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Performance Analysis of Energy Detection Based Spectrum Sensing over Generalized Fading Channels in Cognitive Radio Networks

Hikmat Yousef Mohammad Darawsheh

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Prepared By:

Hikmat Yousef Mohammad Darawsheh

B.Sc.: Electrical Engineering, 2000, An-Najah National University, Palestine

Supervisor: Dr. Ali Jamoos

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Prepared By: Hikmat Yousef Mohammad Darawsheh

Registration No: 21010122

Supervisor: Dr. Ali Jamoos

Master thesis submitted and accepted, Date: ....

The name and signatures of examining committee members are as follows:

1- Head of committee	Dr. Ali Jamoos	Signature: …………….
2- Internal Examiner	Dr. Rushdi Hamamreh	Signature: …………….
3- External Examiner	Dr. Anas Salhab	Signature: …………….

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Declaration:

I certify that this thesis submitted for the degree of Master, is the result of my own research, except where otherwise acknowledged, and that this study (or any part of the same) has not been submitted for a higher degree to any other university or institution.

Signed: ………….............................…

Hikmat Yousef Mohammad Darawsheh

Date: …….August 29, 2013...……….

Dedication

To my mother, father, wife, children and all of my family.

Acknowledgments

All praise be to Allah, the Lord of the worlds, peace and blessings of Allah be upon His Messenger, Prophet Mohammed (SAAW).

Many thanks are given to my thesis supervisor Dr. Ali Jamoos for his appreciated and valuable help; indeed, his directions, guidance and continuing support helped me greatly in accomplishing this thesis.

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Abstract

Cognitive radio is an emerging technology that has gain extensive attention in the last couple decades to solve the problem of radio spectrum under-utilization. Recently, several spectrum access techniques for cognitive radio networks have been addressed, Among the simple techniques which enable an opportunistic spectrum access is the energy-based detection.

In this thesis, the problem of energy detection of an unknown deterministic signal over generalized fading channels is revisited. More particularly, a new closed-form mathematical expression is derived for the average probability of detection of the energy detector over α-μ generalized fading channels with selection combining diversity reception. The derived expression is a generalization for the special cases of Nakagami-m, Weibull, Gamma, Rayleigh and Exponential fading distributions. This expression is then used to quantify the performance improvement of the energy detector with selection combining diversity reception.

The effect of various parameters of the derived expression on the complementary receiver operating characteristics of the energy detector is discussed. Namely, the effect of number of diversity branches, average signal-to-noise ratio, environment non- linearity parameter α, and the number of multipath clusters μ on the performance of the energy detector is plotted and discussed.

Keywords: cognitive radio networks, energy detection, selection combining, diversity reception, fading channels, α-μ generalized fading distribution model.

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Chapter 1

Introduction

1.1 Overview

This chapter introduces the main concepts of wireless communications related to the subject of this thesis. Starting from the electromagnetic spectrum frequency bands and their assignments to specific licensed users. The concept of cognitive radio is introduced, and the need for spectrum sensing techniques are addressed to make the cognitive radio more intelligent and aware of its environment. Then we look at the propagation of signals in wireless fading channels, and how they influence the parameters of the propagating signal. To overcome fading, diversity reception techniques are addressed to enhance the overall detection of the transmitted signals using multiple antennas, frequencies, and time delays. A general view of the thesis and its contributions are illustrated in the following sections.

1.2 Literature Review

Electromagnetic spectrum, as a natural resource, is limited. It has been divided into specific frequency bands that are assigned to different applications and usages. The assignment process, which is organized by regulatory bodies (international and/or local), allows some of these frequency bands to be used for free by the public community (e.g. Amateur) but under some conditions including the maximum allowed power to be transmitted and making no interference to the adjacent bands. While the other frequency bands are either sold to primary users or reserved for scientific researches, military, etc. These primary users are also called licensed subscribers, and they have the full permission to use the bands they bought whenever and wherever they need to, as long as their licenses are valid. However, vast amounts of the spectrum frequency bands are not used efficiently, indeed, they are under-utilized [Aky06].

Cognitive radio (CR), as a clever telecommunication system that can sense and adapt its parameters to avoid interference on licensed users [Hay05], is one solution to this underutilization problem. The CR user is considered as a rental or secondary user of the spectrum and it has to decide when it can access the spectrum and what band to use without causing any kind of interference to licensed user. This leads to the fact that cognitive radio network must accurately sense the spectrum and adapts its transmission parameters in accordance with the results of its sensing operation and the situation of the channel to be used. Several spectrum sensing techniques are proposed in the literature to enhance the sensing process of the spectrum, and so enhancing its utilization [Yuc09]. The ultimate goal of these techniques is to enable rental (secondary, CR) users to benefit from the white spaces in the spectrum that are spatially/temporally free of primary (licensed) users. The energy detector (ED) proposed in [Urk67], is one of the main and simplest techniques frequently used in cognitive radio networks to enable opportunistic spectrum access. The CR user may work alone with its own sensing, adaptation and transmission decision, or it can be part of a group of cognitive radio network that uses cooperative spectrum sensing techniques and different decision-making criteria for accessing the spectrum. The cooperation of multiple CR users enhances the overall utilization of the spectrum and decreases the chance of making interference to the primary users, this is because some problems that face the single CR user case, like hidden terminal problem and channel uncertainty, are overcome with the use of cooperative spectrum sensing in a cognitive radio network [Mis06], since a terminal maybe hidden for one or more senders, but not to all of them.

In wireless communications, signals propagating over certain channels suffer from fading, which is a deviation in the signal's envelope due to multipath propagation or shadowing effects. Fading can be modeled as a random process that can have statistically known distribution. Several fading distribution models have been suggested to describe the statistics of the received signal envelope [Sim05]. Indeed, the short-term signal envelope variation is properly depicted by several main distributions such as Rayleigh, Rice, Nakagami-*m*, Weibull, Hoyt and others. Each of these fading distributions is suitable for certain channel conditions. In some situations, no distributions satisfactorily match experimental observations, although one of them may produce moderate fitting. This motivates the need for a general distribution that can give better fitting to real measurements and can include several fading distributions as special cases. One of these general fading distributions is the $\alpha - \mu$ distribution proposed in [Yac02]. It is an umbrella distribution and involves as special cases several main distributions such as Nakagami-m, Rayleigh, Gamma, Weibull, exponential, and one sided Gaussian. In addition, its probability density function, cumulative distribution function and moment generating function come-out in uncomplicated closed-form formulas. Furthermore, it can describe the non-linearity of the wireless propagation environment. These features make the $\alpha - \mu$ distribution very attractive.

Fading channels can extremely affect the transmitted signals and decrease the overall signal to noise power ratio (SNR) at the receiver. In this case, antenna diversity reception techniques, that combine the outputs of multiple fading branches together, can be used to boost the SNR at the receiver. Selection combining (SC), equal gain combining (EGC), switch and stay combining (SSC), switch and examine combining (SEC), and maximum ratio combining (MRC) are some examples of combining methods used in antenna diversity reception [Stü11]. In this thesis, we will focus on SC technique since there is only one circuit at the receiver combiner, also since it is a starting point for series of researches that will cover most of the diversity combining techniques mentioned above. In diversity reception techniques, redundant copies of the same signal are received via two or more radio paths and combined together to increase the overall received SNR. Indeed, using multiple copies of the same signal can avoid the deep fades that may suffer from. Extracting multiple copies of the same signal can be achieved by using different schemes such as time diversity, frequency diversity, space diversity, etc. [Rap02].

1.3 Motivation

During the last decade, a lot of interest has been paid to the issue of detecting unknown deterministic signals over a variety of fading channel models with or without diversity reception at the receiver [Kos02][Dig07][Her11]. In [Kos02] the average detection probability (ADP) of the ED is derived for Rayleigh, Rician and Nakagami-*m* faded signals. An alternative analytical approach have been proposed by Digham *et al.* in [Dig07], where closed-form expressions are obtained for the average detection probability undergoing Rayleigh and Nakagami-m fading with square law combining and square law selection diversity methods. In [Her11], the moment generating function (MGF) technique and the probability density function (PDF) technique are used to evaluate the performance of energy detector undergoing Rician and Nakagami-m fading with several diversity combining techniques. However, this yields a wide collection of performance expressions that are applicable only for certain fading models with specific model parameters. To avoid this drawback, Fathi *et al.* have recently proposed a versatile performance expression for energy detector over the $\alpha-\mu$ generalized fading channels [Fat12a]. Nevertheless, no diversity combining techniques are considered.

1.4 Thesis Contribution

In this thesis, we suggest to extend the results in [Fat12a] by considering selection combining diversity reception at the receiver. A new closed-form expression is derived for the average detection probability of the energy detector over $\alpha-\mu$ generalized fading channels with selection combining diversity reception [Dar13a] [Dar13b].

Some special cases of the derived expression are extracted to coincide with other corresponding distribution models, but they are shown in new mathematical forms that are not found in literature as per authors' knowledge.

1.5 Thesis Outline

The remainder of this thesis is structured as follows. Chapter 2 reviews the definitions and techniques of CR and spectrum sensing. Chapter 3 discusses the types of fading channels, fading distributions, and antenna diversity combining techniques. Chapter 4 introduces the system model for the ED, and the main contribution of this thesis, which is the derivation of equations for the average probability of detection of the ED over the generalized $\alpha - \mu$ fading model with selection combining diversity technique. Numerical examples and figures are presented and discussed in Chapter 5. Finally, key points of the thesis and some suggestions for future works are reported in Chapter 6.

Chapter 2

Cognitive Radio and Spectrum Sensing

In this chapter, we will introduce the terms of cognitive radio and cognitive radio networks in more details, over which spectrum utilization enhancements are gained. Scanning the spectrum for available holes or white spaces is achieved by spectrum sensing techniques embedded into the CR. Non-cooperative, Cooperative, and interference-based spectrum sensing are presented and caparisoned with each other for the sack of knowing which one is suitable for which case.

2.1 Cognitive Radio Networks

CR is a software defined radio coupled with cognitive capabilities, where it can sense the spectrum, understand the radio frequency (RF) environment, learn from past experience, decide when and how it can access the spectrum and then act accordingly [Mit99]. CR is also defined by Haykin as an "intelligent wireless communication system that is aware of its surrounding environment, learns from the environment and adapts its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters in real time" [Hay05].

An official definition of CR was put first by the Federal Communications Commission (FCC) in the year 2003, namely the ET Docket No. 03-108 [FCC03], defined CR as "a radio that can change its transmitter parameters based on interaction with the environment in which it operates". After that, some IEEE working groups that are related to CR and its proposed applications were established like the IEEE P1900.1 and its subsequent ones. The first official standard adopted by the IEEE for CR was the IEEE 802.22 for the wireless regional area network (WRAN) operating in unused television channels, it was published in 2011, but other versions of the IEEE 802.22.x were subsequently released [IEE11].

2.2 Objectives of Cognitive Radio Networks

Since the electromagnetic spectrum is very crowded and almost fully assigned to licensed users as shown in Figure 2.1, and simultaneously, it is under-utilized as shown in Figure 2.2, CR is a proposed solution to this under-utilization problem by making use of the spectrum holes that are available sometimes in some geographical

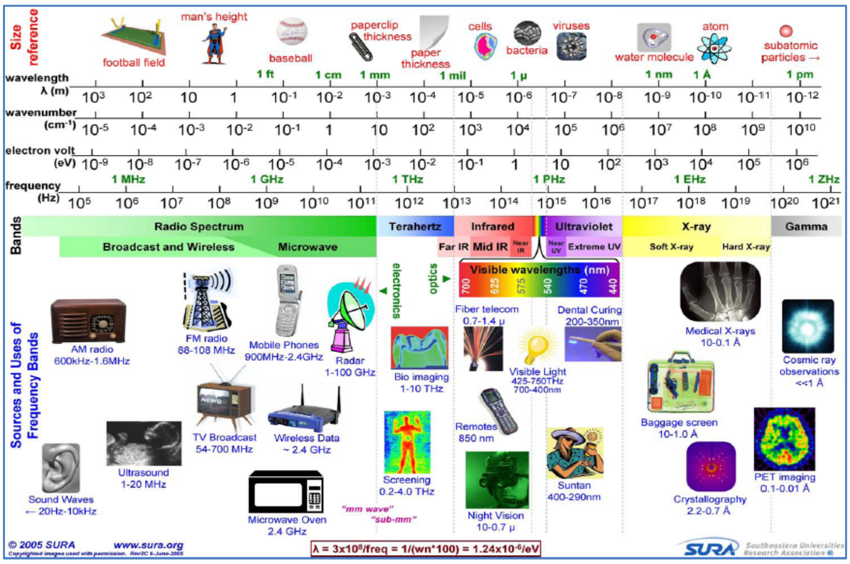

Figure 2.1: The electromagnetic spectrum [Sur05].

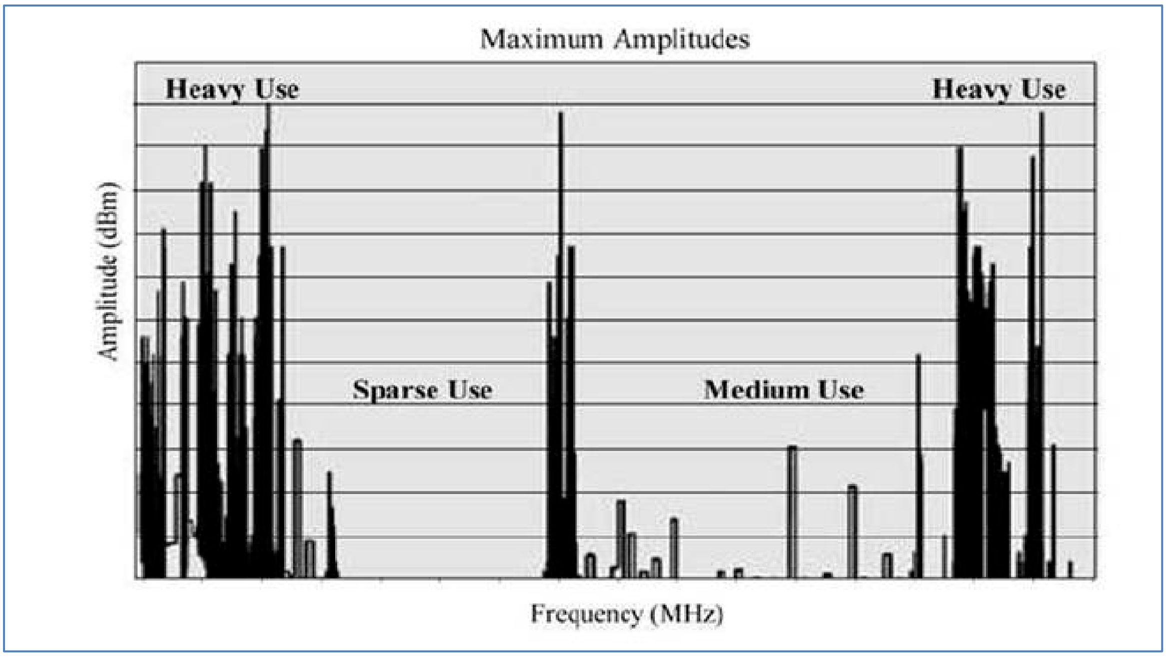

Figure 2.2: Spectrum utilization [Aky06]

locations. Figure 2.3 shows some white spaces or holes of the spectrum, where CR users can dynamically access as it is free of primary users.

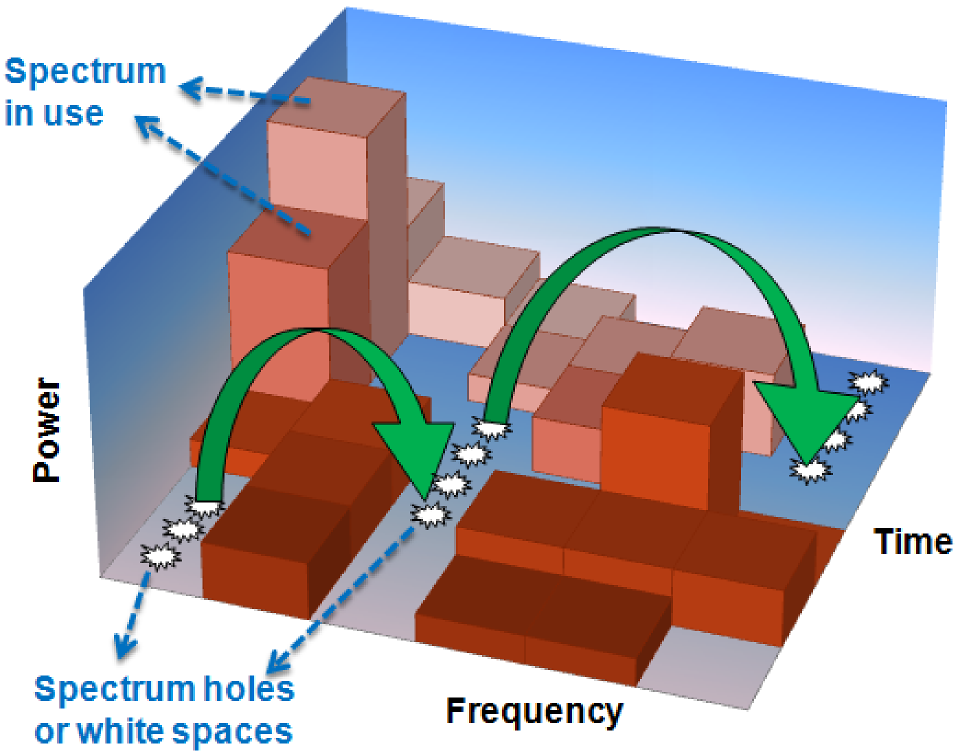

Figure 2.3: Spectrum holes

According to Haykin, the primary objectives of the CR as providing highly reliable communications and utilizing the radio spectrum efficiently [Hay05]. This emphasizes that the key point in any CR users' communications is to ensure that they are cognitive enough to maximizing the utilization of the spectrum while avoiding any kind of interference to the primary (licensed) users. In addition, they have to optimize the sensing/throughput trade-off [Lia08]. CR users may work alone where each user can decide when and how to start transmission based on its own spectrum sensing for white spaces or holes, or they can work in networks as collaborative groups either centralized or decentralized [Aky06].

The major functional blocks of CR are spectrum sensing, spectrum management, spectrum sharing, and spectrum mobility. The most important functional block of them is the spectrum sensing, since detecting the holes in the spectrum so as not to cause interference with existing licensed users is the key point in building any CR standalone user or network.

2.3 Spectrum Sensing Techniques

In literature, there are many spectrum sensing techniques for CR that are frequently used and discussed [Sub11], [Yuc09], and [Ari09]; they can be classified as shown in Figure 2.4. Non-cooperative spectrum sensing techniques include ED, matched filter detector (MFD) and cyclo-stationary feature detector (CSFD). Each one of these detectors has advantages and disadvantages that will be discussed in a while, but we will focus on the ED technique since the main contributions of this thesis are based on the performance analysis of this type of spectrum sensing technique over generalized fading channels. Other sensing techniques will be covered in general.

