left(\frac{\Pi}{n}\right)\right]^2}} \quad (4.18)$$

The stream function may take an arbitrary additive constant, in order to start the iteration, a constant value of 0.5 is guessed fore the stream function at all interior points. The absolute value of the difference between this and the previous value of Ψ_1 at the same point is calculated and is added to the value of a variable called ERROR, whose starting value at the beginning of an iteration is zero. At the end of one iteration, when the values of Ψ_1 at all interior points have been updated, the value of ERROR is compared with ERRMAX. If ERROR is less than or equal to ERRMAX, then calculate the value of ZETA1 Whose starting value at the beginning of iteration is zero, the absolute value of the difference between this and the previous value of ZETA1 At the same point is calculated and is added to ERROR. At the end of one iteration, when the values of ZETA1 At all interior points have been updated, the value of ERROR is compared with ERRMAX. If ERROR is less than or equal to ERRMAX, then desired accuracy has been reached, so the iterating process can be stopped. Otherwise, the iteration counter ITER is increased by 1 and a new iteration is started. In our program we let ERRMAX = 0.001.

After convergence, the fluid and dust velocity components are calculated in terms of Ψ_1 by applying forward and backward difference schemes to the equations:

$u_1 = \Psi_{1y}$, $u_2 = \Psi_{2y}$, $v_1 = -\Psi_{1x}$ and $v_2 = -\Psi_{2x}$. The fluid–phase vorticity is then calculated using a finite difference form of equation (4.13).

Figure one and Figure four illustrate the effect of the permeability on the horizontal velocity component along the vertical centerline and the boundary lower of the channel respectively. They demonstrate the increase of this velocity component with increasing permeability in the lower region of the channel. In the upper regions, the effect of the sink becomes more noticeable in attracting the fluid faster for lower values of permeability.

Figure two demonstrates the effect of the Forchheimer drag coefficient on the fluid–phase horizontal velocity component along the vertical centerline of the channel. It demonstrates the increase in this velocity component with increasing Forchheimer drag coefficient.

Figure five illustrates the effect of the Forchheimer drag coefficient on the fluid–phase horizontal velocity component along the boundary lower of the channel. It demonstrates the decrease in this velocity component with increasing coefficient in the lower regions of the channel.

Figure three and Figure six illustrate the effect of the Reynolds number on the fluid–phase horizontal velocity component along the vertical centerline and the boundary lower of the channel. They demonstrate the decrease in this velocity component with increasing Reynolds number in the lower regions of the channel.

Figure seven describes the effect of the permeability on the horizontal velocity component along the horizontal centerline of the channel. It demonstrates the increase of this velocity component with increasing permeability.

Figure 8 illustrates the effect of the Forchheimer drag coefficient on the fluid–phase horizontal velocity component along the horizontal centerline of the channel. It demonstrates the increase in this velocity component with increasing coefficient Forchheimer drag C_d.

Figure nine demonstrates the effect of the Reynolds number on the fluid–phase horizontal velocity component along the horizontal centerline of the channel. It demonstrates the decrease in this velocity component with increasing Reynolds number.

The increase of the velocity component in the lower part of the channel, and its decrease in the upper part as C_d increases sheds some light on the appropriate value of C_d. Although a critical value of C_d has not been determined in this study, the above behavior indicates that C_d should be less than 0.75.

Conclusions

In this work, an attempt has been made to develop a set of partial differential equations describing the flow of a dusty fluid in variable porosity media. The developed equations take into account the effect of the porous microstructure on the flowing phases, but leave undetermined diffusion and mass transfer coefficients, which have worthy of further investigation. The developed models make a distinction between flow in consolidated and in granular porous media in the manner in which the fluid-phase momentum equations and the dust-phase continuity equations are expressed. The difference in the type of media clearly plays a role in the mass transfer between the dust-phase and the porous matrix, as witnessed by the dust-phase continuity equations. It also plays a role in distinguishing the kind of resistance that the porous structure is exerting on the fluid, and the inertial effects caused by the porous microstructure, as witnessed in the fluid-phase momentum equations. Depending on the type of flow and the porous microstructure, the developed equations may be cast in different forms that parallel the equations governing single-phase flow in porous media.

We presented an overview of the equations governing the flow of a dusty fluid in various type media, including that in naturally occurring media. The dust parameters of interest are the mass of each dust particle and the number density. The porous media parameters of interest in naturally occurring porous media are the Forchheimer drag coefficient and the permeability. The drag coefficient C_d is dependent on the Reynolds number, and this study shows that the value of C_d should be less than 0.75.

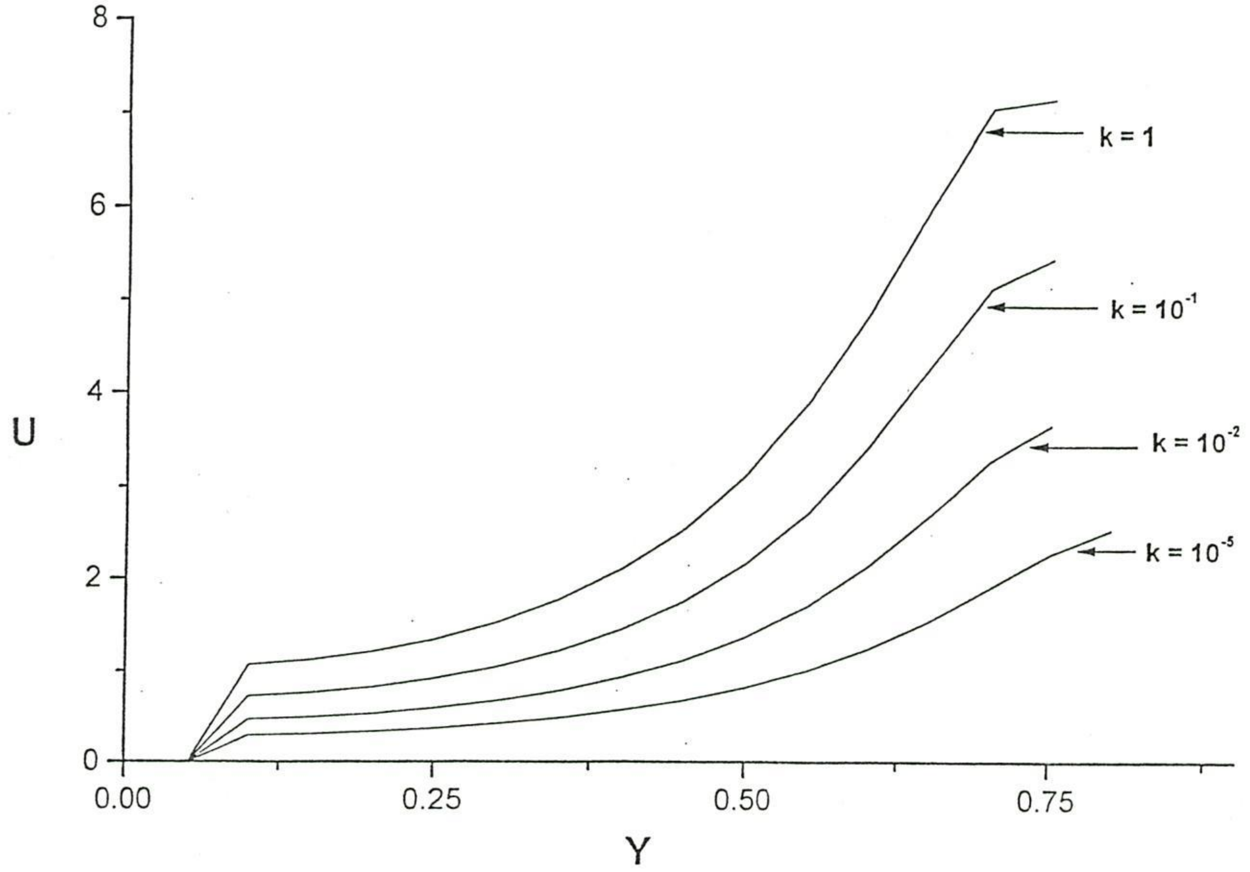


Fig. 1. Horizontal velocity component along the vertical center line of the channel for different permeability and