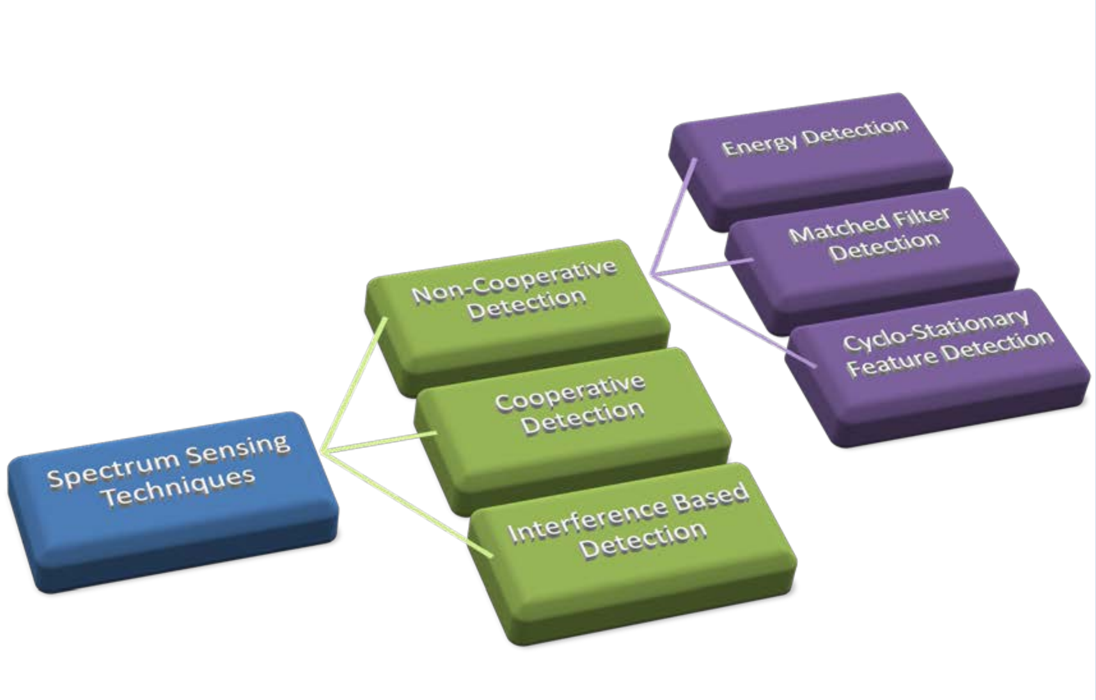

Figure 2.4: Spectrum sensing techniques

2.3.1 Non-Cooperative Spectrum Sensing

Matched filter detection technique depends on apriori knowledge of the transmitted signal; the detector retrieves the sent data by applying a convolution between the received signal and the pilot data of the transmitted signal.

In cyclo-stationary feature detection technique, a partial pilot data of the transmitted data is used to recover the transmitted signal, the main idea behind this technique is the periodicity of signal mean and autocorrelation. This periodicity is not found when receiving only noise signal as it is randomly distributed. Transmitted signal usually uses carrier signal and/or code excited signal which are periodic in nature, and so, this periodicity is reflected on autocorrelation and mean. Thus, these features of transmitted signal can be extracted and distinguished from other noise signals.

Energy detector will be discussed in more details later in chapter 5. It is simply an energy-harvesting device that consists of a band pass filter (BPF), an analogue to digital converter (ADC), a square-law device and an integrator. ED usually collects statistics of received signal amplitude squares within a specific time interval and averages them to measure the energy of the signal [Urk67]. Then it decides whether there is a transmitted signal or not (i.e. just the additive white Gaussian noise (AWGN)). This is achieved by comparing the measured energy with a predetermined threshold that distinguishes between two hypotheses of primary user existence or not.

The following table summarizes the main advantages and disadvantages of non-cooperative spectrum sensing techniques.

Table 1: advantages and disadvantages of spectrum sensing techniques

	Advantages	**Disadvantages**
Energy Detector	• Simple to implement • Low computational cost • No prior information needed about the primary signal	• Sensing time is high • Performance degrades with noise power uncertainty • Can't distinguish primary signals from secondary ones • Can't work in low SNR • Can't be used to detect spread spectrum signals

Matched Filter Detector	• Optimal and high performance detection • Low computational cost	• Requires dedicated receiver for every primary system • A prior knowledge of the primary system is needed
Cyclo-Stationary Feature Detector	• Robust in low SNR and in interference	• Prior partial knowledge of the primary system is needed • Number of samples needed are high • Computational cost is high

In non-cooperative systems, cognitive user works alone in detecting spectrum holes. Although these system techniques seem to be faster than cooperative ones since they don't need to wait for other nodes' cooperation and decision, they suffer severely from hidden terminal problem and channel uncertainty caused by either multipath fading or shadowing, in which the cognitive user is unaware of the presence of primary user. Hence, it may take wrong decision about the spectrum state, yielding in interference with the licensed or primary user. Such problems are suitably solved using the cooperative techniques [Yuc09], where some distributed and spaced secondary users cooperate in taking the decision about the spectrum, causing more accurate results than the standalone techniques. The penalty to be paid is more waiting time and comparatively more complex implementation.

2.3.2 Cooperative Spectrum Sensing

Cooperative spectrum sensing is a scheme where a CR user cooperates with other users in a group by sharing its collected information and parameters, like interference temperature levels measured at each user, current SNR, results of its detection decisions, etc. This yields in accurate spectrum estimation.

There are three main categories for cooperative spectrum sensing groups; centralized coordinated, decentralized coordinated, and decentralized uncoordinated as shown in Figure 2.5. The centralized coordinated group includes, in addition to other regular CR's, one main CR called the fusion centre (FC), where it collects all information from all nodes of the group, takes a decision on the state of the spectrum, and broadcasts the results for all nodes in order to organize the spectrum access task, ensuring that no interference to primary users may occur.

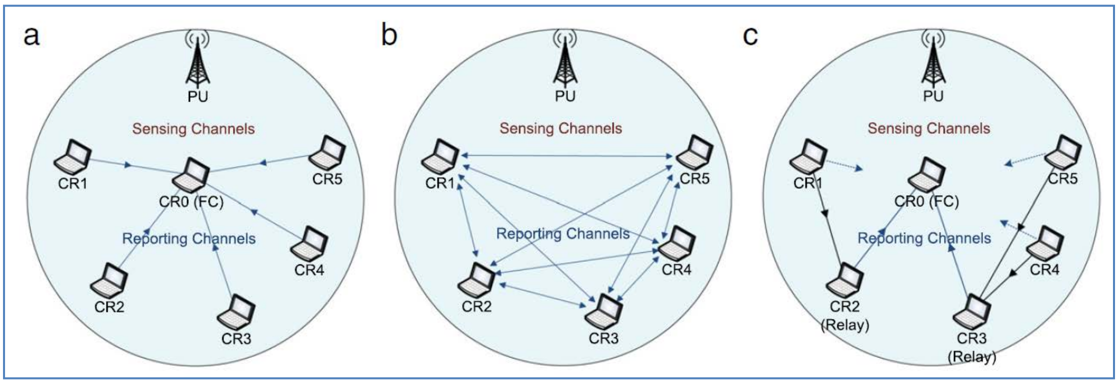

Figure 2.5: Cooperative sensing techniques (a) centralized coordinated (b) decentralized coordinated (c) decentralized uncoordinated [Aky11]

In decentralized coordinated groups, no master CR or FC exists, but information between all users is exchanged, and the decision to access the spectrum is made by an iterative algorithm distributed in each CR [Gha05].

While in decentralized uncoordinated, there is a FC, but the reporting channels from the FC to some CR’s are not perfect and can’t be used to get final decisions directly from the FC. In this case, some CR’s that have good connections with the FC can serve as relays for other weak-reporting-channel CR’s to pass them decisions from the main FC.

2.3.3 Interference Temperature Based Detection

This spectrum sensing technique depends on measuring the total noise power at the receiver antenna. Figure 2.6 shows the FCC's ET Docket No. 03-237 interference temperature model adopted in 2003 that suggested a metric to quantify and manage interference. This model shifts the concept of interference from the transmitter side to the receiver side. Interference temperature is calculated by dividing the total power at the receiver antenna on the associated RF bandwidth and the Boltzman’s constant (1.38 x 10-23 watt-sec/K, where K refers to Kelvin). The maximum allowed interference on the receiver antenna is referred to as interference temperature limit, where unlicensed transmitters are not allowed to exceed this limit in order not to cause any harmful interference on the receiver of the licensed transmitter [FCC03].

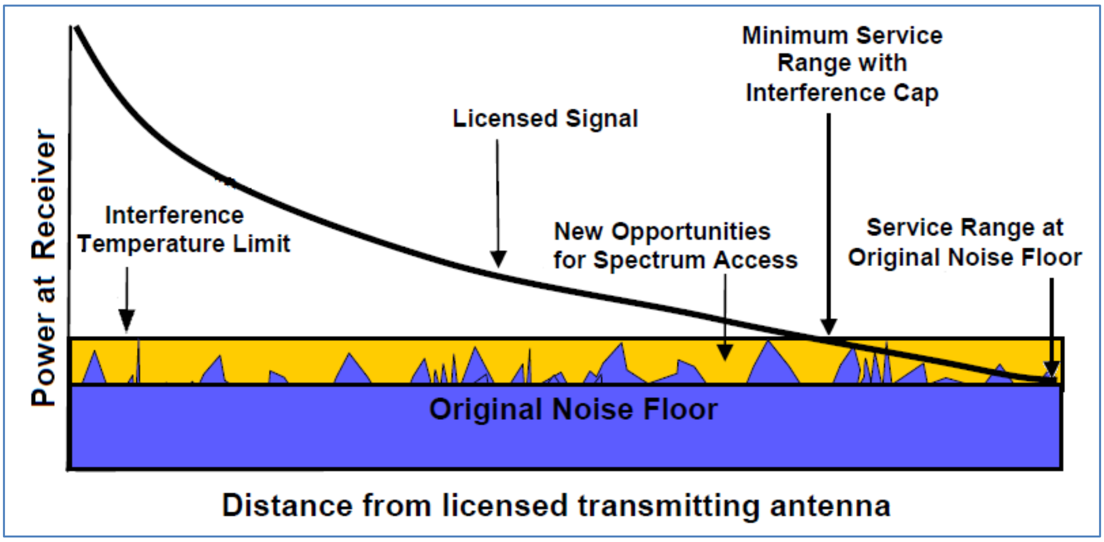

Figure 2.6: Interference temperature model [FCC03].

According to path loss models, power received at primary receiver decreases exponentially with distance, reaching the original noise floor level at which the receiver ignores any communications and consider them as noisy signals.

Any transmission above the interference temperature limit should be removed and considered as noise by the primary receiver, this means that having larger interference temperature limits will reduce the maximum range of the licensed primary transmission, but at the same time, allow for CR's to work under these limits. This is called underlay transmission, where both primary and secondary transmissions operate simultaneously without having secondary transmission cause interference to the primary ones [Men05].

Other cooperative spectrum sensing techniques are available in literature that are less famous than those addressed above, such as the filter bank based spectrum sensing, wavelet based detection, random Hough transform based detection, radio identification based detection [Sub11]. They are beyond the scope of this study, and are mentioned for general knowledge only.

Chapter 3

Fading Channels and Diversity Reception

This chapter addresses the problem of fading, where many fluctuations on wireless signal's amplitude, phase, polarization, and angle of arrival may happen. We discuss many statistical fading models that are known in literature by showing their PDF expressions and plotting many curves for different values of included parameters. Then we will see some diversity combining techniques that are used mainly to enhance the SNR of the system, and so, achieve much reliable communication channel.

3.1 Fading Channels

When electromagnetic signals propagate in most channels on earth's atmosphere and near the ground, they suffer from fading, in which the amplitude, phase, and angle of arrival of the received signal fluctuate due to shadowing and multipath phenomena. since there are a lot of obstacles such as trees, buildings, terrain or hills facing the propagation path of the radio signal, in which reflection, diffraction, or scattering of

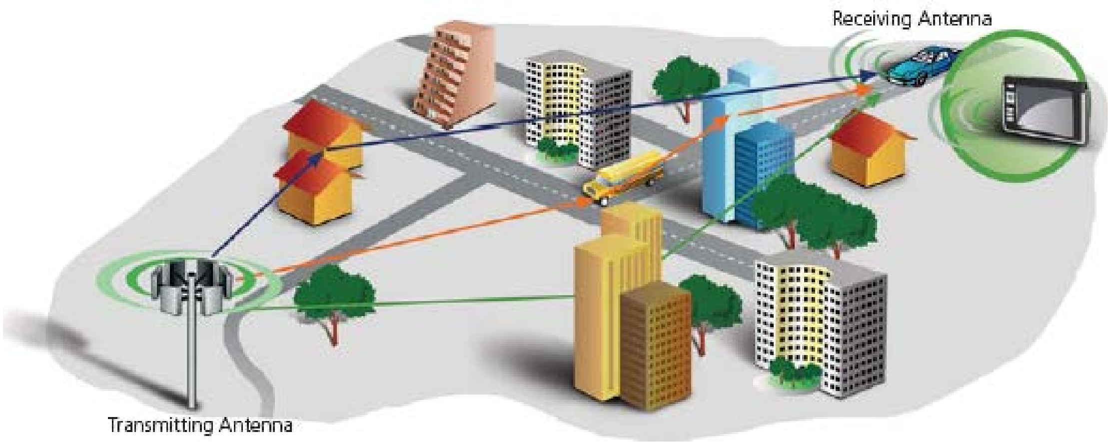

Figure 3.1: Multiple replicas of the transmitted signal arrive at the receiver [save9]

the signal occurs [Stü11]. Figure 3.1 shows the multipath phenomenon, in which multiple replicas of the transmitted signal arrive at the receiver's antenna from more than one path; with each replica has different amplitude, phase shift, and time delay, resulting in either constructive or destructive effects on the original signal.

Those channels are called fading channels, and their effects on the transmitted radio signals are often statistically modeled into mathematical expressions that describe the behavior of the signals on these channels. Each model can be applicable on some environmental conditions depending on the propagation paths of the signals.

For example, in non-line of site (NLOS) propagation, Rayleigh distribution model can mostly fits the statistics of the received signal envelope. While in line of site (LOS) case, the Rician distribution model can be more accurate. Deep or severe fading occurs when the communication channel suffers from destructive effects, in which the received SNR decreases drastically and may lead to losing of the transmitted signal [Sim05].

3.1.1 Slow Fading and Fast Fading

In order to distinct between slow and fast fading channels, we must understand the coherence time of the channel, which is equal to the period of time where the fading process is correlated [Sim05], or alternatively, it can be defined as the minimum time period in which the magnitude's change of the channel is uncorrelated from its previous value. The coherence time is roughly equal to the reciprocal of the channel Doppler spread frequency.

The degradation categories of fading; slow or fast fading, refer to the time-variant nature of the channel. If the time duration of a transmitted symbol is larger than the channel coherence time, then the channel is said to be fast fading channel which has little frequency dispersion into the received signal, and imposes a roughly constant change on the amplitude and phase of the signal over the period of use. Else if the time duration of a transmitted symbol is smaller than the channel coherence time, the channel is said to be slow fading channel, which introduces severe frequency dispersion into the received signal, and also exhibits considerable change in the amplitude and phase of the received signal over the period of use [Pro08].

3.1.2 Flat and Frequency-Selective Fading

The channel can be considered as flat or frequency-nonselective fading if the signal's bandwidth is smaller than the coherence bandwidth of the channel, where the coherence bandwidth of a channel is the range of frequencies that can be passed through the channel without deformation. Such a flat channel will let all the frequency components in the signal to experience the same attenuation and phase shift during transmission [Big98].

On the other hand, the channel is said to be frequency-selective fading if the signal's bandwidth is larger than the coherence bandwidth of the channel. In this case, the frequency components of the signal with separation larger than the coherence bandwidth of the channel undergo different amplitude gains and different phase shifts.

3.1.3 Fading Distribution Models

It is almost impossible to investigate the behaviour of communication fading channels using one rigid mathematical expression for all environmental and physical conditions. Thus, we need to perform a lot of field measurements on the received signals' amplitudes in different surroundings and different locations in order to develop some statistical models for multipath fading [Dob96].

Several fading distributions have been proposed in literature to describe the statistics of the mobile radio signal [Sim05]. Indeed, the short-term signal variation is well described by several main distributions such as Nakagami-*m*, Rayleigh, Rice, Weibull, Hoyt and others. Each of these fading distributions is suitable for certain channel conditions. In some situations, no distributions adequately fit experimental data, although one or another may yield a moderate fitting. This motivates the need for a general fading distribution that can yield better fitting to experimental data and can include several fading distributions as special cases. One of these general fading distributions is the $\alpha - \mu$ distribution recently proposed in [Yac07]. It is an umbrella distribution and includes as special cases important distributions such as Nakagami-*m*, Rayleigh, Gamma, exponential, Weibull, and one-sided Gaussian. In addition, its PDF, CDF, and moments appear in simple closed form expressions. Furthermore, it can explore the nonlinearity of the propagation medium. These features make the $\alpha - \mu$ distribution very attractive. Other general fading models can be addressed in future researches, like the $\alpha - \eta - \mu$ and the $\alpha - \lambda - \mu$ models [Pap09].

In Chapter 5, we will focus on the $\alpha - \mu$ distribution as a general fading distribution, when deriving the expression for the APD of unknown signals using energy-based detection technique with SC diversity. Now, we will introduce some well-known fading distribution models in more details. This will include Rayleigh, Nakagami-*m*, Exponential, Weibull, Gamma, one-sided Gaussian and the $\alpha - \mu$ fading distribution.

3.1.3.1 Rayleigh Distribution

If a and b are two independent and identically distributed (i.i.d) Gaussian random variables with each has zero mean and the same variance, then the random variable c, where $c = \sqrt{a^2 + b^2}$ is statistically said to be Rayleigh distributed [Pro08].

In communications theory, Rayleigh fading is a statistical model used in small scale multipath fading where there are dense scatters, and so, it is more applicable in crowded city centres in which a lot of buildings and obstacles exist between transmitter and receiver, where there is no dominant LOS path between them. Figure 3.2 shows a plot of Rayleigh fading over one second and a maximum Doppler shift of 10Hz.

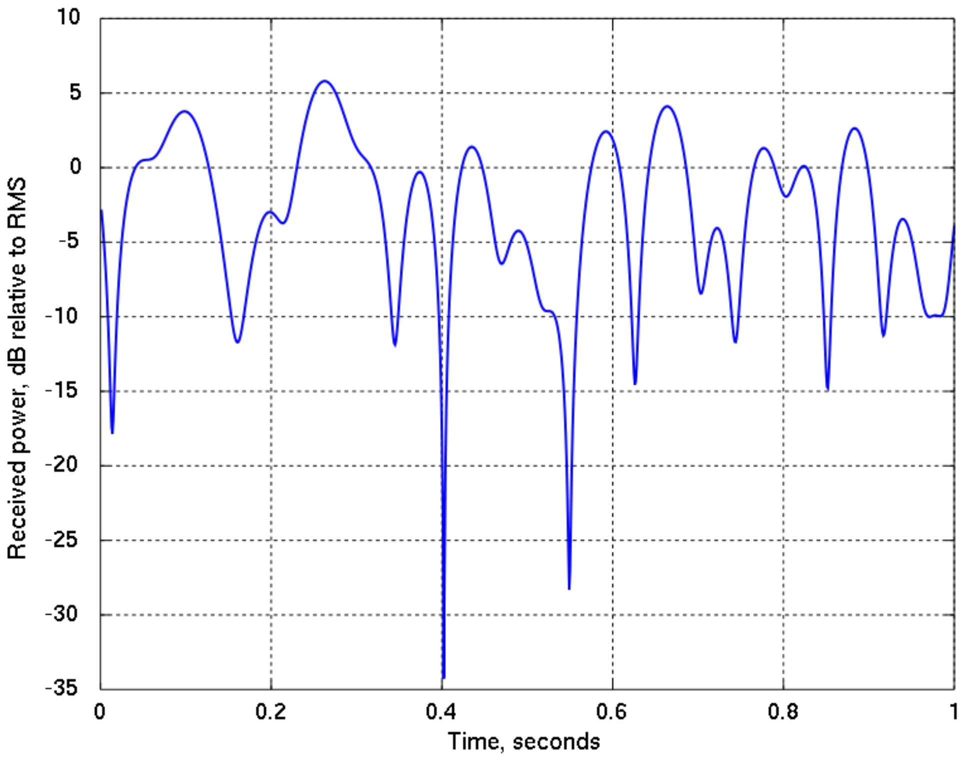

Figure 3.2: One second of Rayleigh fading with a maximum Doppler shift of 10Hz [Wik06]

If h is defined to be the envelope of a Rayleigh fading channel, and Ω is the mean-square, then the PDF $f_h(h)$ of the random variable h is given by (3.1). Figure 3.3 shows curves for $f_h(h)$ vs. channel fading gain h for different values of Ω.

$$f_h(h) = \begin{cases} \frac{2h}{\Omega} \; e^{-\frac{h^2}{\Omega}}, & h \geq 0; \\ 0 & , \; h < 0. \end{cases} \tag{3.1}$$

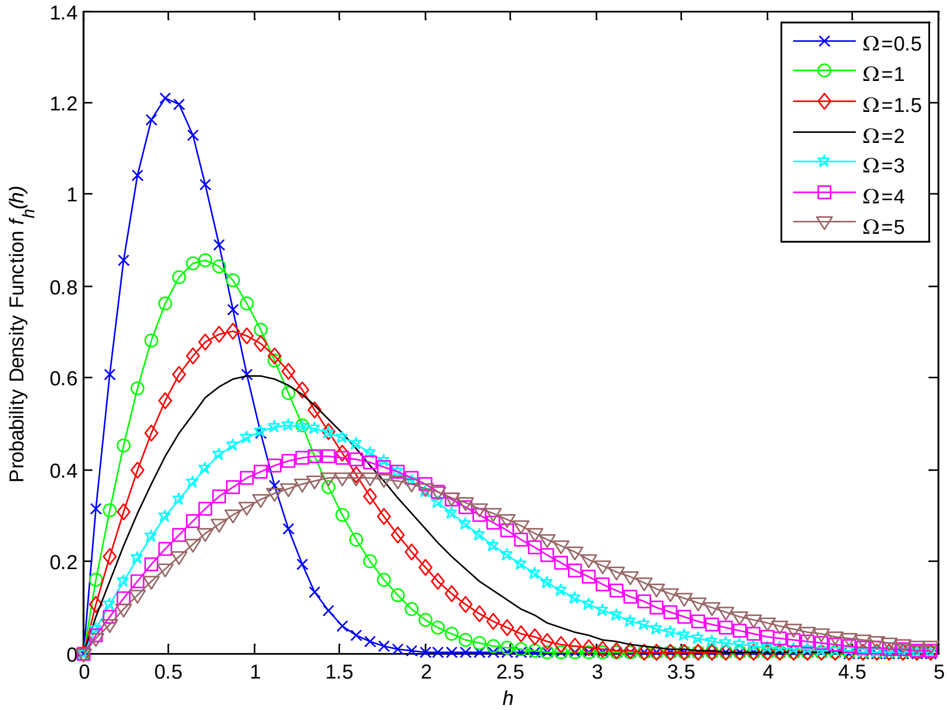

Figure 3.3: Rayleigh PDF $\boldsymbol{f_h(h)}$ vs. fading coefficient h for different values of $\boldsymbol{\Omega}$

*3.1.3.2 Nakagami-*m *Distribution*

The Nakagami-*m* distribution which has been first introduced by Nakagami in 1960 [Nak60], is a widely used statistical model, in multipath fading channels, to investigate fading envelopes of signals because of its generality that covers different fading scenarios. This is achieved by changing the severity parameter *m* in the Nakagami-*m* distribution from 0.5 to infinity, where less severe fading is obtained when increasing the value of *m*. Nakagami-*m* fading distribution model is a general distribution where several other distributions can be obtained when setting the severity parameter *m* to specific values. Indeed, Rayleigh distribution model is obtained by setting *m*=1, and one-sided Gaussian distribution results when setting *m*=0.5. The PDF $f_h(h)$ of Nakagami-*m* distribution model is given by (3.2). Figure

2.4 shows curves for $f_h(h)$ vs. channel fading coefficient h for different values of m and Ω.

$$f_h(h) = \begin{cases} \frac{2m^m h^{2m-1}}{\Omega^m \, \Gamma(m)} \, e^{-\frac{m \, h^2}{\Omega}}, & h \geq 0 \\ 0 & , \, h < 0 \end{cases} \tag{3.2}$$

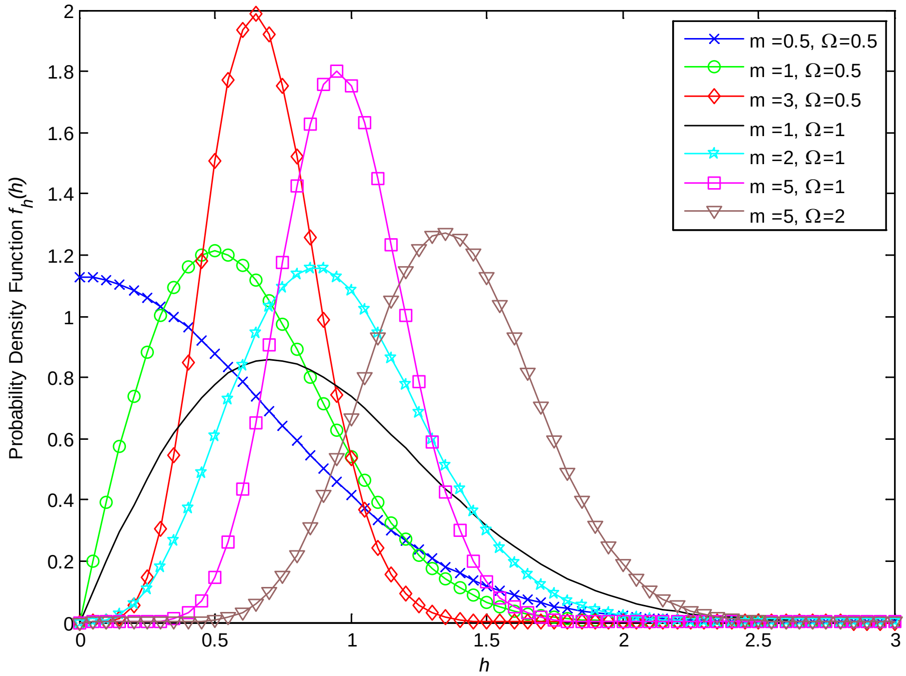

Figure 3.4: Nakagami-*m* PDF $\boldsymbol{f_h(h)}$ vs. fading coefficient *h* for different values of *m* and Ω.

3.1.3.3 Weibull Distribution

Weibull distribution is a statistical model that can properly fit field measurements of fading channels envelopes in both indoor and outdoor environments [Has93], [Ada88]. The PDF of Weibull distribution is given by (3.3). It can be reduced to an exponential distribution by setting *K*=1 and to a Rayleigh distribution when *K*=2.

Curves for the PDF of Weibull distribution for different values of K and Ω are shown in Figure 3.5.

$$f_h(h) = \begin{cases} K\left(\frac{\Gamma(1+\frac{2}{K})}{\Omega}\right)^{K/2} h^{K-1}\, e^{-\left(\frac{\Gamma(1+\frac{2}{K})}{\Omega} h^2\right)^{K/2}} & , \ h \geq 0\,; \\ 0 & , \ h < 0. \end{cases} \tag{3.3}$$

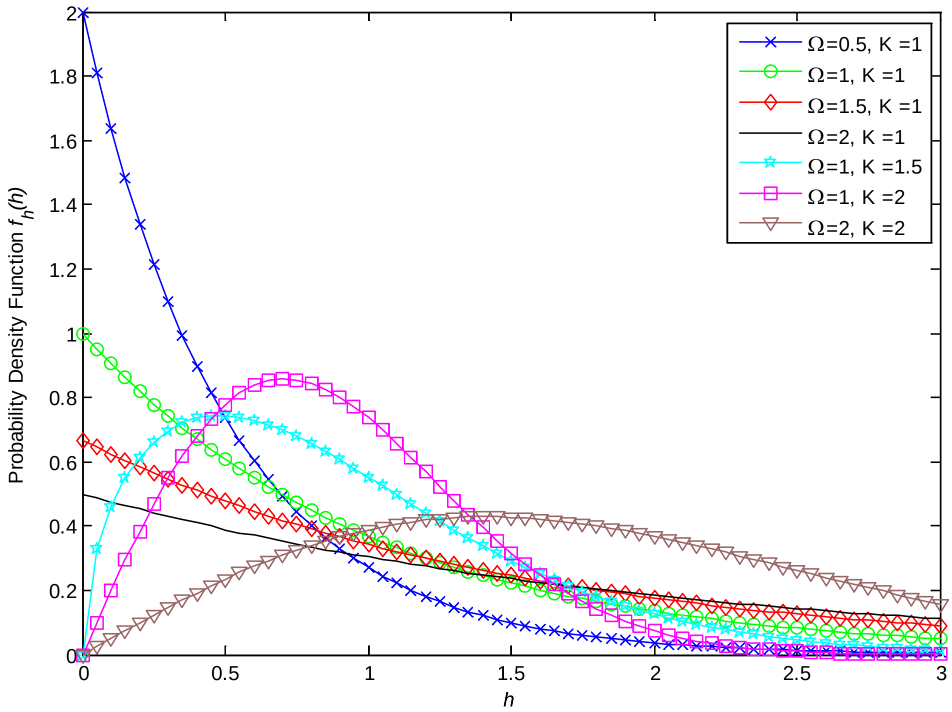

Figure 3.5: Weibull PDF $\boldsymbol{f_h(h)}$ vs. fading coefficient h for different values of K and Ω

3.1.3.4 Exponential Distribution

Originally, exponential distribution is usually used to characterize time periods rate of uncommon happening events and the expected lifetime of some devices. In communication systems, it is the distribution of the instantaneous SNR of a signal if it traverses through a Rayleigh distributed fading channel. It is a special case of Weibull distribution (K=1) and the Gamma distribution (a=1) [Olv10].

The PDF of the exponential distribution is given by (3.4), and it is plotted in Figure 3.6 with different values of Ω.

$$f_h(h) = \begin{cases} \frac{1}{\Omega}\ e^{-\frac{h}{\Omega}}, & h \geq 0; \\ 0 & , h < 0. \end{cases} \tag{3.4}$$

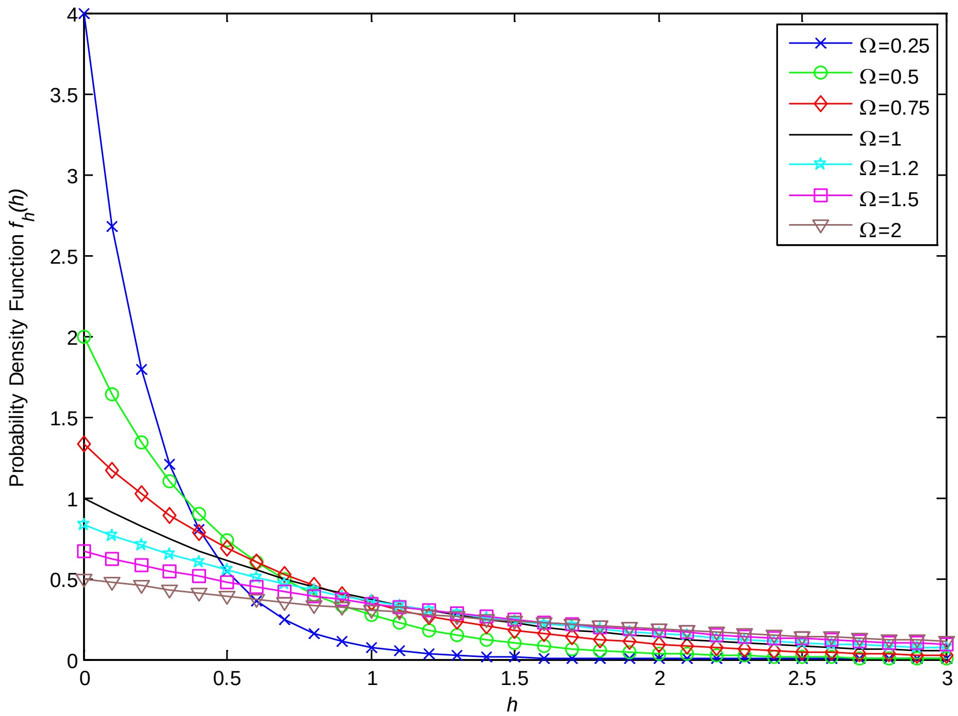

Figure 3.6: Exponential PDF $\boldsymbol{f_h(h)}$ vs. fading coefficient *h* for different values of Ω.

3.1.3.5 Gamma Distribution

The PDF of Gamma distribution is given by (3.5). It includes other distributions as special cases like the Chi-square distribution when setting the a parameter to $v/2$, and the exponential distribution when setting $a = 1$. Several curves for Gamma distribution are shown in Figure 3.7 with different values of a and Ω [Sta62].

$$f_h(h) = \begin{cases} \frac{a^a h^{a-1}}{\Omega^a \, \Gamma(a)} \, e^{-\frac{a\,h}{\Omega}}, & h \geq 0; \\ 0 & , \; h < 0. \end{cases} \tag{3.5}$$

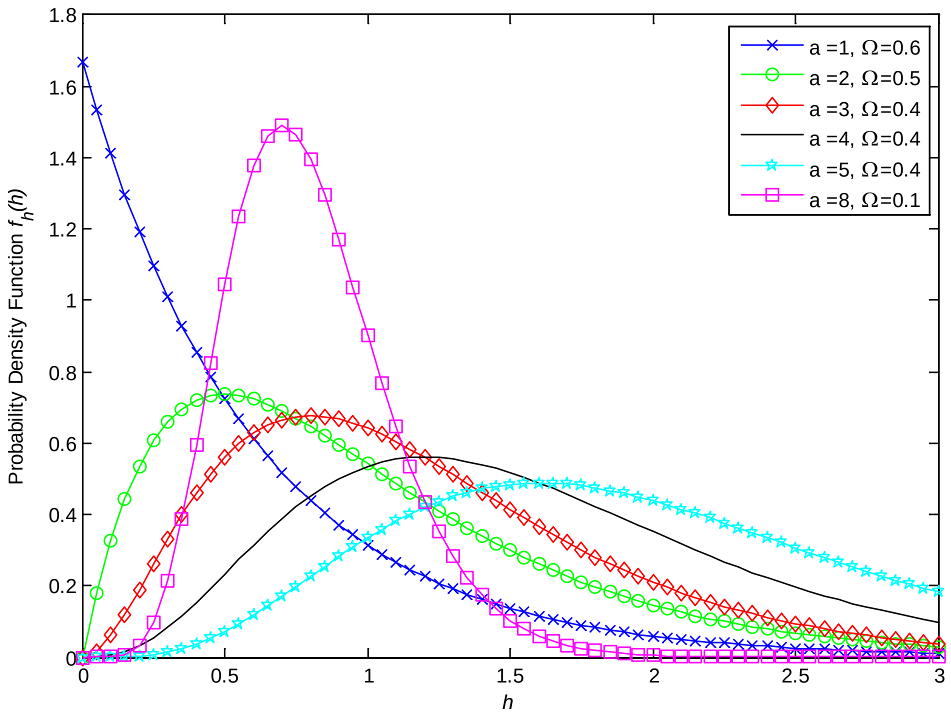

Figure 3.7: Gamma PDF $\boldsymbol{f_h(h)}$ vs. fading coefficient *h* for different values of *a* and $\boldsymbol{\Omega}$.

3.1.3.6 One-Sided Gaussian Distribution

One-sided Gaussian distribution is a half of regular which has zero mean, i.e. it can be viewed as the right or left half of the normal distribution bell-shaped curve. It can be obtained from the Nakagami-*m* distribution by setting *m*=0.5. The one-sided Gaussian distribution has a PDF given by (3.6), and some curves are plotted in Figure 3.8 [Fat12b].

$$f_h(h) = \begin{cases} \sqrt{\frac{2}{\pi\Omega}} \, e^{-\frac{h^2}{2\Omega}}, & h \geq 0; \\ 0 & , \; h < 0. \end{cases} \tag{3.6}$$

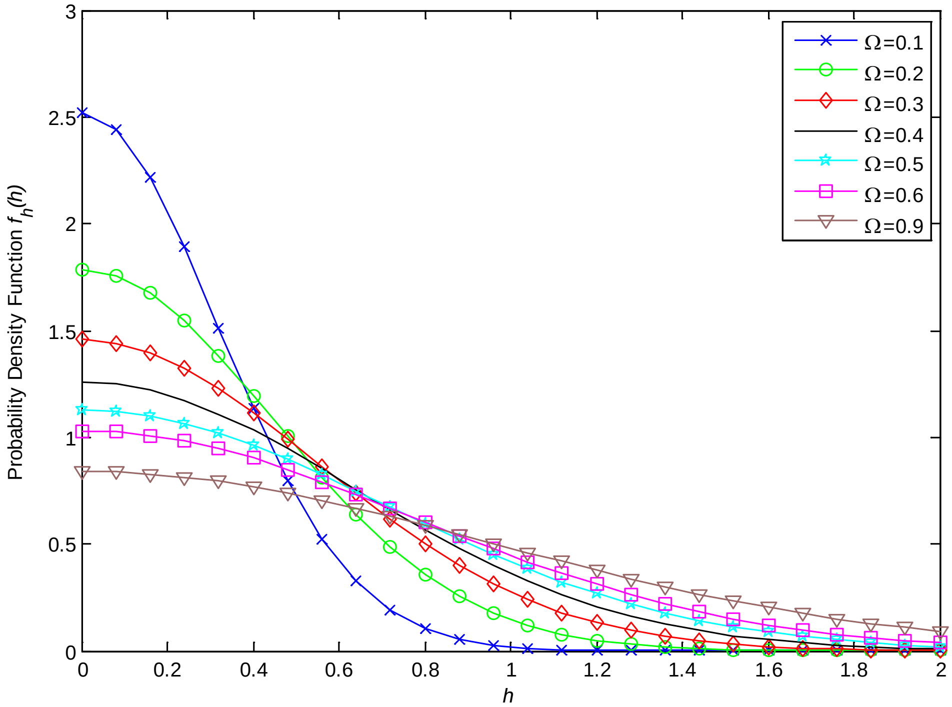

Figure 3.8: One-sided Gaussian PDF $\boldsymbol{f_h(h)}$ vs. fading coefficient *h* for different values of Ω.