$C_d = 0.5$

$R_e = 1$

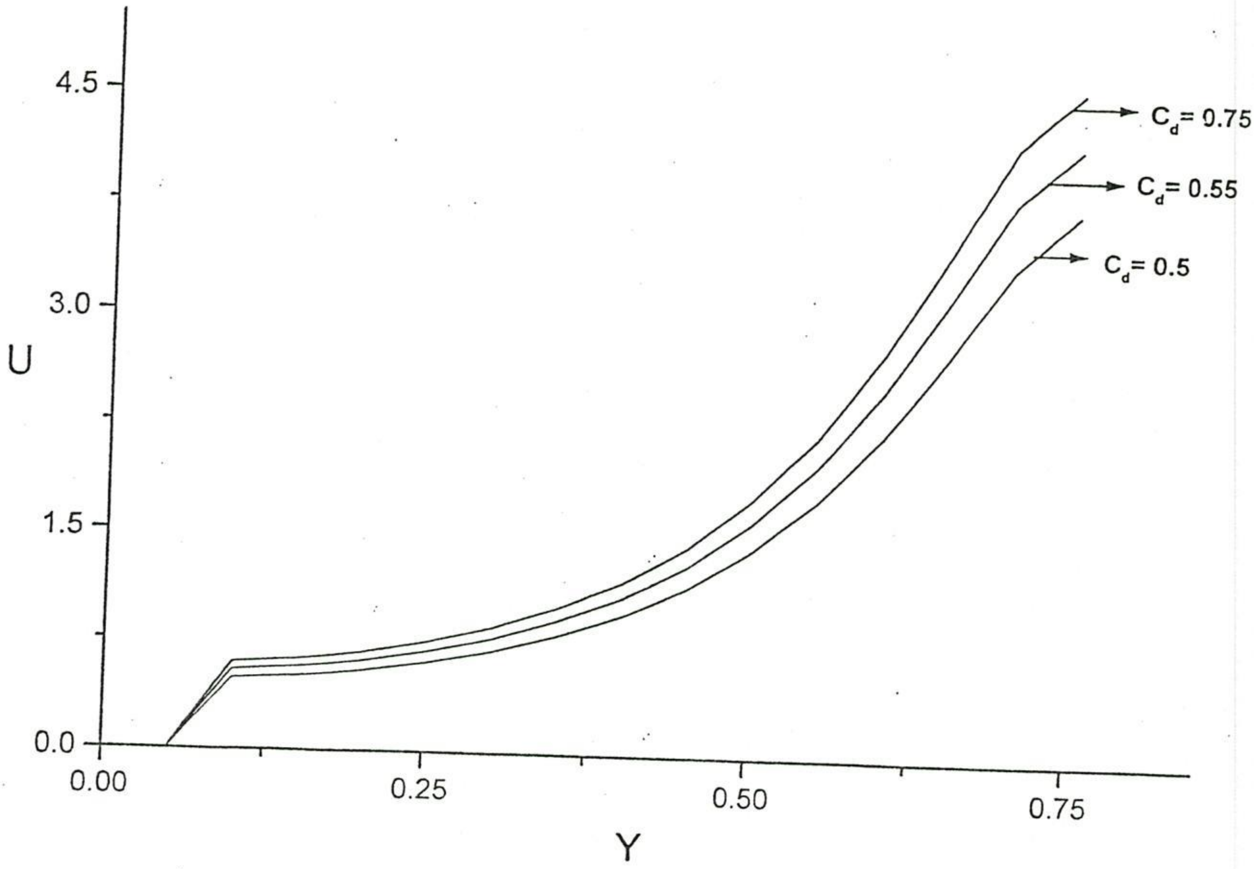


Fig. 2. . Horizontal velocity component along the vertical center line of the channel for different Forchheimer drag coefficient and

$K = 10^{-2}$

$R_e = 1$

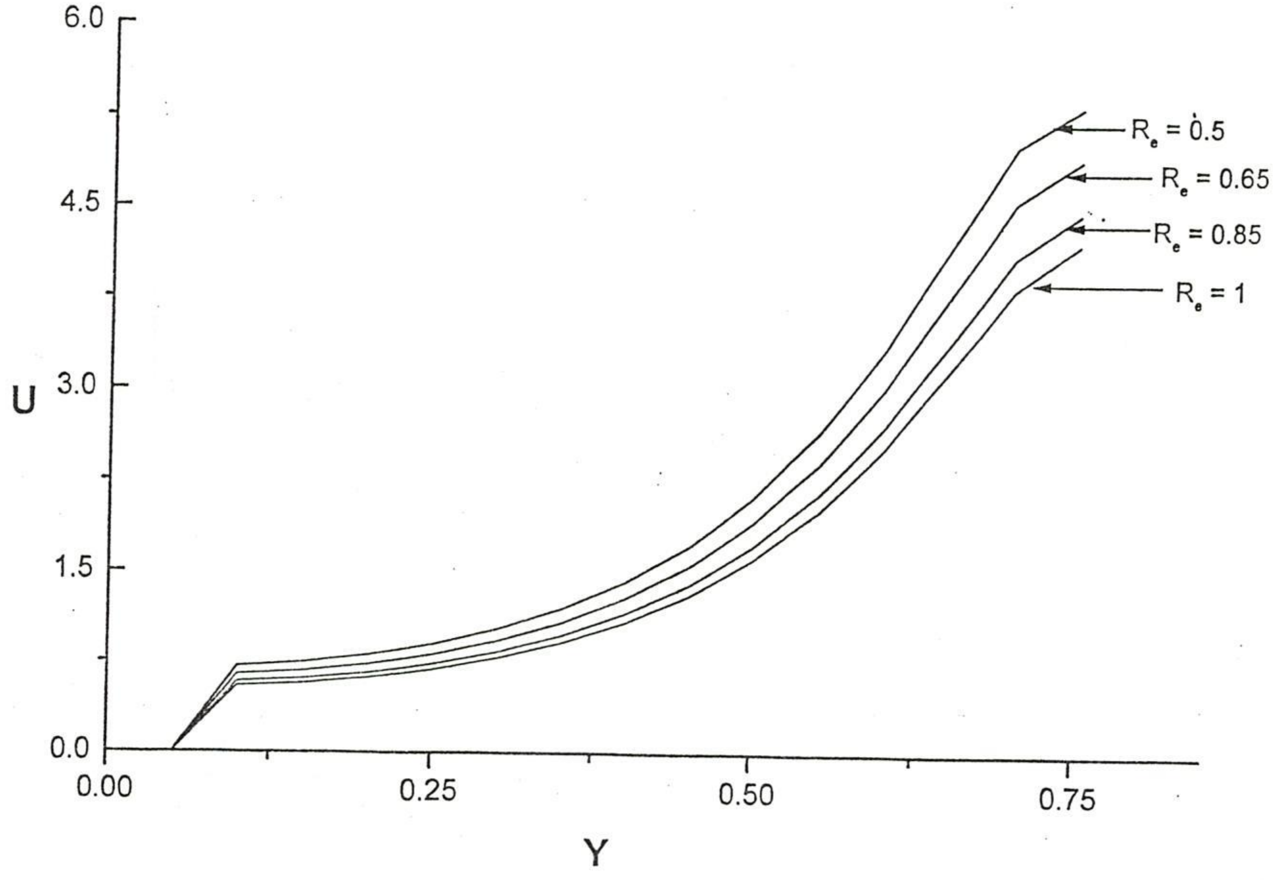


Fig. 3. Horizontal velocity component along the vertical center line of the channel for different Reynolds number and