3.1.3.7 The $\alpha-\mu$ General Fading Distribution

When the fading channel is characterized by the $\alpha-\mu$ generalized fading distribution, the envelope h of the fading signal has the following probability density function (PDF) $f_h(h)$ [Yac07]:

$$f_h(h) = \frac{\alpha\mu^{\mu}\, h^{\alpha\mu-1}}{\bar{h}^{\alpha\mu}\Gamma(\mu)}\; e^{-\mu\left(\frac{h}{\bar{h}}\right)^{\alpha}}, \tag{3.7}$$

where α is a positive arbitrary parameter, and $\mu > 0$ is the inverse of the normalized variance of h^{α}, $\bar{h} = \sqrt[\alpha]{E(h^{\alpha})}$ is the α-root mean value of h, and $\Gamma(\mu)$ is the gamma function. By setting $\alpha = K$ and $\mu = 1$, the $\alpha-\mu$ distribution reduces to the Weibull distribution with parameter K. Setting $K = 1$ results in exponential distribution. In addition, Nakagami-*m* distribution can be obtained by setting $\alpha = 2$ and $\mu = m$, where m is the Nakagami-*m* severity parameter. Furthermore, Rayleigh and one-sided Gaussian distributions are obtained from the Nakagami-*m* distribution by setting $m = 1$ and $m = 1/2$, respectively. Moreover, Gamma distribution is obtained by setting $\alpha = 1$ and $\mu = a$, where a is the parameter of Gamma distribution. Figure 3.9 and Figure 3.10 show the PDF $f_x(x)$ of the normalized envelope $x = h/\bar{h}$ of the $\alpha - \mu$ general fading distribution for several values of α and μ, resulting in several well known fading distributions. Indeed, the $\alpha - \mu$ distribution is general, flexible and covers vast range of fading situations [Yac07].

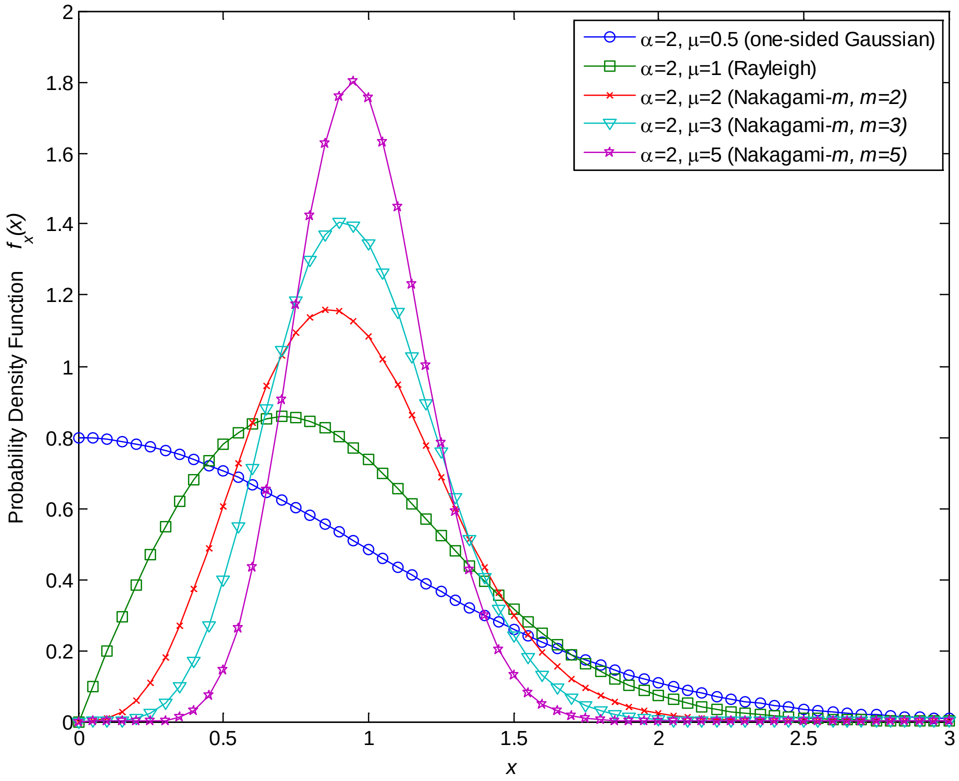

Figure 3.9: The PDF $\boldsymbol{f_x(x)}$ of the $\boldsymbol{\alpha - \mu}$ generalized fading model for $\boldsymbol{\alpha = 2}$ with several values of $\boldsymbol{\mu}$.

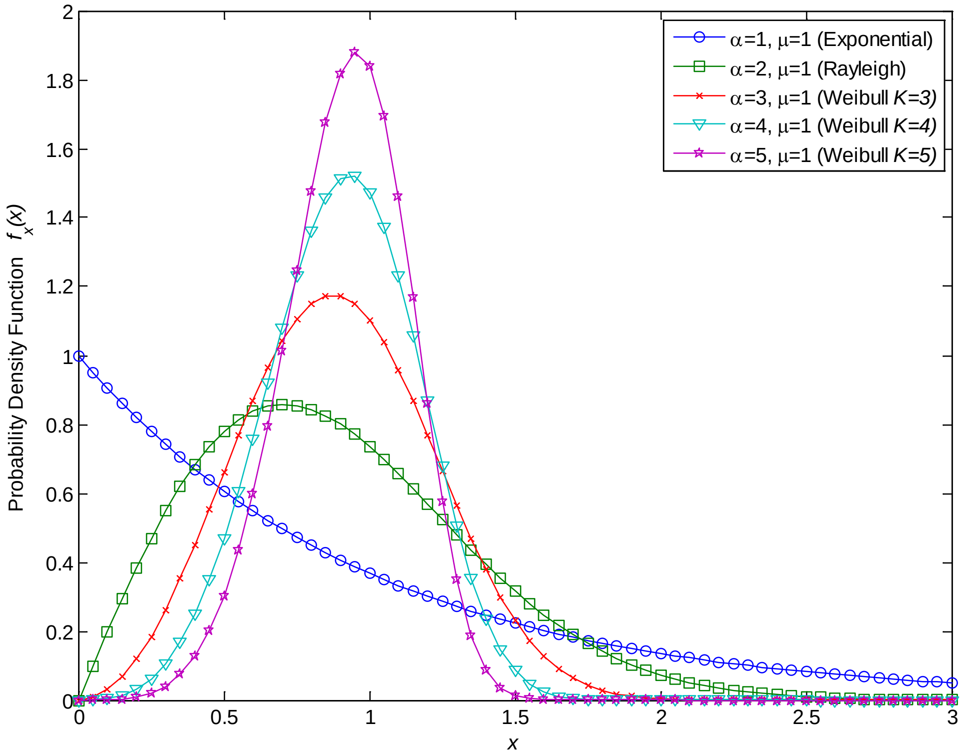

Figure 3.10: The PDF $\boldsymbol{f_x(x)}$ of the $\boldsymbol{\alpha - \mu}$ generalized fading distribution for $\boldsymbol{\mu = 1}$ with several values of $\boldsymbol{\alpha}$.

As we can see from Figure 3.10, the environment nonlinearity parameter α affects the shape of the normalized PDF of the $\alpha - \mu$ generalized fading distribution. Indeed, the exponent term of $f_x(x)$ is solely determined by the α parameter that affects the curve tail of $f_x(x)$. This means that the probability of detection for some given threshold is enhanced when choosing larger values of α and so, the overall complementary ROC of the ED receiver is enhanced [Fat12b], as we will see in Chapter 5.

The other parameter, μ, in the $\alpha - \mu$ generalized fading distribution, which represents the number of multipath clusters, plays a significant role in enhancing the performance of the ED receiver, since larger values of this parameter means more multipath clusters, consequently, larger probability of detection.

3.2 Diversity Reception

In diversity reception, multiple replicas of the same transmitted signal are received via some combining techniques, where independent signal paths have a low probability that they will undergo deep fades at the same time. This leads to enhancing the average SNR at the receiver and combating severe fading that may result in strong destruction of the signal or even interruption of the communications channel.

Diversity combining can be accomplished by making use of different mechanisms; either spatially combining multiple copies of the signal using multiple antennas that are spaced by some physical displacements, or combining multiple copies of the signal that are repeatedly sent in different consecutive times, or by using different frequency bands to receive multiple copies of the signal, or by using impulse (multipath) diversity with RAKE combining [Seb12].

3.2.1 Diversity Combining Techniques

Several diversity combining techniques can be used to combat fading of radio channels. Among the most popular combining techniques, that were frequently used in literature, are equal gain combining (EGC), selection combining (SC), and maximal ratio combining (MRC) [Rap02]. Figure 3.11 shows examples of the diversity combining receivers, where S1,...,SN are the received signals, g is the gain used in EGC, g1,...,gN are multiplication gains of channel baths from 1,...,N respectively. Rx is the receiving circuit.

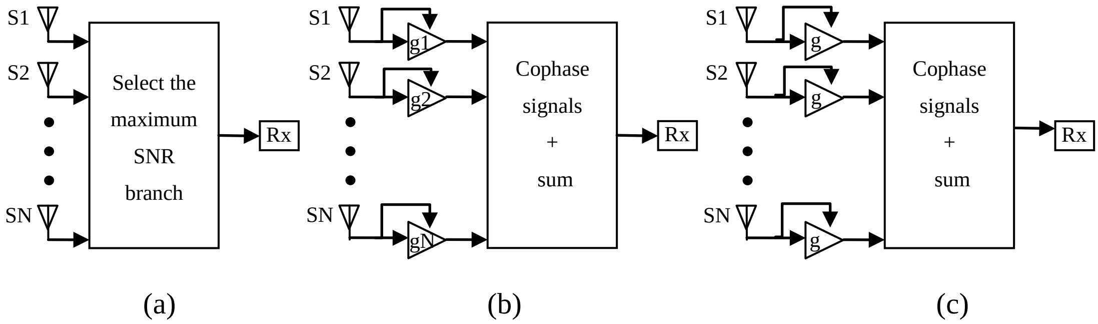

Figure 3.11: Diversity combining techniques (a) SC (b) MRC (c) EGC

3.2.1.1 Selection Combining (SC)

In this technique of diversity combining, the combiner chooses the branch with the highest SNR (or highest channel gain if all branches experience same average AWGN power) and receives the data from that branch. So, monitoring all channels SNRs is supposed to be carried out by using training data before sending actual data. The output SNR in this case is equal to the SNR of the selected branch [Ko00]. In case of L i.i.d. Rayleigh fading channels, for instance, the PDF of the SNR at the combiner's output, $f_{\gamma_t}(\gamma)$, is given by (3.8), assuming each channel has the same average SNR of $\bar{\gamma}$ [Rap02],

$$f_{\gamma_t}(\gamma) = \frac{L}{\bar{\gamma}}\left(1 - e^{-\left(\frac{\gamma}{\bar{\gamma}}\right)}\right)^{L-1} e^{-\left(\frac{\gamma}{\bar{\gamma}}\right)} \qquad (3.8)$$

where $\gamma_t = \max\{\gamma_1, \gamma_2, \ldots, \gamma_L\} = \bar{\gamma}\sum_{i=1}^{L}\frac{1}{i}$.

3.2.1.2 Maximal Ratio Combining (MRC)

MRC is an optimal diversity scheme with the most complicated structure, in which the receiver is assumed to have enough channel state information (CSI) so as it can multiply the received data from each branch with its gain conjugate. Then all multiplied data are added together to maximize the output SNR, which then is equal to the sum of SNRs of all the diversity branches [Gib12]. In case of L i.i.d. Rayleigh fading channels, for instance, the PDF of the SNR at the combiner's output, $f_{\gamma_t}(\gamma)$, is given by (3.10), assuming each channel has the same average SNR of $\bar{\gamma}$ [Rap02],

$$f_{\gamma_t}(\gamma) = \frac{\gamma^{L-1}}{(L-1)!\,\bar{\gamma}^{L}}\ e^{-\left(\frac{\gamma}{\bar{\gamma}}\right)}, \qquad (3.10)$$

where $\gamma_t = \sum_{i=1}^{L} \gamma_i$ and γ_i is the i^{th} branch SNR.

3.2.1.3 Equal Gain Combining (EGC)

In ECG, the amplitude of each signal branch is amplified with the same gain, but they are aligned in phase to achieve co-phasing and avoid signal cancellation. The EGC diversity receiver has reduced complexity comparative with other diversity combining receivers, such as the optimal MRC receiver, and at the same time, it can achieve as good performance as the optimal ones. This makes it cost effective and hardware simple to implement. In this type of combiners, and in case of L i.i.d. Rayleigh fading channels, for instance, the PDF of the SNR of the combiner's output, $f_{\gamma_t}(\gamma)$, does not exist in closed form for $L > 2$ [Pro08]. Nevertheless, $f_{\gamma_t}(\gamma)$ in case of two uncorrelated i.i.d. Rayleigh fading channels is given by (3.8), assuming each channel has the same average SNR of $\bar{\gamma}$,

$$f_{\gamma_t}(\gamma) = \frac{1}{\bar{\gamma}}\, e^{-\frac{2\gamma}{\bar{\gamma}}} - \sqrt{\pi}\, e^{-\frac{2\gamma}{\bar{\gamma}}} \left(\frac{1}{2\sqrt{\gamma\bar{\gamma}}} - \frac{1}{\bar{\gamma}}\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)\left(1 - 2Q\left(\sqrt{\frac{2\gamma}{\bar{\gamma}}}\right)\right) \tag{3.8}$$

where $\gamma_t = \bar{\gamma}\left(1 + (L-1)\frac{\pi}{4}\right)$.

Chapter 4

Energy Detection over $\alpha - \mu$ Fading Channels with Selection Combining

This chapter discusses the concept of energy detection, where a receiver collects the energy of number of received symbols after being digitized and squared. Then ED is used as the system model to carry out our thesis research on. But sense there are many fading distribution models, we select the $\alpha - \mu$ generalized fading model to cover several other models. Also we introduce herein the SC as one of the diversity reception techniques. A new expression for the APD of unknown signals of the ED over $\alpha - \mu$ generalized fading model with SC is obtained.

4.1 The Energy Detector

The ED is a threshold-based decision device. Its output is one of two hypotheses H_0 and H_1 denoting, respectively, signal absence and signal presence. The decision is made by comparing the aggregated energy of a band-pass-filtered (BPF) received signal, over an observation period of time Ts, against a predetermined detection threshold λ as shown in Figure 4.1.

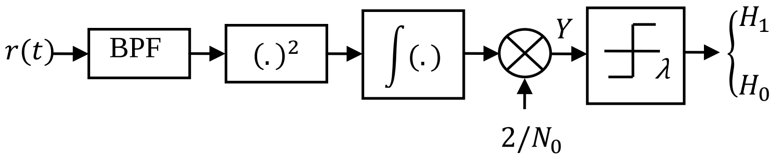

Figure 4.1: Block diagram of the energy detector

Thus, the received signal $r(t)$ can be interpreted as a binary hypothesis test and it is given by (4.1), where $n(t)$ is an additive AWGN process with one-sided power spectral density N_0 Watt/Hz, $s(t)$ is the transmitted signal, and h is the channel coefficient amplitude having mean-square value of $\overline{h^2}$ and PDF $f_h(h)$.

$$r(t) = \begin{cases} n(t), & H_0\,; \\ h\, s(t) + n(t), & H_1. \end{cases} \tag{4.1}$$

The instantaneous SNR at the receiver antenna can be expressed as $\gamma = |h|^2 E_s/N_0$, where E_s is the energy of the signal accumulated over the observation period. It is well known that the PDF of the decision variable Y can be expressed in terms of the central and non-central Chi-square distributions with $u = TW/2$ degrees of freedom, where W is the BPF bandwidth and TW is the time-bandwidth product.

Figure 4.2 shows the conditional probability of false alarm $P_f = Pr(Y > \lambda|H_0)$ and the conditional probability of miss detection $P_m = Pr(Y < \lambda|H_1)$. Note that the probability of detection $P_d = 1 - P_m = Pr(Y > \lambda|H_1)$.

Based on the statistics of Y and given a fixed threshold λ, the conditional probabilities of false alarm and detection for a certain value of γ can be expressed as [Dig03],

$$P_f = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)}, \tag{4.2}$$

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}), \tag{4.3}$$

where $\Gamma(.,.)$ is the upper incomplete gamma function, and $Q_u(.,.)$ is the generalized Marcum Q-function [Nut74].

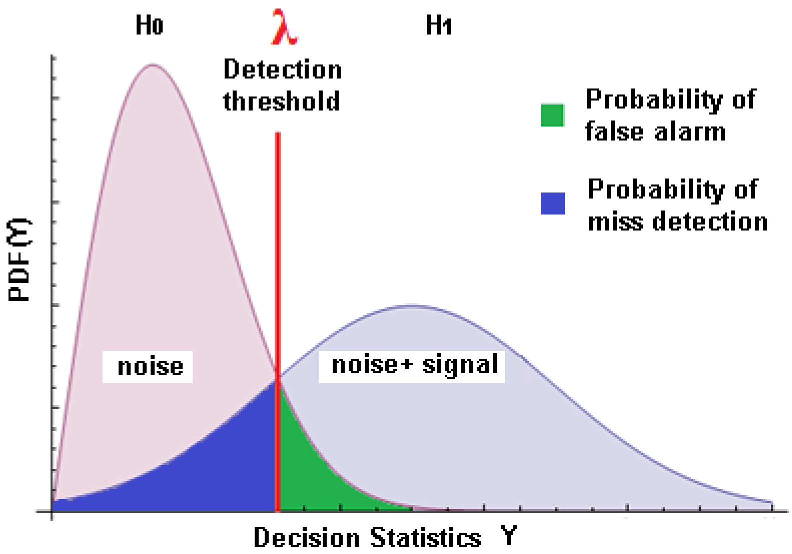

Figure 4.2: Conditional probabilities of false alarm and miss detection

When considering the $\alpha-\mu$ generalized fading channel, with an envelope h, the PDF of the fading signal envelope $f_h(h)$ is given by (3.7),

$$f_h(h) = \frac{\alpha\mu^{\mu}\, h^{\alpha\mu-1}}{\bar{h}^{\alpha\mu}\Gamma(\mu)}\, e^{-\mu\left(\frac{h}{\bar{h}}\right)^{\alpha}} \tag{4.4}$$

Now, to obtain the average probability of detection $\bar{P}_d$ when considering AWGN and $\alpha - \mu$ fading channel, equation (4.3) should be averaged over the PDF $f_\gamma(\gamma)$ of the SNR $\gamma = |h|^2 E_s/N_0$ as follows,

$$\bar{P}_d = \int_0^\infty P_d(\gamma)\, f_\gamma(\gamma) d\gamma, \tag{4.5}$$

where $f_\gamma(\gamma)$ is derived from $f_h(h)$ by transformation of variables as shown in [Sim05] eq.(2.3),

$$f_\gamma(\gamma) = \frac{\alpha\mu^\mu\, \gamma^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\, \bar{\gamma}^{\frac{\alpha\mu}{2}}} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \tag{4.6}$$

where $\bar{\gamma} = \overline{h^2} E_s/N_0$ is the average SNR. Note that the probability of false alarm given by (4.2) has no terms relating to fading channel parameters, and so, doesn't change.

4.2 Performance of Energy Detector over $\alpha - \mu$ Fading with Selection Combining

In [Fat12a], the average probability of detection for the ED is obtained over the $\alpha - \mu$ general fading distribution. However, no diversity combining techniques are considered. Therefore, the performance of the ED over the $\alpha - \mu$ fading channel is revisited to derive a mathematical expression for the average probability of detection when SC diversity technique is employed at the receiver.

When considering SC diversity technique, the diversity branch with the highest SNR is chosen by the selection combiner. The PDF of the instantaneous SNR for a single branch over $\alpha - \mu$ fading channel is given by (4.6). For L diversity branches, the instantaneous SNR of the SC would be equal to the maximum of $\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$, where γ_i is the *i*-th branch instantaneous SNR [Sim05]. Assuming that the average SNRs for all branches are equal, let's denote it by $\bar{\gamma}$, then for any single branch, the probability that its SNR γ_i is less than some value γ is given by:

$$Pr\ (\gamma_i \leq \gamma) = \int_0^{\gamma} f_{\gamma_i}(\gamma_i) d\gamma_i \qquad (4.7)$$

Upon substituting (4.6) into (4.7) we get:

$$Pr\ (\gamma_i \leq \gamma) = \int_0^{\gamma} \frac{\alpha \mu^{\mu}\, \gamma_i^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\, \bar{\gamma}^{\frac{\alpha\mu}{2}}} e^{-\mu\left(\frac{\gamma_i}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma_i$$

$$= 1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)} \qquad (4.8)$$

The CDF ($F_{\gamma_{SC}}$) of the output SNR of L i.i.d. branches of a selection combiner is derived as follows:

$$F_{\gamma_{SC}}(\gamma) = Pr(\gamma_1 \leq \gamma, \gamma_2 \leq \gamma, \ldots, \gamma_L \leq \gamma) \qquad (4.9)$$

$$F_{\gamma_{SC}}(\gamma) = \prod_{i=1}^{L} \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma\ (\mu)} \right) \qquad (4.10)$$

$$F_{\gamma_{SC}}(\gamma) = \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma\,(\mu)}\right)^{L} \quad (4.11)$$

Now, the PDF of the combiner's output SNR, denoted by $f_{\gamma_{SC}}(\gamma)$, is the derivative of $F_{\gamma_{SC}}(\gamma)$, hence, it is calculated as:

$$f_{\gamma_{SC}}(\gamma) = \frac{dF_{\gamma_{SC}}(\gamma)}{d\gamma} \quad (4.12)$$

$$= \frac{d}{d\gamma}\left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma\,(\mu)}\right)^{L} \quad (4.13)$$

$$= \frac{L\alpha\mu^{\mu}}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{\alpha\mu}{2}}}\left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \quad (4.14)$$

Therefore, the average probability of detection for the SC can be evaluated by averaging (4.3) over (4.14) as follows:

$$\bar{P}_{d,\alpha\mu,sc} = \int_{0}^{\infty} Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \times \frac{L\alpha\mu^{\mu}}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{\alpha\mu}{2}}}\left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\, d\gamma$$

(4.15)

The generalized Marcum Q-function $\mathrm{Q_u}\left(\sqrt{2\gamma}, \sqrt{\lambda}\right)$ can be rewritten into series representation using [Ann11] eq. (8), as follows:

$$Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) = \sum_{n=0}^{\infty} \frac{\Gamma\left(n+u, \frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)} \gamma^n e^{-\gamma} \quad (4.16)$$

Also the term $\left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1}$ in (4.15) can be further simplified into series representation by means of binomial and then multinomial expansions respectively as follows:

- *Binomial expansion*

$$\left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1} = \sum_{i=0}^{L-1} (-1)^i \binom{L-1}{i} \left(\frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^i \quad (4.17)$$

Using [Gra07] eq. (8.352-4), and with the help of the equality $\Gamma(\mu) = (\mu - 1)!$ taking into consideration integer values only for the μ parameter; then we can get the following equal terms:

$$\frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)} = e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \sum_{m=0}^{\mu-1} \frac{\left(\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)^m}{m!} \quad (4.18)$$

- *Multinomial expansion*

$$\left(e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \sum_{m=0}^{\mu-1} \frac{\left(\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)^m}{m!}\right)^i = e^{-i\,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \sum_{m=0}^{i(\mu-1)} \beta_{mi}(\mu) \left(\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)^m \quad (4.19)$$

where $\beta_{mi}(\mu)$ is the coefficient of multinomial expansion, and it can be computed recursively as illustrated in [Aal98] eq. (32) and in [Sim05] eq. (9.124). Moreover, it can be computed using computer software like Wolfram Mathematica, for instance, as shown in the following code:

```
Ind[a_,b_,i_]:=If[a≤i≤b,1,0]

B[0,k_,μ_]:=1;
B[1,k_,μ_]:=k;
B[m_,1,μ_]:=1/m!;
B[m_,k_,μ_]:=Sum[(B[n,k-1, μ]/Factorial[m-n])*Ind[0,((k-
1)*( μ-1)),n],{n,m-μ+1,m}]
```

Substituting (4.19) into (4.17) and then into (4.15), also substituting (4.16) into (4.15), we get the final integral form of the average probability of detection as shown below:

$$\bar{P}_{d,\alpha\mu,sc} = \int_0^\infty \sum_{n=0}^{\infty} \frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)} \gamma^n e^{-\gamma} \times \frac{L\alpha\mu^\mu}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{\alpha\mu}{2}}} \sum_{i=0}^{L-1} (-1)^i \binom{L-1}{i} e^{-i\,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \sum_{m=0}^{i(\mu-1)} \beta_{mi}(\mu) \left(\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)^m \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma \quad (4.20)$$

After some manipulation and rearranging of the terms in (4.20), and solving the integral by formulating the mathematical expression inside it in-terms of the general Laplace transform as illustrated in [Pru90] eq. 2.2.1-22, it follows that:

$$\bar{P}_{d,\alpha\mu,sc} = C \sum_{n=0}^{\infty} a_n \sum_{i=0}^{L-1} (-1)^i \binom{L-1}{i} \sum_{m=0}^{i(\mu-1)} \frac{\mu^m l^{\frac{\alpha m}{2}+n}}{\bar{\gamma}^{\frac{\alpha m}{2}}} \beta_{mi}(\mu) G_{l,k}^{k,l}\left(z \middle| \begin{matrix}\Delta(l,-w)\\ \Delta(k,0)\end{matrix}\right), \quad (4.21)$$

where $C = \frac{\alpha\mu^\mu L \sqrt{k}\, l^{\frac{1}{2}(\alpha\mu-1)}}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{1}{2}(\alpha\mu)} (2\pi)^{\frac{k+l}{2}-1}}$, $a_n = \frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)}$, $z = l^l \left(\frac{(i+1)\mu}{k\bar{\gamma}^{\alpha/2}}\right)^k$, $w = \frac{1}{2}\alpha(\mu+m)+n-1$, $G_{p,q}^{q,p}\left(z \middle| \begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ is the Meiger-G function [Olv10] eq. (16.17.1), $\Delta(k,a) = \frac{a}{k}, \frac{a+1}{k}, \dots, \frac{a+k-1}{k}$, l and k are some integers such that $\frac{l}{k} = \frac{\alpha}{2}$.

To the best of our knowledge, (4.21) is being derived for the first time.

To obtain the optimum value of the threshold λ for the term in (4.21), we can differentiate $\bar{P}_{d,\alpha\mu,sc}$ with respect to λ, and equating the result with 0, and solving for the value for λ, hence, the resulting value is the optimum threshold.

4.3 Some Special Cases

It should be noted that the result found in (4.21) is general for any number of diversity branches L. In addition, the derived expression in (4.21) is general in the $\alpha - \mu$ parameters that can cover several fading distribution models. Thus, some special cases can be extracted from that general expression as illustrated in the following subsections.

Note that this general expression is valid for integer values of μ only, since we use the multinomial expansion of a term containing μ as a limit of a summation.

4.3.1 $\alpha - \mu$ Generalized Fading, No Diversity

In the case of no diversity reception, i.e. single diversity branch $L = 1$, the derived expression for the probability of detection of the ED with SC diversity reception in (4.21) reduces to a previously known result found in [Fat12a] eq.(8), as follows:

$$\bar{P}_{d,\alpha\mu} = A\ \sum_{n=0}^{\infty} l^n\ a_n G_{l,k}^{k,l}\left(S;\ {}^{\Delta(l,-v)}_{\Delta(k,0)}\right) \tag{4.22}$$

where $= \frac{\alpha\mu^{\mu}\sqrt{k}\ l^{(\alpha\mu-1)/2}}{2\Gamma(\mu)\ \bar{\gamma}^{\left(\frac{\alpha\mu}{2}\right)}(2\pi)^{\frac{k+l}{2}-1}}$, $S = \left(\frac{\mu}{k\bar{\gamma}^{\alpha/2}}\right)^{k} l^{l}$, and $v = n + \frac{\alpha\mu}{2} - 1$.

Note that a typo in [Fat12a] eq. (8) is fixed in (4.22) by adding the missing term l^n.

4.3.2 Rayleigh Fading

For the special case of Rayleigh fading channels, the probability of detection for the ED with SC diversity reception can be found from (4.21) by setting $\alpha = 2$ and $\mu = 1$, in which (4.21) reduces to:

$$\bar{P}_{d,Ray,SC,L} = \frac{L}{\bar{\gamma}} \sum_{n=0}^{\infty} \frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{\Gamma(n+u)} \sum_{i=0}^{L-1} (-1)^i \left(\frac{\bar{\gamma}}{1+i+\bar{\gamma}}\right)^{n+1} \binom{L-1}{i} \tag{4.23}$$

It should be noted that (4.23) is a new alternative form to the one derived in [Dig03] eq. (30). For the no diversity case, i.e. $L = 1$, equation (4.23) reduces to:

$$\bar{P}_{d,Ray,L=1} = \frac{1}{1+\bar{\gamma}} \sum_{n=0}^{\infty} \frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{\Gamma(n+u)} \left(\frac{\bar{\gamma}}{1+\bar{\gamma}}\right)^{n} \tag{4.24}$$

The result in (4.24) is equivalent to (11) in [Fat12b] for the Rayleigh fading without diversity; it is also an alternative form to (9) in [Dig07].

Chapter 5

Numerical Results and Discussion

This chapter presents some numerical results obtained from the derived expression in the previous chapter. Various figures are plotted with different assumptions for variables of that expression other than those given at both x and y axes. Enhancements on the complementary ROC curves are achieved using SC diversity. Comparisons between figures and discussions are presented to clarify the obtained results.

5.1 Pre-Assumptions for the Numerical Results

The performance of the ED is quantified by depicting the complementary receiver operating characteristics (ROCs) ($P_m = 1 - \bar{P}_d$ versus P_f) with the effect of the various parameters α, μ, L, and $\bar{\gamma}$. From the $\alpha - \mu$ general fading distribution, other fading distributions can be derived based on specific values for both α and μ. The following { α, μ } pairs are taken as test cases for the subsequent complementary ROCs [Fat12b]: {2, 5} Nakagami-*m* (*m*=5), {1, 5} Gamma (Chi-Square *a*=5), {1.5, 1} Weibull (*K*=1.5), {1, 1} exponential and {2, 1} Rayleigh. Since the result in (4.21) contains an infinite summation, then we must truncate the summation after certain number of iterations when calculating the APD using computer software. This depends on how much resolution of the calculated APD do we need. For example, achieving 15 digits accuracy needs about 3000 iterations, while truncating the summation after 1000 iterations yields 10 digits of accuracy. But for few number of accuracy digits, let's say 4 digits, only 55 iterations needed. Table 2 shows values for probability of false alarm P_f vs. threshold λ obtained by making use of (4.2) in Sec 4.1, where u is assumed to be equal 5.

Table 2: P_f vs. threshold λ, u=5.

λ	P_f
25.18818	0.005
19.0758	0.03931
17.03374	0.073621
15.72007	0.107931
14.72945	0.142241
13.92259	0.176552
13.23436	0.210862
12.6288	0.245172
12.08387	0.279483
11.58496	0.313793
11.12186	0.348103
10.68705	0.382414
10.2748	0.416724
9.880555	0.451034
9.500575	0.485345
9.131668	0.519655
8.771003	0.553966
8.41596	0.588276
8.063992	0.622586
7.712498	0.656897
7.358657	0.691207
6.999234	0.725517
6.63027	0.759828
6.246588	0.794138
5.8409	0.828448
5.402005	0.862759
4.910607	0.897069
4.327245	0.931379
3.540949	0.96569
0	1

5.2 No Selection Combining

Figure 5.1 shows the complementary ROCs of the ED over $\alpha - \mu$ fading channel without SC; it is the special case when setting L=1 in (4.21) yielding (4.22). By selecting different values of α and μ, the results for several well known distributions are shown in Figure 5.1. One can notice from this figure that the performance of energy detector is degraded when going from the non-fading case (only AWGN channel) to the fading case with several distributions. Exponential faded channel results in the worst performance while Nakagami-*m* faded channel with severity parameter *m*=5 yields the best performance. Note that Rayleigh faded channel (special case when *m*=1) yields worse performance than Nakagami-*m* faded channel with *m*=5. Thus, the higher is the *m*, the higher is the detection probability.

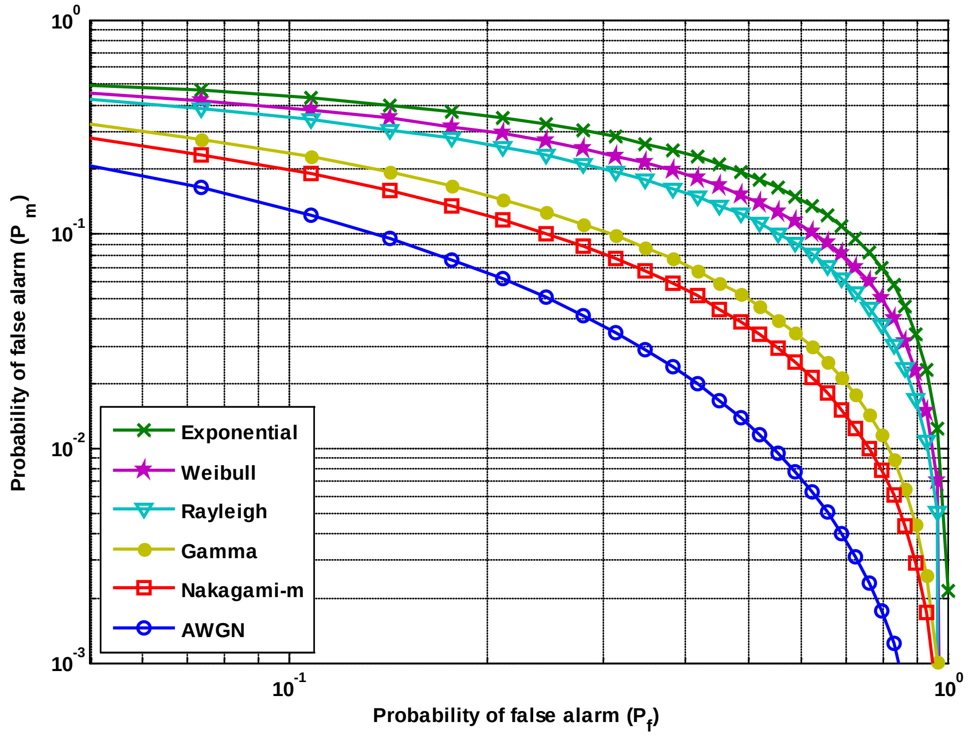

Figure 5.1: Complementary ROCs of the ED over different fading channels without SC (L=1), u=5, and average SNR $\bar{\gamma} = 9$dB.

5.3 Selection Combining wirh L=2

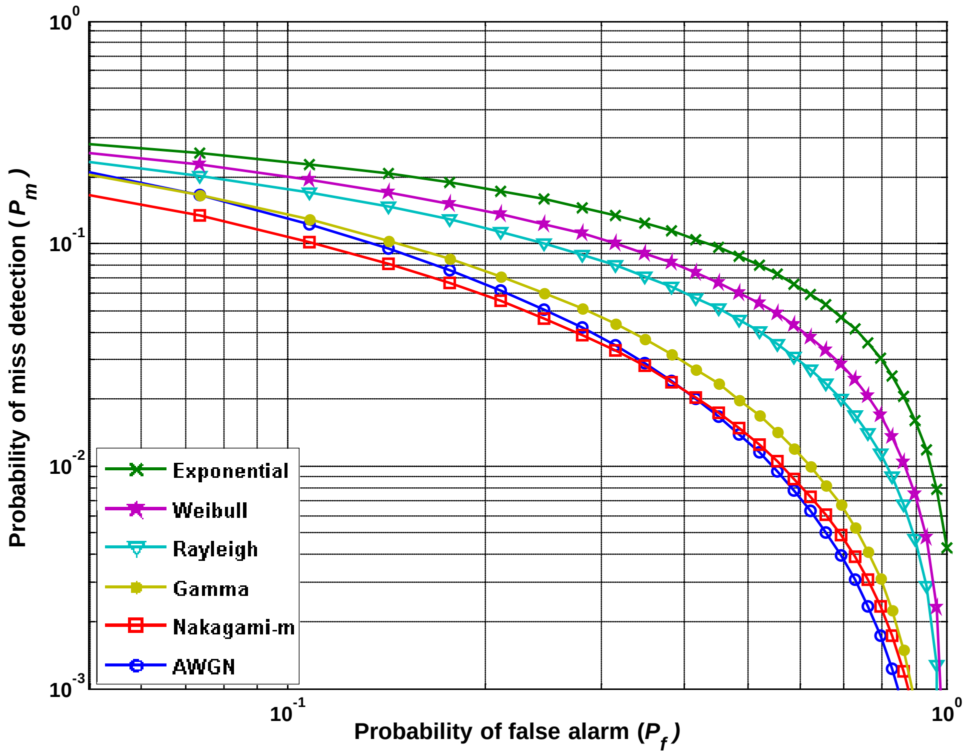

Figure 5.2: Complementary ROCs of the ED over different fading channels with SC (L=2), u=5, and $\bar{\gamma} = 9$dB.

Complementary ROCs of the ED over $\alpha - \mu$ fading with SC diversity (L=2) are shown in Figure 5.2. When comparing Figure 5.1 and Figure 5.2, one can notice that the SC antenna diversity technique greatly enhances the performance of the ED. For example, when P_f=0.2, all curves in Figure 5.1 have P_m less than 0.4, while in Figure 5.2 they are less than 0.2. This means that the more is the diversity branches, the less is the miss detection probability.

5.4 Effect of Average Signal-to-Noise Ratio

The effect of increasing the average SNR $\bar{\gamma}$ on the complementary ROCs of the ED is depicted in Figure 5.3. One can notice that the miss detection probability improves greatly when increasing $\bar{\gamma}$ form 10dB to 25dB. This is true, since increasing the SNR makes the signal more distinguishable from AWGN, and so, increasing the P_d and hence decreasing the P_m.