$K = 10^{-2}$

$C_e = 0.75$

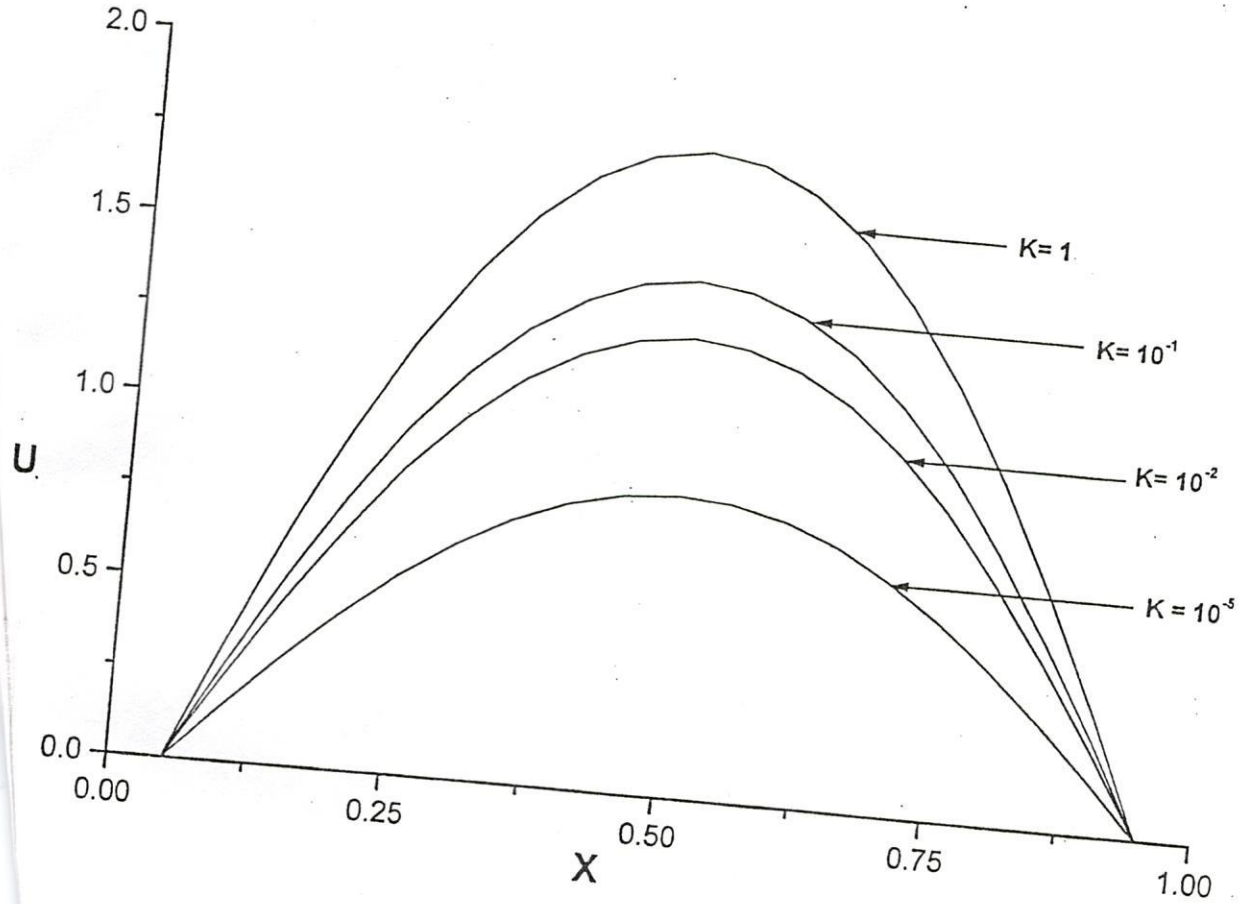


Fig.4. Horizontal velocity component along the lower boundary of the channel for different permeability and

$C_d = 0.75$

$R_e = 1$

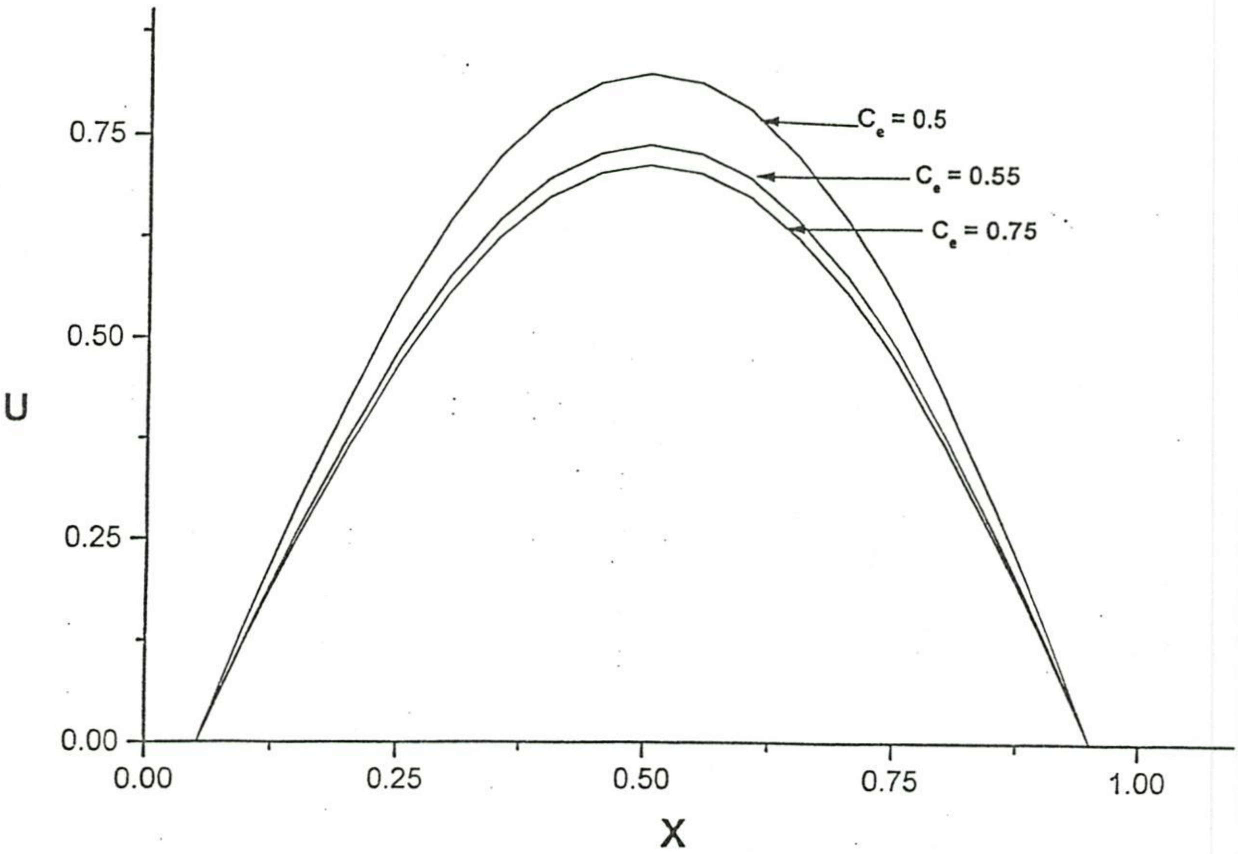


Fig.5. Horizontal velocity component along the lower boundary of the channel for different Forchheimer drag coefficient and

$K = 10^{-2}$

$R_e = 1$

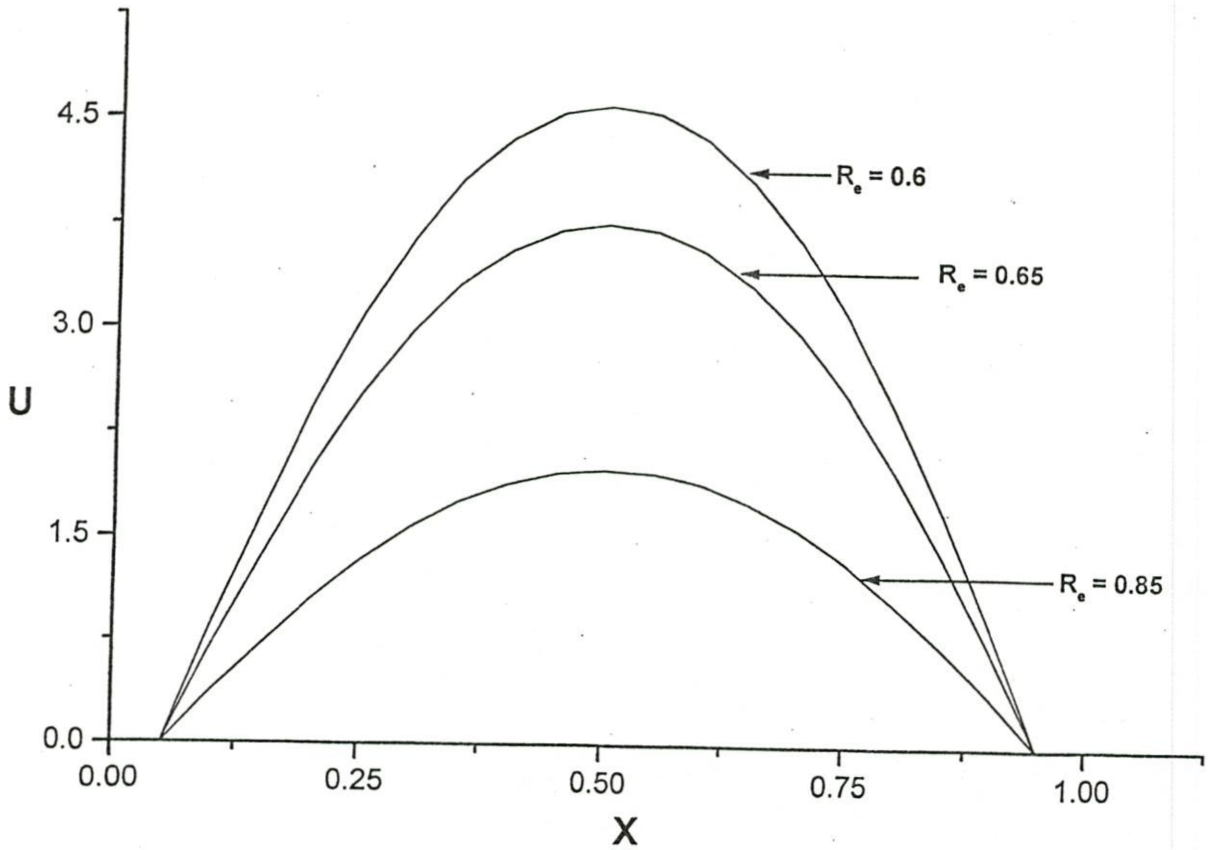


Fig.6. Horizontal velocity component along the lower boundary of the channel for different Reynolds number and

$K = 10^{-2}$

$C_d = 0.75$

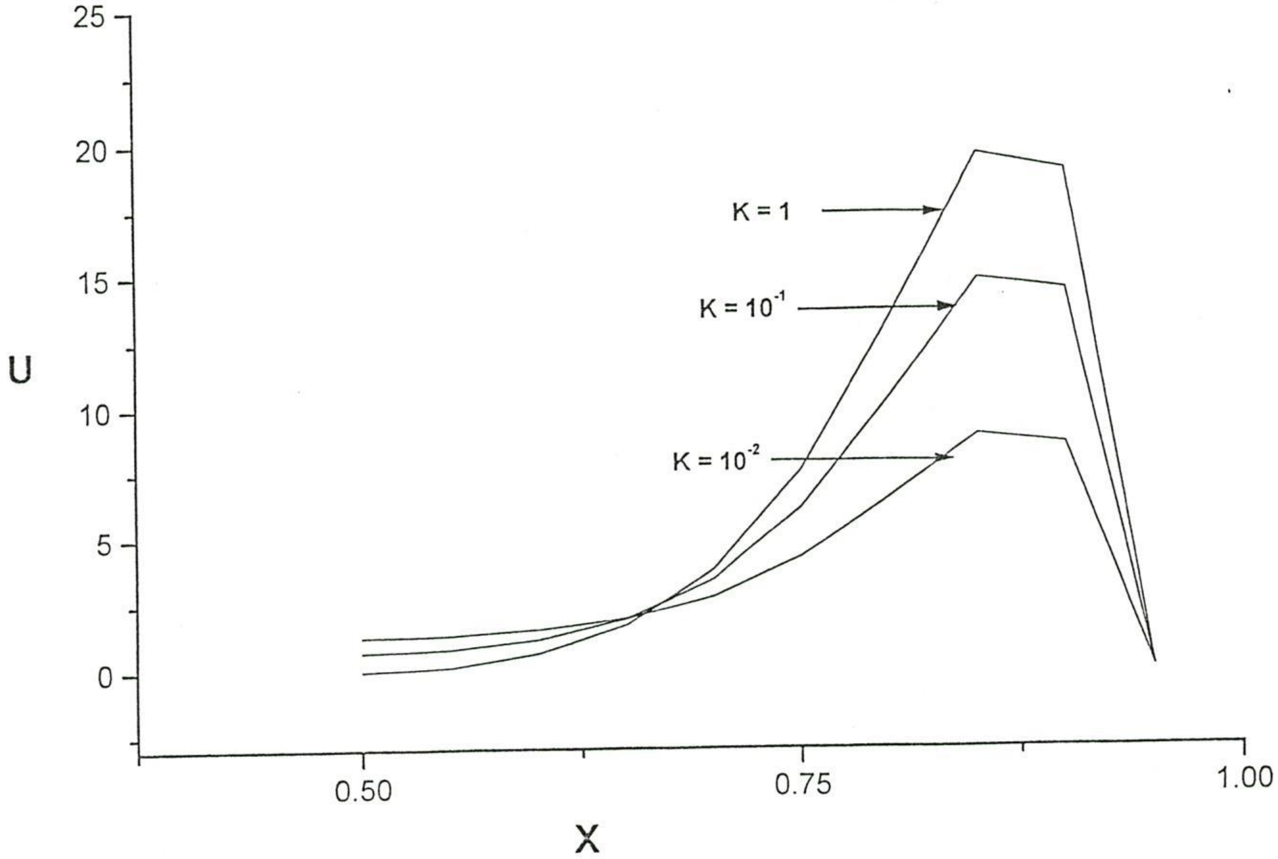


Fig. 7. Horizontal velocity component along the Horizontal center line of the channel for different permeability and