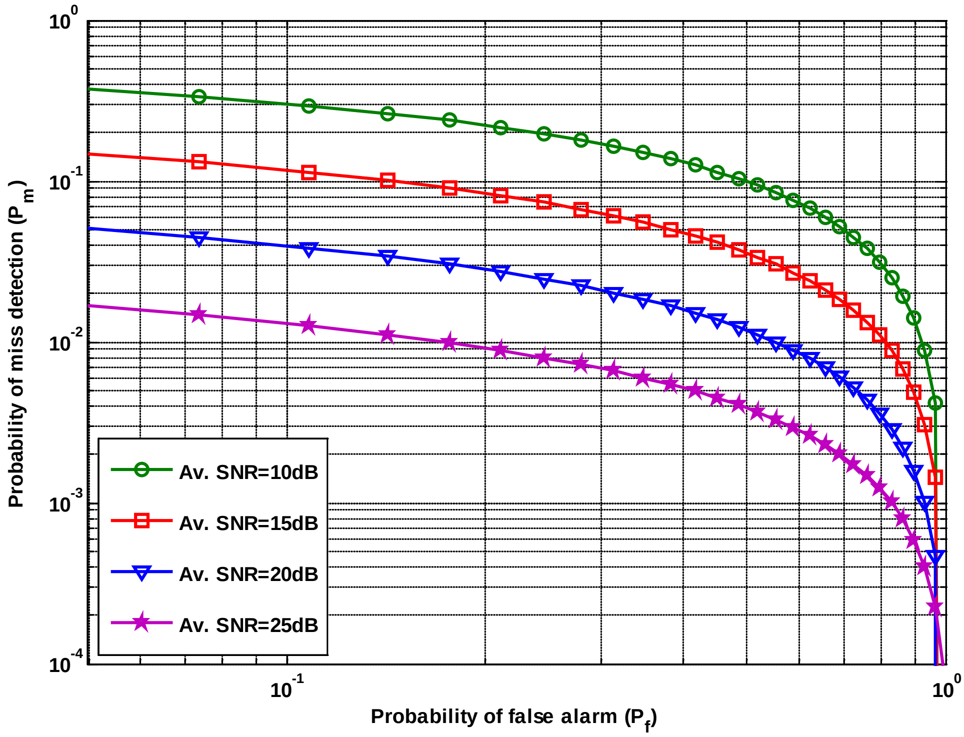

Figure 5.3: Complementary ROCs of the ED for Rayleigh fading ($\alpha = 2, \mu = 1$) without SC (L=1), u=5, and different values of $\bar{\gamma}$.

Figures 4.4, 4.5 and 4.6 illustrate the effect of increasing the number of SC diversity branches (from L=1 to L=5) on the complementary ROCs of the ED for Rayleigh, Nakagami-m and Weibull faded signals, respectively. It is noticed that the miss detection probability is greatly reduced when increasing the number of diversity branches.

5.5 Effect of Number of Diversity Branches

As mentioned in Chapter 3, diversity reception techniques are used to combat signal fading, where the probability that all copies of the signal will undergo the same fading is small, thus, the use of diversity combining schemes will enhance the overall SNR and therefore enhancing the probability of detection P_d at specific value of probability of false alarm P_f, this is what is shown clearly in Figures 5.4, 5.5 and 5.6. In Figure 5.4, the complementary ROCs of the ED are plotted for Rayleigh fading channel ($\alpha = 2, \mu = 1$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB. The same settings are applied for both Nakagami-m fading channel ($\alpha = 2,\ \mu = m = 5$) and Weibull fading channel ($\alpha = K = 1.5,\ \ \mu = 1$), then curves are plotted in Figures 5.5 and 5.6 respectively.

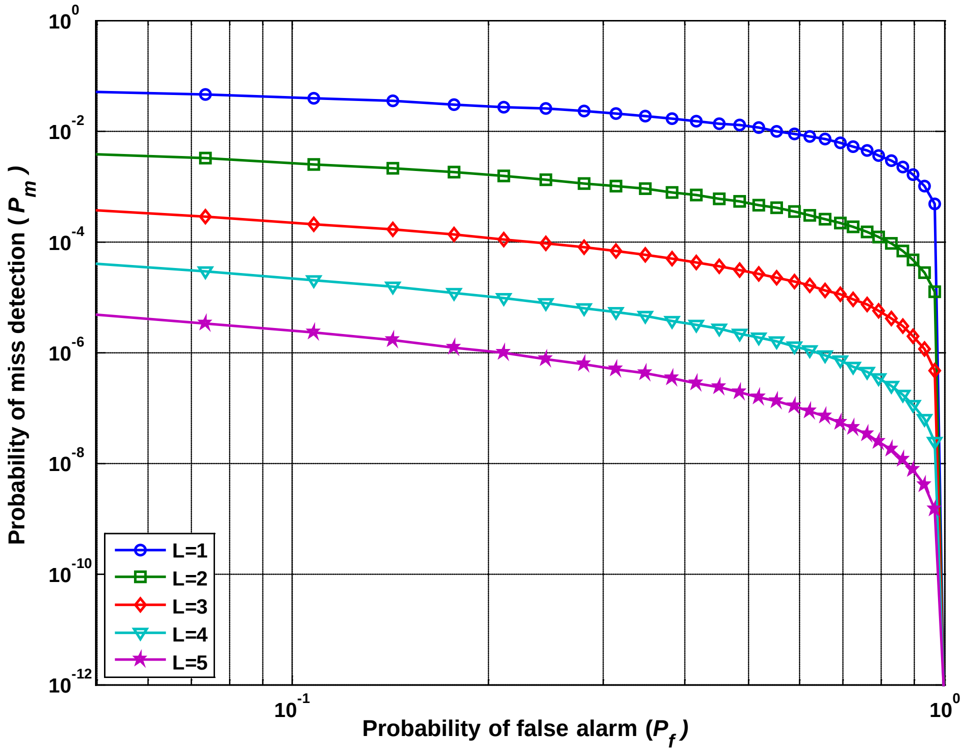

Figure 5.4: Complementary ROCs of the ED for Rayleigh fading channel ($\alpha = 2, \mu = 1$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB.

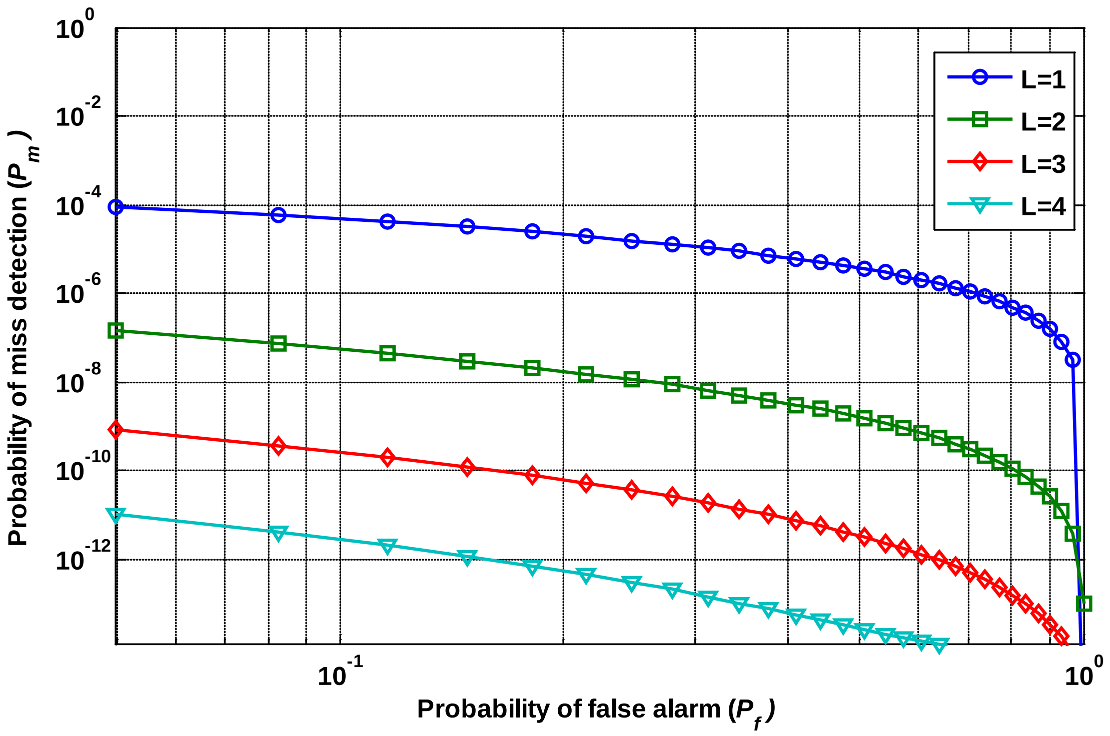

Figure 5.5: Complementary ROCs of the ED for Nakagami-m fading channel ($\alpha = 2, \mu = m = 5$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB.

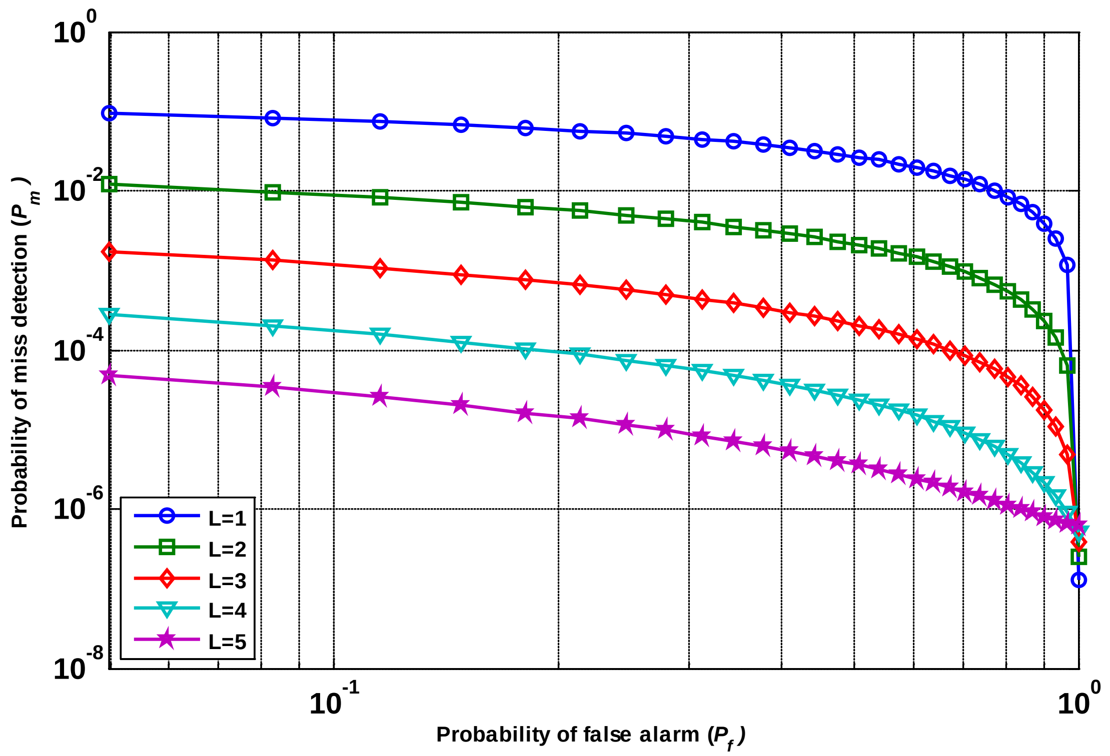

Figure 5.6: Complementary ROCs of the ED for Weibull fading channel ($\alpha = K = 1.5$, $\mu = 1$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB.

5.6 Effect of α

Figure 5.7 shows the complementary ROC of the ED over $\alpha - \mu$ fading channel with different values of α when L=1, 3. Again, we can notice that the performance of the ED is greatly improved when increasing the number of SC diversity branches from L=1 to L=3. In addition, increasing the value of the nonlinearity parameter α improves the performance of the ED by decreasing the probability of miss detection. Indeed, increasing α enhances the tail under the PDF as illustrated in Figure 3.10, and hence for a given fixed threshold it decreases the miss detection probability.

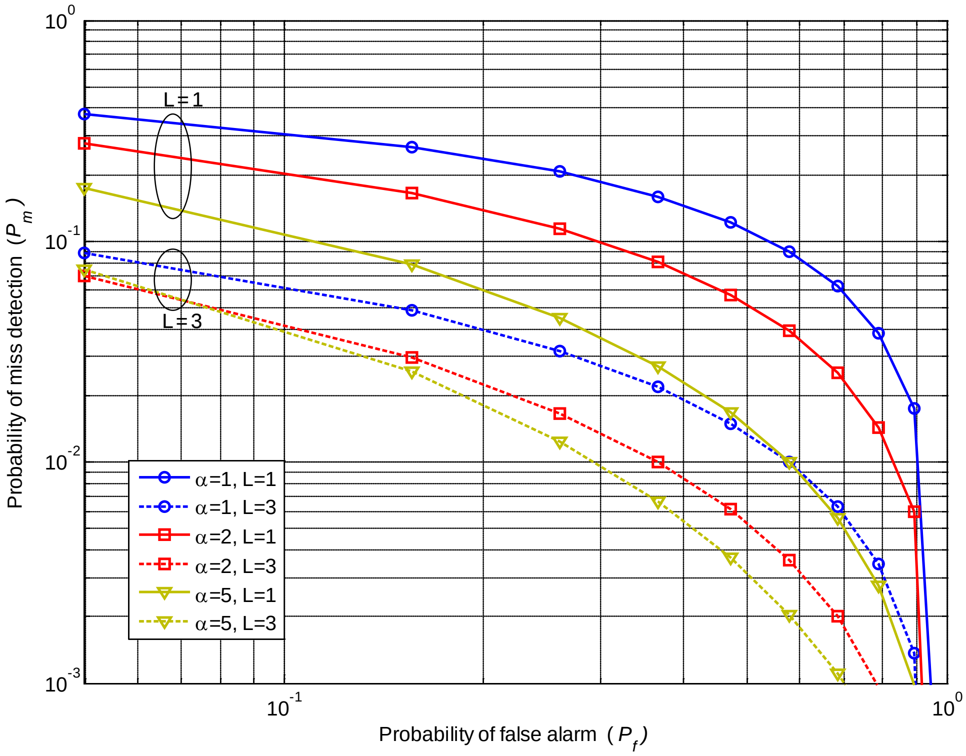

Figure 5.7. Complementary ROC curves of the ED for $\alpha - \mu$ fading channel with SC and different values of α, $\mu = 2$, and $\bar{\gamma} = 10$dB.

5.7 Effect of μ

According to Figure 5.8, the complementary ROC of the ED is greatly enhanced when L is increased from L=1 to L=3 with the several values of μ. In addition, increasing the value of μ increases the number of multipath clusters contributing to the envelope of the received signal, and hence increases the diversity gain resulting in lower miss detection probability.

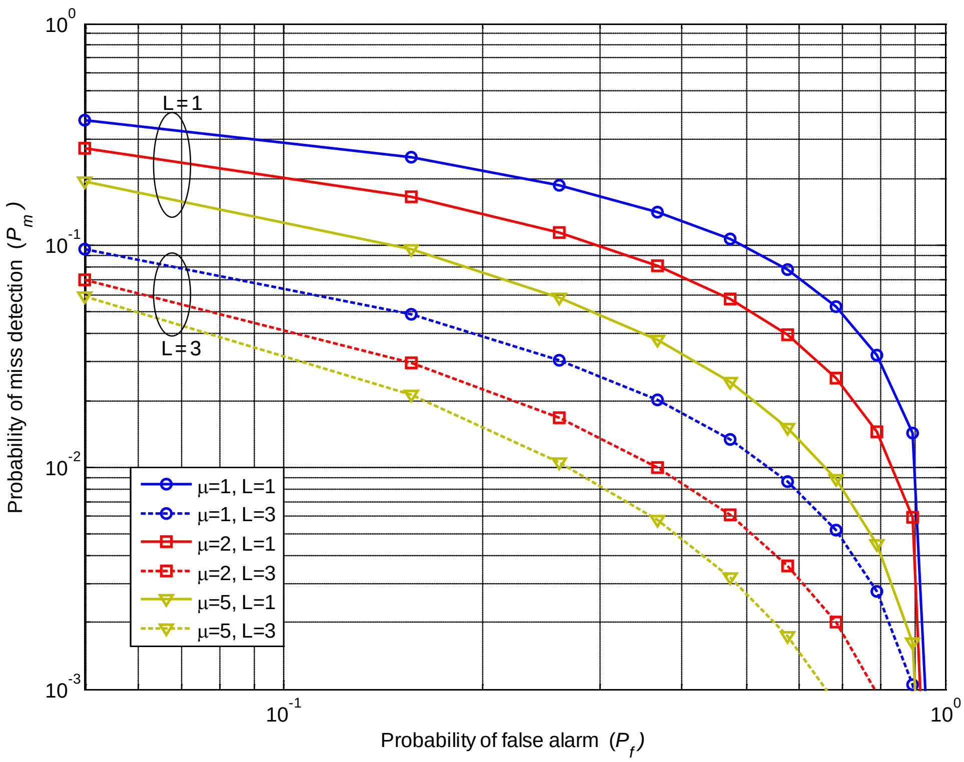

Figure 5.8: Complementary ROC curves of the ED for $\alpha-\mu$ fading channel with SC and different values of μ, $\alpha = 2$, and $\bar{\gamma} = 10$dB.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, we have shed light on “Cognitive Radio (CR)”, a term coined by J. Mitola [Mit00]. We’ve seen that this field of study attracted huge number of researches in the last two and half decades. The CR has several spectrum sensing techniques which make it an intelligent SDR. Cooperation between several CR users and the use of diversity combining techniques can combat fading effects on wireless signals.

In this thesis, we have discussed the system model of the ED. The performance analysis of the ED, as one simple and famous technique used in spectrum sensing for CR, is obtained. We have examined the performance of the ED over several known fading distribution models covered by the umbrella of $\alpha - \mu$ fading model. Namely, a new expression for the average detection probability of the ED over $\alpha - \mu$ generalized fading model with SC diversity reception is derived in this thesis. The derived expression covers many fading distribution models as special cases; in addition, it can be used with and without SC diversity reception. Some new forms for the APD of the ED with and without SC diversity combining are extracted from the general derived expression. Complementary ROCs are drawn for the ED where enhancement of the probability of detection was achieved by using SC as one of the familiar diversity reception techniques.

We have also seen the effect of many parameters contained in the derived expression on the overall performance of the ED. Those parameters include the number of diversity branches L, the average signal-to-noise power ratio $\bar{\gamma}$, the environment non-linearity parameter α, and the number of multipath clusters μ.

6.2 Future Work

Currently, we are analyzing the detection probability of the energy detector undergoing $\alpha - \mu$ fading with other diversity combining techniques such as EGC, MRC, etc.

Then, we will analyze the performance of energy detection based spectrum sensing over other generalized fading distributions, such as the $\alpha - \eta - \mu$ and the $\alpha - \lambda - \mu$ models [Pap09], with and without diversity reception for cognitive radio networks.

We can also extend our study to account for the case of correlated fading channels by using the results obtained in [Ibr10]-[Ibr04] for instance.