C_d = 0.75

R_e = 1

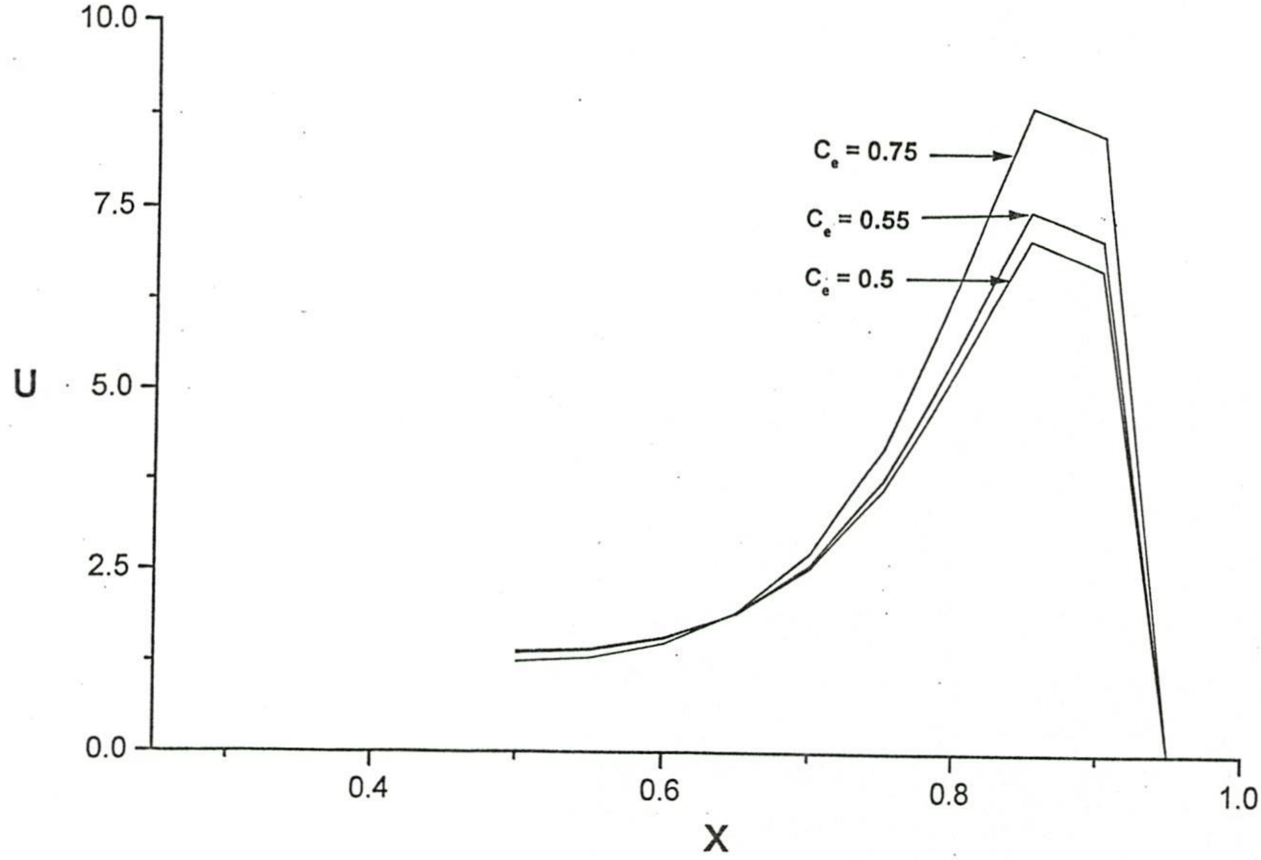


Fig. 8 . Horizontal velocity component along the Horizontal center line of the channel for different Forchheimer drag coefficient and

$K = 10^{-2}$

$R_e = 1$

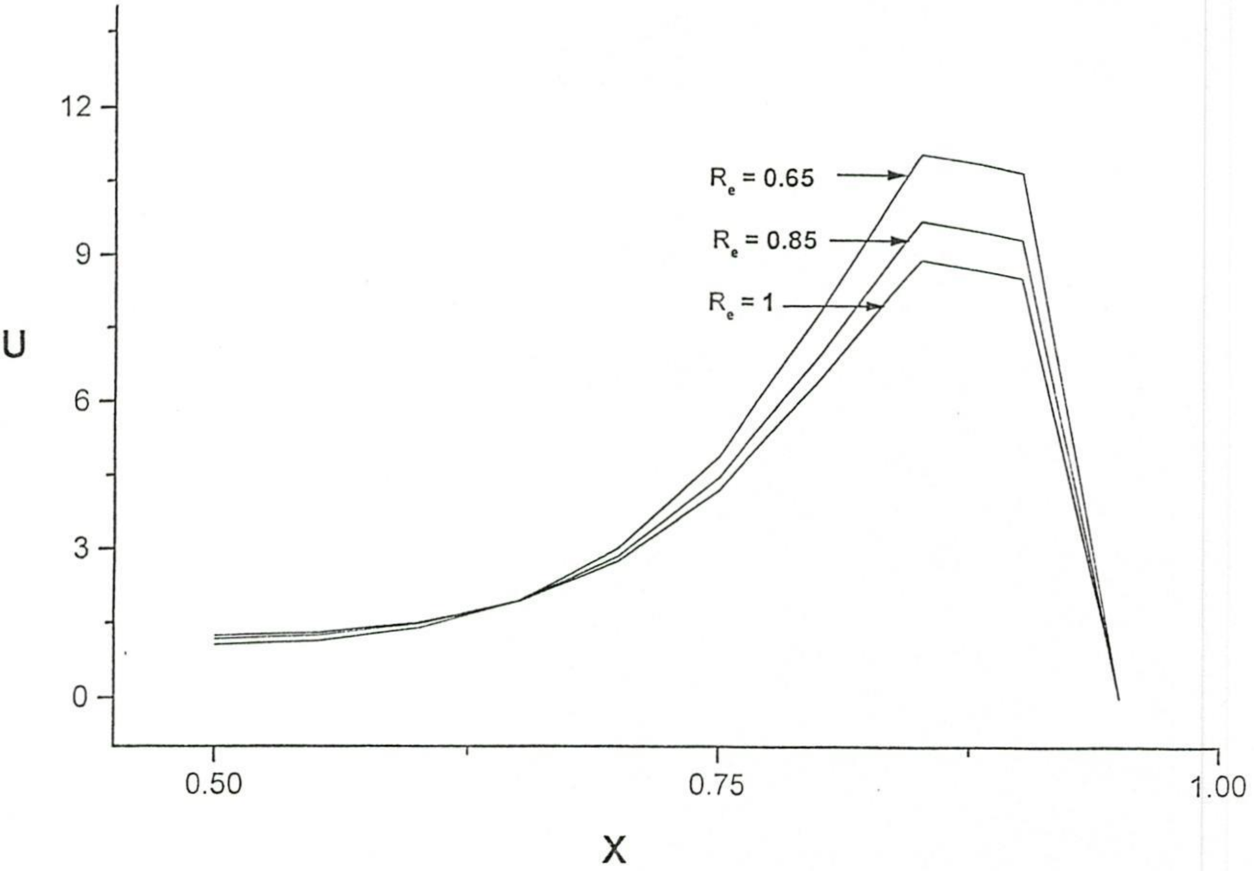


Fig. 9. Horizontal velocity component along the Horizontal center line of the channel for different Reynolds number and

$K = 10^{-2}$

$C_e = 0.75$

Appendices

Appendix 1

Averaging the dusty–fluid equations:

i) The dust–phase continuity equation:

$$\nabla \, . \, (N \, \mathrm{v}) = 0 \tag{A.1.1}$$

Applying rules (vi), (vii), (i), (iii), (i) respectively,

Rule (vi): $\langle \nabla \, . \, (N \, \mathrm{v}) \rangle = \nabla \, . \, \langle N \, \mathrm{v} \rangle + (1/V) \int_S n \, . \, (N \, \mathrm{v}) \, ds$. (A.1.2)

Rule (vii): $\langle \nabla \, . \, (N \, \mathrm{v}) \rangle = \nabla \, . \, \langle N \, \mathrm{v} \rangle$ (A.1.3)

Rule (i): $\nabla \, . \, \langle N \, \mathrm{v} \rangle = \nabla \, . \, \Phi \langle N \, \mathrm{v} \rangle_{\Phi}$ (A.1.4)

Rule (iii): $\nabla \, . \, \Phi \langle N \, \mathrm{v} \rangle_{\Phi} = \nabla \, . \, \Phi \left(\langle N \rangle_{\Phi} \langle \mathrm{v} \rangle_{\Phi} + \langle N^{\circ} \, \mathrm{v}^{\circ} \rangle_{\Phi} \right)$ (A.1.5)

Rule (i): $\nabla \, . \, \Phi \left(\langle N \rangle_{\Phi} \langle \mathrm{v} \rangle_{\Phi} + \langle N^{\circ} \, \mathrm{v}^{\circ} \rangle_{\Phi} \right) = \nabla \, . \, (\Phi \langle N_{\Phi} \, \mathrm{v}_{\Phi})$

$+ \nabla \, . (\Phi \langle N^{\circ} \, \mathrm{v}^{\circ} \rangle_{\Phi})$ (A.1.6)

Thus the averaged dust–phase continuity equation is given by

$$\nabla \, . \, (\Phi \langle N_{\Phi} \, \mathrm{v}_{\Phi}) + \nabla \, . (\Phi \langle N^{\circ} \, \mathrm{v}^{\circ} \rangle_{\Phi}) = 0 \tag{A.1.7}$$

ii) The dust–phase linear momentum equation is:

$$\nabla \, . \, (\mathrm{v} \, \mathrm{v}) = (k/m) (\mathrm{u} - \mathrm{v}) \tag{A.1.8}$$

Applying rules (vi,vii,i,iii,i)

Averaging the left hand side:

Rule (vi): $\langle \nabla \, . \, (\mathrm{vv}) \rangle = \nabla \, . \, \langle \mathrm{vv} \rangle + (1/V) \int_S n \, . \, (\mathrm{vv}) \, ds$. (A.1.9)

Rule (vii): $\langle \nabla \, . \, (\mathrm{vv}) \rangle = \nabla \, . \, \langle \mathrm{vv} \rangle$. (A.1.10)

Rule (i): $\nabla \, . \, \langle \mathrm{vv} \rangle = \nabla \, . \, \Phi \langle \mathrm{vv} \rangle_{\Phi}$. (A.1.11)

Rule (iii): $\nabla \, . \, \Phi \langle \mathrm{vv} \rangle_{\Phi} = \nabla \, . \, \Phi \left(\langle \mathrm{v} \rangle_{\Phi} \langle \mathrm{v} \rangle_{\Phi} + \langle \mathrm{v}^{\circ} \mathrm{v}^{\circ} \rangle_{\Phi} \right)$ (A.1.12)

Rule (i): $\nabla \, . \, \Phi \left(\langle \mathrm{v} \rangle_{\Phi} \langle \mathrm{v} \rangle_{\Phi} + \langle \mathrm{v}^{\circ} \mathrm{v}^{\circ} \rangle_{\Phi} \right) = \nabla \, . \, (\Phi \, \mathrm{v}_{\Phi} \, \mathrm{v}_{\Phi})$

$+ \nabla \, . \, (\Phi \langle \mathrm{v}^{\circ} \, \mathrm{v}^{\circ} \rangle_{\Phi})$ (A.1.13)

Averaging the right hand side:

Rule (viii): $\langle k/m \, (\mathrm{u} - \mathrm{v}) \rangle = k/m \langle \mathrm{u} - \mathrm{v} \rangle$ (A.1.14)

Rule (i): $k/m \langle \mathrm{u} - \mathrm{v} \rangle = k \, \Phi / m \langle \mathrm{u} - \mathrm{v} \rangle_{\Phi}$ (A.1.15)

Rule (ii): $k \, \Phi / m \langle \mathrm{u} - \mathrm{v} \rangle_{\Phi} = k \, \Phi / m \left(\langle \mathrm{u} \rangle_{\Phi} - \langle \mathrm{v} \rangle_{\Phi} \right)$ (A.1.16)

Rule (i): $k\,\Phi/m\,(\langle \mathrm{u} \rangle_{\Phi} - \langle \mathrm{v} \rangle_{\Phi}) = k\,\Phi/m\,(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi})$ (A.1.17)

Thus, the averaged dust–phase leaner momentum equation is given by

$$\nabla.(\Phi\, \mathrm{v}_{\Phi}\, \mathrm{v}_{\Phi}) + \nabla.(\Phi \langle \mathrm{v}^{\circ}\, \mathrm{v}^{\circ} \rangle_{\Phi}) = k\,\Phi/m\,(\mathrm{u}_{\Phi} - \mathrm{v}_{\Phi}) \quad \text{(A.1.18)}$$

iii) The fluid–phase continuity equation is

$$\nabla.\,\mathrm{u} = 0 \quad \text{(A.1.19)}$$

In order to average equation (A.1.19), we apply the rules (vi), (vii), (i) respectively,

Rule (vi): $\langle \nabla.\,\mathrm{u} \rangle = \nabla.\langle \mathrm{u} \rangle + (1/V)\int_S n.\,\mathrm{u}\ ds$. (A.1.20)

Rule (vii): $\nabla.\langle \mathrm{u} \rangle + (1/V)\int_S n.\,\mathrm{u}\ ds = \nabla.\langle \mathrm{u} \rangle$ (A.1.21)