Acronyms and Abbreviations

ADC: Analogue to Digital Converter

APD Average Probability of Detection

AWGN: Additive White Gaussian Noise

BPF: Band Pass Filter

CDF: Cumulative Distribution Function

CR : Cognitive Radio

CSFD: Cyclo-Stationary Feature Detector

CSI: Channel State Information

ED : Energy Detector

EGC: Equal Gain Combining

FC: Fusion Centre

FCC: Federal Communications Commission

i.i.d.: Independent and Identically Distributed

LOS: Line of Site

MFD: Matched Filter Detector

MGF: Moment Generating Function

MRC: Maximum Ratio Combining

NLOS: Non-Line Of Site

PDA: Personal Digital Assistants

PDF: Probability Density Function

RF: Radio Frequency

ROC: Receiver Operating Characteristics

SC: Selection Combining

SEC Switch and Examine Combining

SNR : Signal-to-Noise Ratio

SSC: Switch and Stay Combining

TW: Time-Bandwidth Product

WRAN: Wireless Regional Area Network

Notations

α: Positive Arbitrary Parameter, Environment Non- Linearity Parameter

μ: Inverse of the Normalized Variance of h^{α}, Number of Multipath Clusters

h: Fading Channel Envelope

$f_h(h)$: PDF of the Fading Channel Envelope

$f_\gamma(\gamma)$: PDF of the SNR of the Fading Channel Envelope

$f_x(x)$: PDF of the Normalized Fading Channel Envelope

Ω: Expectation of h^2, Variance

e: Base of the Natural Logarithm=2.71828

m: Nakagami Distribution Severity Parameter

K: Weibull Parameter

π: 3.14159

S: Radio Signal

g: Amplifier Gain

H_0: Hypothesis 0, Signal Absent

H_1: Hypothesis 1, Signal Present

λ: Detection Threshold

T: Observation Time Period

$r(t)$: Received Signal

N_0: One-Sided Noise Power Spectral Density

$n(t)$: Additive White Gaussian Noise (AWGN)

$\overline{h^2}$: Fading Channel Envelop Mean-Square Value

γ: Instantaneous SNR

γ_i: i-th Branch Instantaneous SNR

E_s: Energy of the Signal Accumulated over the Observation Period

Y: Decision Statistics Random Variable

u: half Time-Bandwidth Product

TW: Time-Bandwidth Product

P_f: Probability of False Alarm

P_d: Probability of Detection

$\bar{P}_d$: Average Probability of Detection

P_m: Probability of Miss Detection

Pr: Probability

$\Gamma(.)$: Gamma Function

$\Gamma(.,.)$: Upper Incomplete Gamma Function

$Q_u(.,.)$: Generalized Marcum Q-function

$F_{\gamma_{SC}}(\gamma)$: Cumulative Distribution Function of the SNR of the Combiner Output

$f_{\gamma_{SC}}(\gamma)$: PDF of the combiner's output SNR

L: Number of diversity Branches

$\bar{\gamma}$: Average PDF of the SNR

$\beta_{mi}(\mu)$: Coefficient of Multinomial Expansion

$\bar{P}_{d,\alpha\mu}$: Average Probability of Detection of the $\alpha - \mu$ Fading Model

$\bar{P}_{d,\alpha\mu,sc}$: Average Probability of Detection of the $\alpha - \mu$ Fading Model with SC

$\bar{P}_{d,Ray,SC,L}$: Average Probability of Detection of the Rayleigh Fading Model with SC

$\bar{P}_{d,Ray,L=1}$: Average Probability of Detection of the Rayleigh Fading Model

$G_{p,q}^{q,p}\left(z\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)$: Meiger-$G$ Function

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Appendix A

Published Paper

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Selection Diversity Combining Analysis of Energy Detector Over α-μ Generalized Fading Channels

Hikmat Y. Darawsheh
Information and Communication Technology Center
Al-Quds Open University
Ramallah, Palestine
hdarawsheh@qou.edu

Ali Jamoos, *member, IEEE*
Department of Electronic Engineering
Al-Quds University
Jerusalem, Palestine
ali@eng.alquds.edu

***Abstract* — This paper addresses the problem of energy detection of an unknown deterministic signal over a general fading channel model. More particularly, a closed-form mathematical expression is derived for the energy detector's probability of detection over α-μ generalized fading channel with selection combining diversity reception. The derived expression can be used to study the performance of energy detector in many known fading channel models with and without selection combining; this can be achieved by choosing some specific values for both α and μ parameters in the proposed general expression. Nakagami-*m*, Weibull, Gamma, Rayleigh and Exponential fading distributions are special cases of the derived general expression.**

Keywords—energy detector; selection combining diversity; fading channels; α-μ generalized fading distribution.

I. INTRODUCTION

Electromagnetic spectrum, as a natural resource, is limited. It has been divided into specific bands that are assigned to suitable applications. The licensed subscriber has the full permission to use the band they bought whenever and wherever they need to, as long as their licenses are valid. However, vast amounts of the spectrum are not used efficiently, indeed, they are under-utilized. Cognitive radio (CR), which is a clever telecommunication system that can sense and adapt its parameters to avoid interference on licensed users [1], is one solution to this underutilization problem. The CR user is considered as a rental or secondary user of the spectrum and it has to decide when it can access the spectrum and what band to use in order not to cause any kind of interference to any licensed user. This leads to the fact that cognitive radio network must accurately sense the spectrum and adapts its transmission in accordance with the results of its sensing operation and the situation of the channel to be used. There are several spectrum sensing techniques used to enhance the spectrum utilization [2]. The ultimate goal of these techniques is to enable rental (secondary) users benefitting from the white spaces in the spectrum that are spatially/temporally free of primary users. The energy detector proposed in [3], is one of the main and simplest techniques frequently used in cognitive radio networks to enable opportunistic spectrum access.

Several fading distribution models have been suggested to describe the statistics of the received mobile signal envelope [4]. Indeed, the short-term signal envelope variation is properly depicted by several main distributions such as Rice, Nakagami-*m*, Weibull, Rayleigh, Hoyt and others. Each of these fading distributions is suitable for certain channel conditions. In some situations, no distributions satisfactorily match experimental observations, although one of them may produce moderate fitting. This motivates the need for a general distribution that can give up better fitting to real measurements and can include several fading distributions as special cases. One of these general fading distributions is the $\alpha - \mu$ distribution recently proposed in [5][6]. It is an umbrella distribution and involves as special cases several main distributions such as Nakagami-*m*, Rayleigh, Gamma, Weibull, exponential, and one sided Gaussian. In addition, its probability density function, cumulative distribution function, and moments come-out in uncomplicated closed-form formulas. Furthermore, it can describe the non-linearity of the wireless propagation environment. These features make the $\alpha - \mu$ distribution very attractive.

Fading channels can extremely affect the transmitted signals and decreasing the overall signal to noise power ratio (SNR) at the reception end. In this case, antenna diversity reception techniques, that combine the outputs of multiple fading branches together, can be used to boost the SNR at the receiver. Selection diversity combining (SC), equal gain combining (EGC), switch and stay combining (SSC), and maximum ratio combining (MRC) are examples of combining methods used in antenna diversity reception [7].

During the last decade, a lot of interest has been paid to the issue of detecting unknown deterministic signals over a variety of fading channel models with or without diversity reception at the receiver [8][9][10]. Indeed, in [8] the average detection probability of energy detector is derived for Rayleigh, Rician and Nakagami-*m* faded signals. An alternative analytical approach have been proposed by Digham et al. in [9], where closed-form expressions are obtained for the average detection probability undergoing Rayleigh and Nakagami-*m* fading with square law combining and square law selection diversity methods. In [10], the moment generating function (MGF) technique and the probability density function (PDF) technique are employed to evaluate the performance of energy detector undergoing Rician and Nakagami-*m* fading with several diversity combining techniques. However, this yields a wide collection of performance expressions that are applicable only for certain fading models with specific model parameters. To avoid this drawback, Fathi and Tawfik have recently proposed

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a versatile performance expression for energy detector over the $\alpha-\mu$ generalized fading channels [11]. Nevertheless, no diversity combining techniques are considered.

In this contribution, we suggest to extend the results in [11] by considering selection combining diversity reception at the receiver. A new closed-form formula is derived for the average detection probability of the energy detector over $\alpha-\mu$ generalized fading channels with selection combining diversity reception.

The remainder of the paper is structured as follows. In section 2, the system model is described. Section 3 exposes the derived mathematical expression for the probability of detection of the energy detector with SC. Section 4 presents some special cases of the derived general expression. Numerical examples are presented and discussed in Section 5. Conclusions are reported in Section 6.

II. System Model and Performance Analysis

A. The Energy Detector (ED)

The ED is simply a threshold-based binary decision device; its output is one of two hypotheses H_0: white space, i.e. only Additive White Gaussian Noise (AWGN), or H_1: occupied, i.e. existence of primary user. The decision is made by comparing the aggregated energy of a Band-Pass-Filtered (BPF) received signal, in an observation period of time against a predetermined detection threshold λ. Fig. 1 exhibit a block diagram of the well known ED.

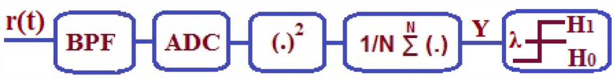

Fig. 1. Block diagram of the ED.

The received signal $r(t)$ can be either AWGN denoted by $n(t)$, or unknown transmitted signal $s(t)$ faded by a channel gain h and added to a noise $n(t)$ as follows:

$$r(t)=\begin{cases}n(t) & H_0\\ h\,s(t)+n(t) & H_1\end{cases} \qquad (1)$$

If the fading channel is characterized to have small-scale variations with nonlinear propagation medium, then the envelope h of the fading signal obeys the $\alpha-\mu$ general fading distribution, where α denotes a positive parameter, and $\mu>0$ denotes the inverse of normalized variance of h^α. The PDF of the signal envelope h is expressed as

$$f_h(h)=\frac{\alpha\mu^\mu h^{\alpha\mu-1}}{\bar{h}^{\alpha\mu}\Gamma(\mu)}\,e^{-\mu\left(\frac{h}{\bar{h}}\right)^\alpha} \qquad (2)$$

where $\bar{h}=\sqrt[\alpha]{E(h^\alpha)}$ denotes the α-root mean value of h, and $\Gamma(\mu)$ denotes the gamma function and is defined by $\Gamma(\mu)=\int_0^\infty t^{\mu-1}e^{-t}dt$.

By letting $\alpha=K$ and $\mu=1$, the $\alpha-\mu$ distribution reduces to the Weibull distribution. Setting $K=1$ results in exponential distribution. In addition, Nakagami-m density is obtained by setting $\alpha=2$ and $\mu=m$. Furthermore, from Nakagami-m density, one sided Gaussian and Rayleigh densities are obtained by setting $m=1/2$ and $m=1$, respectively. Moreover, letting $\alpha=1$ and $\mu=a$ yields Gamma density. Fig. 2 shows the PDF $f_h(h)$ of the $\alpha-\mu$ fading distribution for several values of α and μ, resulting in five well known distributions. Indeed, the $\alpha-\mu$ distribution is general, flexible and covers vast range of fading situations.

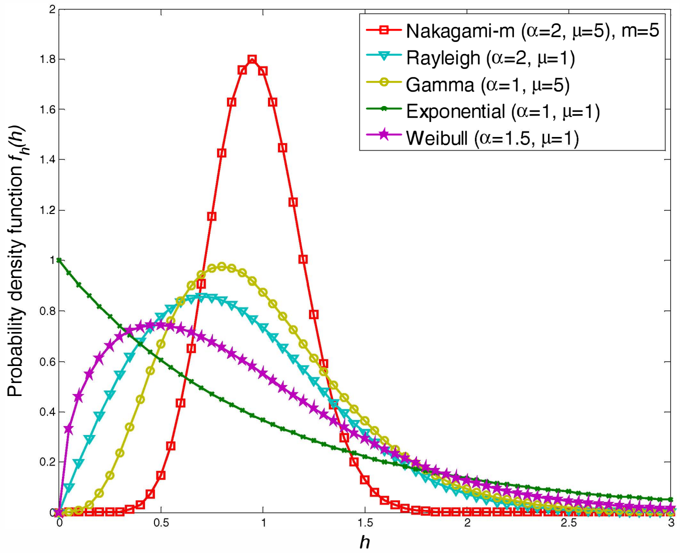

Fig. 2. The PDF $f_h(h)$ of the $\alpha-\mu$ general fading distribution for several values of α and μ.

Since the decision statistics Y is the sum of square values of the received signal amplitudes, its PDF has to be derived from $f_h(h)$ by simple change of variables [11], as follows:

$$f_\gamma(\gamma)=\frac{\alpha\mu^\mu\gamma^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{\alpha\mu}{2}}}\,e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \qquad (3)$$

where $\gamma=|h|^2E_s/N_0$ is the instantaneous signal to noise power ratio (SNR) of the signal envelope, E_s is the energy of the signal accumulated over the observation period, N_0 is the power spectral density of the noise, and $\bar{\gamma}=\bar{h}^2E_s/N_0$ is the average SNR.

B. Conditional probabilities of detection and false alarm

There are two important probabilities to discuss when distinguishing between two hypotheses related to statistically random variates. They are the false alarm probability P_f and the detection probability P_d. Fig. 3 shows the miss detection probability $P_m=1-P_d=Pr(Y<\lambda|H_1)$ and the false alarm probability $P_f=Pr(Y>\lambda|H_0)$.

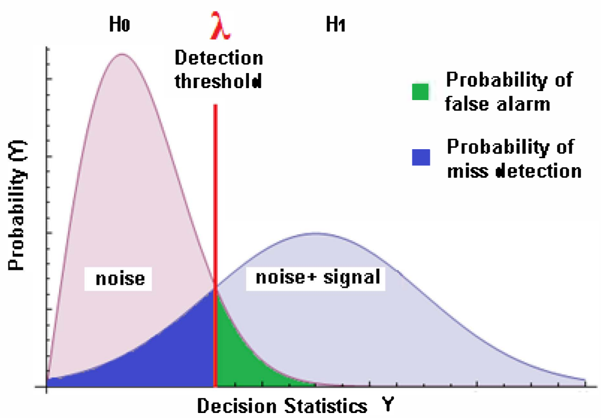

Fig. 3. False alarm and miss detection probabilities.

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The false alarm P_f and detection P_d probabilities can be computed, respectively, in AWGN as follows [9]:

$$P_f = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)} \tag{4}$$

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \tag{5}$$

where $\Gamma(.,.)$ denotes the incomplete gamma function, $Q_u(.,.)$ denotes the generalized Marcum Q-function, and u denotes the time-bandwidth product which is equal to half number of symbols in the observation time period.

III. Energy Detector Performance Analysis Over Generalized Fading Channels

In this section, the analysis of the ED over the $\alpha - \mu$ generalized fading channel is revisited to derive a mathematical formula for the average detection probability when SC diversity technique is employed at the receiver.

To obtain the Receiver Operating Characteristics (ROC) when considering AWGN and $\alpha - \mu$ fading channel, equation (5) should be averaged over the PDF of the fading channel. Note that the false alarm probability given by (4) has no terms relating to fading channel parameters, and so, doesn't change.

In [11], the average detection probability of the ED is obtained over the $\alpha - \mu$ general fading distribution, but with no diversity reception. When diversity is used, multiple branches are combined and the combiner output is then compared to a threshold value to distinguish between the two hypotheses H_0 and H_1. The combining technique used by the combiner determines the shape of the output decision statistics variable. In this paper, we discuss the selection combining diversity technique, where the diversity branch with highest SNR is chosen by the selection combiner. The PDF of the SNR for any single branch in the $\alpha - \mu$ fading channel is given by (3). For L diversity branches, the SNR of the combiner output is equal to the maximum of $\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$, where γ_i is the i-th branch instantaneous SNR. Assuming that the average SNRs for all branches are equal, let's denote it by $\bar{\gamma}$, then for any single branch, the probability that its SNR γ_i is less than some value γ is given by:

$$Pr\,(\gamma_i \le \gamma) = \int_0^{\gamma} f_{\gamma_i}(\gamma_i) d\gamma_i = \int_0^{\gamma} \frac{\alpha\mu^{\mu}\gamma_i^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\bar{\gamma}^{\frac{\alpha\mu}{2}}} e^{-\mu\left(\frac{\gamma_i}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma_i$$

$$= 1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)} \tag{6}$$

The cumulative distribution function (CDF) of the output SNR of L i.i.d selection combiner branches is derived as follows:

$$CDF(\gamma) = Pr(\gamma_1 \le \gamma, \gamma_2 \le \gamma, \ldots, \gamma_L \le \gamma)$$

$$= \prod_{i=1}^{L} \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)$$

$$= \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^L \tag{7}$$

Now, the PDF of the SNR at the output of the combiner $f_{SC}(\gamma)$ is the derivative of $CDF(\gamma)$, and it is calculated as:

$$f_{SC}(\gamma) = \frac{L\alpha\mu^{\mu}}{2\Gamma(\mu)\bar{\gamma}^{\frac{\alpha\mu}{2}}} \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \tag{8}$$

Therefore, the average detection probability for the SC can be calculated by averaging (5) over (8) as follows:

$$\bar{P}_{d,\alpha\mu,sc} = \int_0^{\infty} \frac{L\alpha\mu^{\mu}}{2\Gamma(\mu)\bar{\gamma}^{\frac{\alpha\mu}{2}}} \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \times Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) d\gamma \tag{9}$$

The Q-function $Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right)$ can be rewritten into series representation [9][12] as follows:

$$Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) = \sum_{n=0}^{\infty} \frac{\Gamma\left(n+u, \frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)} \gamma^n e^{-\gamma} \tag{10}$$

Substituting (10) into (9) and using binomial and then multinomial expansion for the term $\left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1}$, it follows after solving the integral in (9) based on the general Laplace transform [13, 2.2.1-22] that:

$$\bar{P}_{d,\alpha\mu,sc} = C \sum_{n=0}^{\infty} a_n \sum_{i=0}^{L-1} (-1)^i \binom{L-1}{i} \sum_{m=0}^{i(\mu-1)} \frac{\mu^m l^{\frac{\alpha m}{2}+n}}{\bar{\gamma}^{\frac{\alpha m}{2}}} \times \beta_{mi}(\mu)\, G_{l,k}^{k,l}\left(z; {\Delta(l, -w) \atop \Delta(k, 0)}\right) \tag{11}$$

where $C = \frac{\alpha\mu^{\mu} L \sqrt{k}\, l^{\frac{1}{2}(\alpha\mu-1)}}{2\Gamma(\mu)\bar{\gamma}^{\frac{1}{2}(\alpha\mu)}(2\pi)^{\frac{k+l}{2}-1}}$, $a_n = \frac{\Gamma\left(n+u, \frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)}$, $z = l^l \left(\frac{(i+1)\mu}{k\bar{\gamma}^{\alpha/2}}\right)^k$, and $w = \frac{1}{2}\alpha(\mu+m) + n - 1$. In addition, $G_{p,q}^{q,p}\left(z; {a_1, \ldots, a_p \atop b_1, \ldots, b_q}\right)$ is the Meiger-G function, $\Delta(k, a) = \frac{a}{k}, \frac{a+1}{k}, \ldots, \frac{a+k-1}{k}$, l and k are some integers such that $\frac{l}{k} = \frac{\alpha}{2}$, $\beta_{mi}(\mu)$ denotes the multinomial expansion coefficient which can be computed recursively as illustrated in [4, 9.124].