Rule (i): $\nabla.\langle \mathrm{u} \rangle = \nabla.\,\Phi\langle \mathrm{u} \rangle_{\Phi} = \nabla.\,\Phi\,\mathrm{u}_{\Phi}$ (A.1.22)

Hence, the averaged fluid–phase continuity equation is given by

$$\nabla.(\Phi\,\mathrm{u}_{\Phi}) = 0 \quad \text{(A.1.23)}$$

iv) The fluid–phase linear momentum equation is

$$\rho\nabla.(\mathrm{u\,u}) = -\nabla p + \mu\nabla^2\,\mathrm{u} + kN\,(\mathrm{v} - \mathrm{u}) \quad \text{(A.1.24)}$$

Equation (A.1.24) is averaged term–wise, hence we can write it at the form

$$\langle \rho\,\nabla.(\mathrm{u\,u}) \rangle = -\langle \nabla p \rangle + \langle \mu\nabla^2\mathrm{u} \rangle + \langle kN(\mathrm{u} - \mathrm{v}) \rangle \quad \text{(A.1.25)}$$

In order to average the term $\rho\,\nabla.(\mathrm{u\,u})$ we apply the rules (viii), (vi), (vii), (i), (iii) respectively,

Rule (viii): $\langle \rho\,\nabla.(\mathrm{u\,u}) \rangle = \rho\langle \nabla.(\mathrm{uu}) \rangle$ (A.1.26)

Rule (vi): $\rho\langle \nabla.(\mathrm{uu}) \rangle = \rho\,\nabla.\langle \mathrm{uu} \rangle + (1/V)\int_S n.(\mathrm{uu})\ ds$. (A.1.27)

Rule (vii): $\rho\,\nabla.\langle \mathrm{uu} \rangle + (1/V)\int_S n.(\mathrm{uu})\ ds = \rho\,\nabla.\langle \mathrm{uu} \rangle$ (A.1.28)

Rule (i): $\rho\,\nabla.\langle \mathrm{uu} \rangle = \rho\,\nabla.\,\Phi\langle \mathrm{uu} \rangle_{\Phi}$ (A.1.29)

Rule (iii):
$$\begin{aligned}\rho\,\nabla.\,\Phi\langle \mathrm{uu} \rangle_{\Phi} &= \rho\nabla.\,\Phi\,(\langle \mathrm{u} \rangle_{\Phi}\,\langle \mathrm{u} \rangle_{\Phi} + \langle \mathrm{u}^{\circ}\mathrm{u}^{\circ} \rangle_{\Phi})\\ &= \rho\nabla.\,\Phi\,(\mathrm{u}_{\Phi}\mathrm{u}_{\Phi} + \langle \mathrm{u}^{\circ}\mathrm{u}^{\circ} \rangle_{\Phi})\\ &= \rho\nabla.(\Phi\,\mathrm{u}_{\Phi}\mathrm{u}_{\Phi}) + \rho\,\nabla.(\Phi\langle \mathrm{u}^{\circ}\mathrm{u}^{\circ} \rangle_{\Phi}) \quad \text{(A.1.30)}\end{aligned}$$

Hence,

$$\langle \rho\,\nabla.(\mathrm{u\,u}) \rangle = \rho\nabla.(\Phi\,\mathrm{u}_{\Phi}\mathrm{u}_{\Phi}) + \rho\,\nabla.(\Phi\langle \mathrm{u}^{\circ}\mathrm{u}^{\circ} \rangle_{\Phi})\ . \quad \text{(A.1.31)}$$

To average the term ∇p, we apply the rules (v), (i) respectively, Rule (v): $\langle \nabla p \rangle = \Phi \nabla (\langle p \rangle_\Phi) + (1/V) \int_S n\, p^\circ \, \mathrm{ds}$. (A.1.32)

Rule (i): $\Phi \nabla (\langle p \rangle_\Phi) + (1/V) \int_S n\, p^\circ = \Phi \nabla p + (1/V) \int_S n\, p^\circ \, \mathrm{ds}$. (A.1.33)

Hence,

$$\langle \nabla p \rangle = \Phi \nabla p_\Phi + (1/V) \int_S n\, p^\circ \, \mathrm{ds} \ . \quad \text{(A.1.34)}$$

In order to average the term $\mu \nabla^2 \mathrm{u}$, we first write it in the form $\mu \nabla . \nabla \mathrm{u}$ and using rules (viii), (vi), (vii), (v), (vii), (i) respectively,

Rule (viii): $\langle \mu \nabla . \nabla \mathrm{u} \rangle = \mu \langle \nabla . \nabla \mathrm{u} \rangle$. (A.1.35)

Rule (vi): $\mu \langle \nabla . \nabla \mathrm{u} \rangle = \mu \nabla . \langle \nabla \mathrm{u} \rangle + (1/V) \int_S n .(\nabla \mathrm{u}) \, \mathrm{ds}$. (A.1.36)

Rule (vii): $\mu \nabla . \langle \nabla \mathrm{u} \rangle + (1/V) \int_S n .(\nabla \mathrm{u}) \, \mathrm{ds}$

$$= \mu \nabla . \Phi \nabla (\langle \mathrm{u} \rangle_\Phi) + (1/V) \int_S n . (\nabla \mathrm{u}) \, \mathrm{ds}$$

$$= \mu \nabla^2 \Phi \, (\langle \mathrm{u} \rangle_\Phi) + (1/V) \int_S n . (\nabla \mathrm{u}) \, \mathrm{ds} \ . \quad \text{(A.1.37)}$$

Hence,

$$\langle \mu \nabla^2 \mathrm{u} \rangle = \mu \nabla^2 (\Phi \mathrm{u}_\Phi) + (1/V) \int_S n . (\nabla \mathrm{u}) \, \mathrm{ds} \ . \quad \text{(A.1.38)}$$

To average the term $kN(\mathrm{v} - \mathrm{u})$, we apply the rules (viii), (i), (ii)

Rule (viii): $\langle kN(\mathrm{v} - \mathrm{u}) \rangle = k \langle N(\mathrm{v} - \mathrm{u}) \rangle$. (A.1.39)

Rule (i): $k \langle N(\mathrm{v} - \mathrm{u}) \rangle = k \Phi \langle N(\mathrm{v} - \mathrm{u}) \rangle_\Phi$. (A.1.40)

Rule (iii): $k\Phi \langle N(\mathrm{v} - \mathrm{u}) \rangle_\Phi = k\Phi (\langle N \rangle_\Phi \langle \mathrm{v} - \mathrm{u} \rangle_\Phi + \langle N^\circ (\mathrm{v} - \mathrm{u})^\circ \rangle_\Phi$ (A.1.41)

Rule (ii): $k\Phi (N_\Phi (\mathrm{v}_\Phi - \mathrm{u}_\Phi) + \langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi) =$

$$k\Phi (N_\Phi (\mathrm{v}_\Phi - \mathrm{u}_\Phi)) + k\Phi (\langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi) \ . \quad \text{(A.1.42)}$$

Hence,

$$\langle kN(\mathrm{v} - \mathrm{u}) \rangle = k\Phi (N_\Phi (\mathrm{v}_\Phi - \mathrm{u}_\Phi)) + k\Phi (\langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi) \ . \quad \text{(A.1.43)}$$

Upon substituting equations (A.1.31), (A.1.34), (A.1.38), (A.1.43) into equation (A.1.25) we have

$$\rho \nabla . (\Phi \, \mathrm{u}_\Phi \mathrm{u}_\Phi) + \rho \nabla . (\Phi \langle \mathrm{u}^\circ \mathrm{u}^\circ \rangle_\Phi) = - \Phi \nabla p + \mu \nabla^2 (\Phi \mathrm{u}_\Phi)$$

$$+ k\Phi N_\Phi (\mathrm{v}_\Phi - \mathrm{u}_\Phi) + k\Phi (\langle N^\circ \mathrm{v}^\circ - N^\circ \mathrm{u}^\circ \rangle_\Phi) .$$

$$+ (1/V) \int_S (-n p^\circ + \mu n . \nabla \mathrm{u}) \, \mathrm{ds} \ . \quad \text{(A.1.44)}$$

Appendix 2

List of Principal Variables in the Computer Program

Program Symbol	Definition
ERROR,ERRMAX	Total error during one iteration and its maximum allowable value
H	Grid size in both x and y directions
ITER, ITMAX	Iteration counter and its maximum allowable value
I0, J0	Location of line vortex in index notation
M, N	Number of grid lines in x and y direction, respectively
PSI, PSIPRV	Stream function at the present and at the previous Iteration, respectively
X, Y	Coordinates of grid points
XMAX, XMIN, YMAX, YMIN	Limiting values on the abscissa and ordinate, respectively
X0, Y0	Coordinates of line vortex
ZETA	Vorticity
ZETA0	Vorticity at center of vortex line

Appendix 3

```
C

C     SOLVING POISSON EQUATION FOR THE FLOW AROUND A VORTEX BOUNDE
C     WITHIN A RECTANGULAR REGION .

      DIMENSION X(31), Y(31), ZETPRV(31,31),PSIPRV(31,31),
     1          ZETA(31,31),PSIX(31,31),PSIXX(31,31),
     1          PSIXY(31,31),PSIY(31,31),PSIYY(31,31),
     1          UMER(31,31),DEN(31,31),PSI(31,31)
      COMMON M,N,H
C     ..... ASSIGN INPUT DATA .....
      DATA XMAX,XMIN,YMAX,YMIN / 1., 0., 1., 0. /
      DATA ZETA0,X0,Y0,ITMAX,ERRMAX / 100., 0., 0., 153, 0.001 /

      H = 0.05
      PI=ACOS(-1.)
      Cd = 0.5
      k = 0.01
      Re = 1
C     ..... COMPUTE COORDINATES FOR GRID POINTS....

      M = INT(XMAX/H)
      N = INT(YMAX/H)
      X(0) = XMIN
      Y(0) = YMIN
      DO 1  I = 1,M
      X(I) = X(I-1) + H
   1  Y(I) = Y(I-1) + H

C     ..... ASSIGN BOUNDARY VALUES TO PSI...
      DO 3  I = 0,M
      PSI(I,0) = 0.
    3 PSI(I,N) = 1.
      DO 4  J = 0,N
      PSI(M,J) = .0
    4 PSI(0,J) = J*H
      PSI(M,M) = 1.
      DO 5  I = 1,M
      DO 5  J = 1,N
    5 ZETA(I,J) = 0.
      ZETA(0,0) = ZETA0

C     ..... ASSIGN GUESSED VALUES TO PSI....

      DO 6  I = 2,M-1
      DO 6  J = 2,N-1
    6 PSI(I,J) = 0.5

C     ..... LET ITERATION COUNTER START FROM ZERO....
      ITER = 0

C     ..... START AN ITERATION BY INCREASING ITERATION COUNTER BY ON
C           AND SETTING ERROR INITIALLY TO ZERO.BEFORE APPLYING
C           LIEBMANN,S FORMULA,THE LOCAL VALUE OF PSI ARE STORED IN
C           THE ARRY PSIPRV.ABSOLUTE DIFFERENCES BETWEEN TWO

C           CONSECUTIVE APPROXIMATIOS TO PSI AT INDIVIDUAL INTERIOR
```

```
C           POINTS ARE SUMMED AND STORED IN ERROR....
       DO 7  I = 0,M
       DO 7  J = 0,N
 7     PSIPRV(I,J) = PSI(I,J)
       ω=4/(2+(4-(COS(PI/M)+COS(PI/N))**2)**0.5)
 9     ITER = ITER + 1
       ERROR = 0.
       DO 41 KKK=1,1000
       DO 31 I = 2,M-1
       DO 31 J = 2,N-1
 31    PSIPRV(I,J)=PSI(I,J)
       DO 11  I = 2,M-1
       DO 11  J = 2,N-1
       PSI(I,J)=PSIPRV(I,J)+(PSI(I-1,J)+PSIPRV(I+1,J)+PSI(I,J-1)+
      1   PSIPRV(I,J+1)-4*PSIPRV(I,J)+ H**2*ZETA(I,J))*W/4.0
 11    ERROR = ERROR + ABS(PSI(I,J)-PSIPRV(I,J))
C      .....PRINT PSI IF FINAL VALUE OF ERROR IS LESS THAN OR EQUAL
C           TO ERRMAX,OR IF THE NUMBER OF ITERATIONS HAS REACHED THE
C           VALUE ITMAX ,OTHERWISE GO BACK FOR ANOTHER ITERATION
       IF( ERROR.LE.ERRMAX )  GO TO 14
       IF( ITER-ITMAX )  9,12,12
C      .....PRINT PSIPRV TO MAKE SURE THAT THE RESULT IS SATISFACTORY
C           AT EVERY GRID POINT, THEN SKIP THE NEXT PAGE....
 14    DO 15 I = 2,M-1
       DO 15 J = 2,N-1
       ZETPRV(I,J)=ZETA(I,J)
       PSIX(I,J)=(PSI(I+1,J)-PSI(I-1,J))/(2*H)
       PSIXX(I,J)=(PSI(I+1,J)-2*PSI(I,J)+PSI(I-1,J))/H**2
       PSIY(I,J)=(PSI(I,J+1)-PSI(I,J-1))/(2*H)
       PSIXY(I,J)=((PSI(I+1,J+1)-PSI(I-1,J+1))/(2*H)-PSIX(I,J))/H
       PSIYY(I,J)=(PSI(I,J+1)-2*PSI(I,J)+PSI(I,J-1))/H**2
       UMER(I,J)=((PSIX(I,J)**2)*(PSIXX(I,J)**2)
      1          +2*PSIX(I,J)*PSIY(I,J)*PSIXY(I,J)
      1          +(PSIY(I,J)**2)*(PSIYY(I,J)**2))*Cd/SQRT(k)
       DEN(I,J)=(SQRT(PSIX(I,J)**2+PSIY(I,J)**2))/k*Re
      1         +(PSIX(I,J)**2+PSIY(I,J)**2)*Cd/SQRT(k)
       ZETA(I,J)=UMER(I,J)/DEN(I,J)
 15    ERR1=ERR1+ABS(ZETA(I,J)-ZETPRV(I,J))
       IF (ERR1 .LE. ERRMAX) GO TO 12
 41    CONTINUE
 12    WRITE(*,*)  M,N,ITER,ERROR
       DO 13  I = 1,N
 13    WRITE(6,*) I*H,PSIY(I,2)
       END
```

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