IV. Some Special Cases

A. No diversity reception with α-μ general fading distribution

In the case of no diversity at the receiver, i.e. single branch $L = 1$, the derived formula for the detection probability of the ED with SC diversity reception in (11) reduces to a previously known result found in [11, Eq. (8)], as follows:

$$\bar{P}_d = A \sum_{n=0}^{\infty} l^n \, a_n G_{l,k}^{k,l}\left(S;\ \begin{matrix}\Delta(l,-v)\\ \Delta(k,0)\end{matrix}\right) \tag{12}$$

where $A = \frac{\alpha\mu^{\mu}\sqrt{k}\, l^{(\alpha\mu-1)/2}}{2\Gamma(\mu)\,\bar{\gamma}^{\left(\frac{\alpha\mu}{2}\right)}(2\pi)^{\frac{k+l}{2}-1}}$, $S = \left(\frac{\mu}{k\bar{\gamma}^{\alpha/2}}\right)^k l^l$, and $v = n + \frac{\alpha\mu}{2} - 1$. Note that a typo in [11, Eq. (8)] is fixed in (12) by adding the missing term l^n.

B. Rayleigh distribution with and without diversity reception

The probability of detection for the ED with SC diversity reception when considering Rayleigh faded signals can be found from (11) by setting $\alpha = 2$ and $\mu = 1$:

$$\bar{P}_{d,Ray,SC,L} = \frac{L}{\bar{\gamma}}\sum_{r=0}^{\infty}\frac{\Gamma\left(r+u,\frac{\lambda}{2}\right)}{\Gamma(r+u)}\sum_{j=0}^{-1+L}(-1)^j\left(\frac{1+j+\bar{\gamma}}{\bar{\gamma}}\right)^{-1-r}\binom{L-1}{j} \tag{13}$$

For the no diversity case $L = 1$, equation (13) reduces to:

$$\bar{P}_{d,Ray,L=1} = \frac{1}{1+\bar{\gamma}}\sum_{r=0}^{\infty}\frac{\Gamma\left(r+u,\frac{\lambda}{2}\right)}{\Gamma(r+u)}\left(\frac{\bar{\gamma}}{1+\bar{\gamma}}\right)^r \tag{14}$$

V. Numerical Results and Discussion

The performance of the ED is quantified by depicting the complementary Receiver Operating Characteristics (ROCs) ($P_m = 1 - \bar{P}_d$ versus P_f) with the effect of the various parameters α, μ, L, and $\bar{\gamma}$. From the $\alpha - \mu$ general fading distribution, other fading distributions can be derived based on specific values for both α and μ. The following $\{\alpha, \mu\}$ pairs were taken as test cases for the subsequent complementary ROCs [11]. {2,5} Nakagami-*m* (*m*=5); {1,5} Gamma (Chi-Square *a*=5); {1.5,1} Weibull (*K*=1.5); {1,1} Exponential; {2,1} Rayleigh.

Fig. 4 shows the complementary ROCs of the ED over $\alpha - \mu$ fading channel without SC; it is the special case when setting *L*=1 in (11) yielding (12). By selecting different values of α and μ, the results for several well known distributions are shown. One can notice from Fig. 4 that the performance of energy detector is degraded when going from the non-fading case (only AWGN channel) to the fading case with several distributions. Exponential faded channel results in the worst performance while Nakagami-*m* faded channel with severity parameter *m*=5 yields the best performance. Note that Rayleigh faded channel (special case when *m*=1) yields better performance than Nakagami-*m* faded channel with *m*=5. Thus, the higher is the *m*, the higher is the detection probability.

Fig. 5 shows the complementary ROCs of the ED over $\alpha - \mu$ fading with SC diversity (*L*=2). When comparing Fig. 4 and Fig. 5, one can notice that the SC antenna diversity technique greatly enhances the performance of the ED. For example, when P_f=0.2, all curves in Fig. 4 have P_m less than 0.4, while in Fig. 5 they are less than 0.2. This means that the more is the diversity branches, the less is the miss detection probability.

The effect of increasing the average SNR ($\bar{\gamma}$) on the complementary ROCs of the ED is depicted in Fig. 6. One can notice that, the miss detection probability improves greatly when increasing $\bar{\gamma}$ form 10dB to 25dB. Fig. 7, Fig. 8 and Fig. 9 illustrate the effect of increasing the number of SC diversity branches (from *L*=1 to *L*=5) on the complementary ROCs of the ED for Rayleigh, Nakagami-*m* and weibull faded signals, respectively. It is noticed that the miss detection probability is greatly reduced when increasing the number of diversity branches.

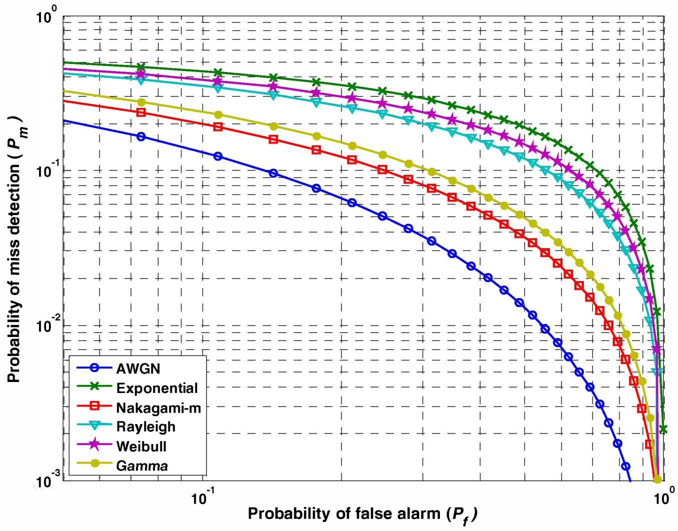

Fig. 4. Complementary ROCs of the ED over different fading channels without SC (*L*=1), *u*=5, and average SNR $\bar{\gamma}$ = 9dB.

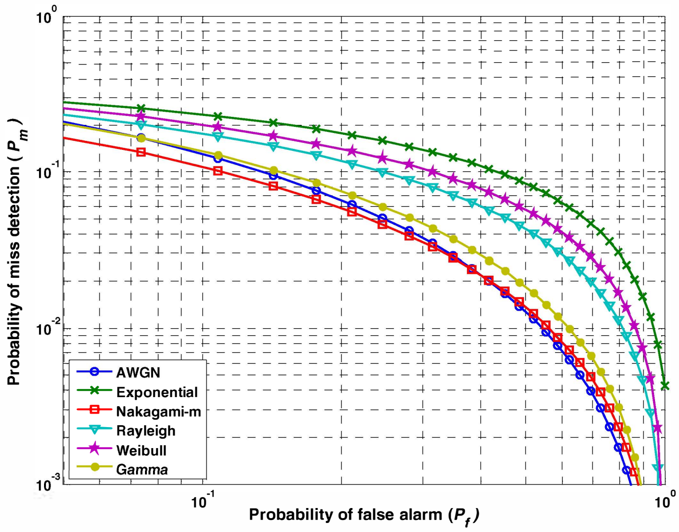

Fig. 5. Complementary ROCs of the ED over different fading channels with SC (*L*=2), *u*=5, and $\bar{\gamma}$ = 9dB.

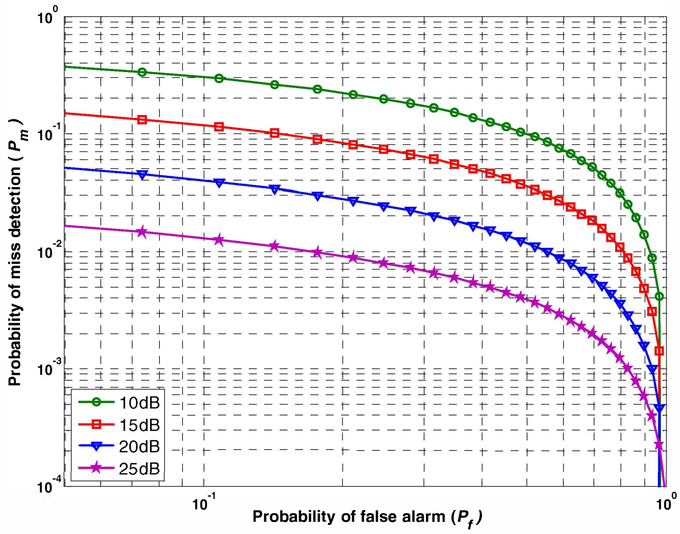

Fig. 6. Complementary ROCs of the ED for Rayleigh fading ($\alpha = 2, \mu = 1$) without SC (*L*=1), *u*=5, and different values of SNR $\bar{\gamma}$.

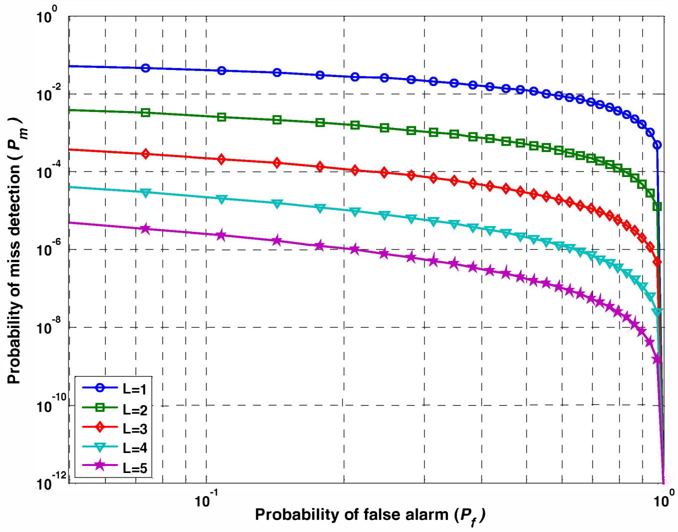

Fig. 7. Complementary ROCs of the ED for Rayleigh fading channel ($\alpha = 2, \mu = 1$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB.

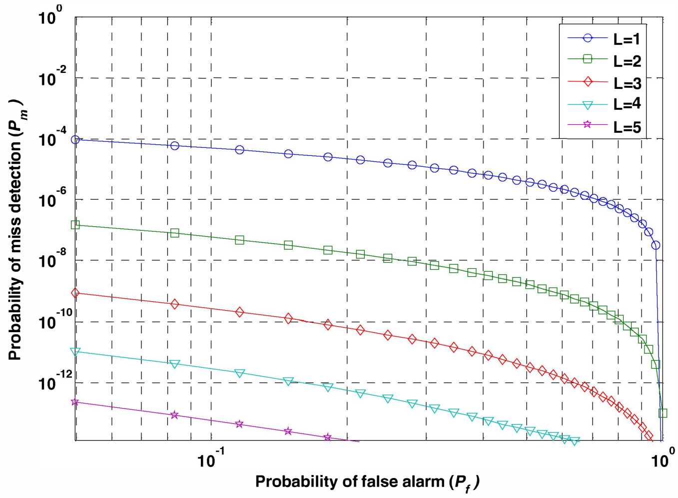

Fig. 8. Complementary ROCs of the ED for Nakagami-m fading channel ($\alpha = 2, \mu = m = 5$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB.

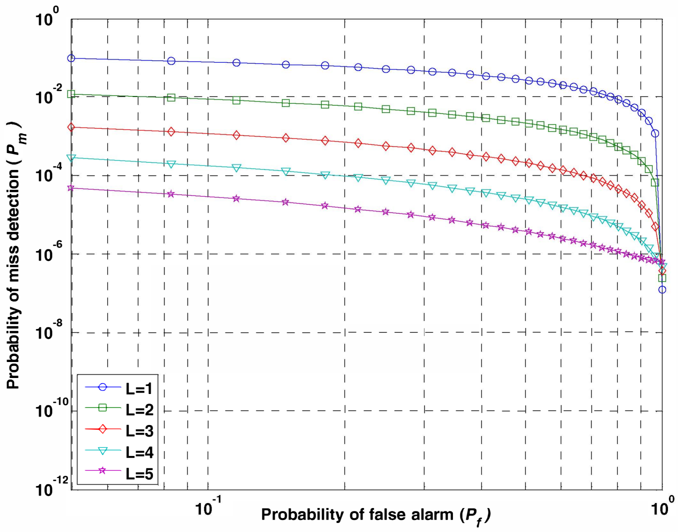

Fig. 9. Complementary ROCs of the ED for Weibull fading channel ($\alpha = K = 1.5, \mu = 1$) with different values of SC diversity branches L, u=5, and $\bar{\gamma} = 20$dB.

VI. CONCLUSION

A new expression for the detection probability of the ED over $\alpha - \mu$ generalized fading model with SC antenna diversity is derived in this paper. The derived expression covers several known fading distribution models as special cases as well as it can be used with and without SC diversity reception. Complementary ROCs were drawn for the ED where enhancement of the probability of detection was achieved by using SC antenna diversity reception. Currently, we are analyzing the detection probability of the energy detector undergoing $\alpha - \mu$ fading with other diversity combining techniques such as EGC, MRC, SLC, etc.

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Appendix B

Submitted Paper

Hikmat Y. Darawsheh and Ali Jamoos, "Performance Analysis of Energy Detector over α-μ Fading Channels with Selection Combining", submitted to the International Journal of Electronics Letters, Taylor & Francis, 2013.

Performance Analysis of Energy Detector Over α-μ Fading Channels With Selection Combining

Hikmat Y. Darawsheh[1] and Ali Jamoos[2]

[1]Information and Communication Technology Center, Al-Quds Open University, Ramallah, Palestine
hdarawsheh@qou.edu

[2]Department of Electronic Engineering, Al-Quds University, Jerusalem, Palestine
ali@eng.alquds.edu

***Abstract*—Energy detection is the most widely used technique in cognitive radio networks to enable opportunistic spectrum access. In this paper, the problem of energy detection of an unknown deterministic signal over fading channels is revisited. More particularly, a new closed-form mathematical expression is derived for the average probability of detection of the energy detector (ED) over α-μ generalized fading channels with selection combining (SC) diversity reception. The derived expression is general and includes as special cases Nakagami-m, Weibull, Gamma, Rayleigh and Exponential distributions. This expression is useful to quantify the performance improvement of the ED with SC diversity reception.**

Keywords*—cognitive radio networks; energy detection; selection combining; diversity reception; fading channels; α-μ generalized fading distribution.

I. Introduction

The static spectrum assignment policy adopted by traditional wireless networks is faced with spectrum scarcity at particular spectrum bands. In addition, a large portion of the assigned spectrum is still under-utilized. To solve these spectrum inefficiency problems, Cognitive Radio (CR) technology is recently proposed [1]. Cognitive Radio is an intelligent communication system that is aware of its environment. It can sense and adapt its parameters to avoid interference on licensed users. This makes spectrum sensing an important requirement for the realization of cognitive radio networks. There are several spectrum sensing techniques proposed in the literature such as energy detection, matched filter detection and feature detection [2]. The ultimate goal of these techniques is to provide more spectrum access opportunities to CR users without causing harmful interference to the primary users. Among the existing spectrum sensing techniques, the energy detector proposed in [3] has the advantage of low cost and simple implementation. It simply measures the received energy on a primary band during an observation interval and declares either a white space if the measured energy is less than a properly set threshold, or occupied if the energy is larger than the threshold.

Several fading distributions have been proposed in the literature to describe the statistics of the mobile radio signal [4]. Indeed, the short-term signal variation is well described by several main distributions such as Nakagami-*m*, Rayleigh, Rice, Weibull, Hoyt and others. Each of these fading distributions is suitable for certain channel conditions. In some situations, no distributions adequately fit experimental data, although one or another may yield a moderate fitting. This motivates the need for a general fading distribution that can yield better fitting to experimental data and can include several fading distributions as special cases. One of these general fading distributions is the $\alpha - \mu$ distribution recently proposed in [5]. It is an umbrella distribution and includes as special cases important distributions such as Nakagami-*m*, Rayleigh, Gamma, exponential, Weibull, and one-sided Gaussian. In addition, its probability density function, cumulative distribution function, and moments appear in simple closed form expressions. Furthermore, it can explore the nonlinearity of the propagation medium. These features make the $\alpha - \mu$ distribution very attractive.

Fading channels can extremely affect the transmitted signals resulting in degrading the received signal-to-noise power ratio (SNR). In this case, antenna diversity reception techniques that combine the outputs of multiple fading branches can be used to enhance the SNR at the receiver. Equal gain combining (EGC), maximal ratio combining (MRC), and selection combining (SC) are the most widely used diversity combining techniques [4].

During the last decade, a great deal of interest has been paid to the problem of detecting unknown deterministic signals over a variety of fading channel distributions with or without diversity reception at the receiver [6][7][8]. Indeed, in [6] the average detection probability performance of energy detector is derived for Rayleigh, Rician and Nakagami-*m* fading channels. An alternative analytical approach have been proposed by Digham et al. in [7], where closed-form expressions are obtained for the average detection probability over Rayleigh and Nakagami-*m* fading channels with square-law combining and square-law selection diversity schemes. In [8], the moment generating function (MGF) method and the probability density function (PDF) method are used to evaluate the performance of energy detector over Rician and Nakagami-*m* fading models with several diversity combining techniques. However, this yields a wide collection of performance expressions that are applicable only for certain fading models with specific model parameters. To avoid this drawback, Fathi and Tawfik have recently proposed a versatile performance expression for energy detector over the $\alpha - \mu$ generalized fading channels [9]. Nevertheless, no diversity combining techniques are considered.

In this paper, we propose to extend the results in [9] by considering selection combining diversity reception at the receiver. A new closed-form expression is derived for the average detection probability of the energy detector over $\alpha - \mu$ generalized fading channels with selection combining diversity reception.

The rest of the paper is organized as follows. Section II introduces the energy detector over $\alpha - \mu$ fading channels. Section III presents the performance of the energy detector with selection combining diversity reception. Numerical results are discussed in Section IV. The conclusions are reported in Section V.

II. ENERGY DETECTION OVER $\alpha-\mu$ FADING

The energy detector (ED) is a threshold-based decision device. Its output is one of two hypotheses H_0 and H_1 denoting, respectively, signal absence and signal presence. The decision is made by comparing the aggregated energy of a band-pass-filtered (BPF) received signal, over an observation period of time T s, against a predetermined detection threshold λ as shown in Fig. 1.

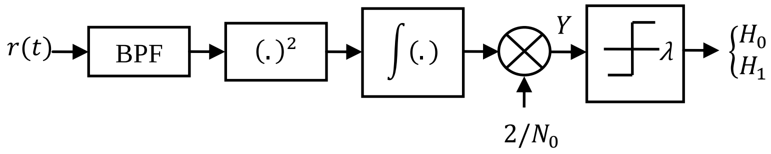

Fig. 1. Block diagram of the energy detector.

Thus, the received signal $r(t)$ can be interpreted as a binary hypothesis test:

$$r(t)=\begin{cases} n(t) & H_0 \\ h\,s(t)+n(t) & H_1 \end{cases} \tag{1}$$

where $n(t)$ is an additive white Gaussian noise (AWGN) process with one-sided power spectral density N_0 Watt/Hz, $s(t)$ is the transmitted signal, and h is the channel coefficient amplitude having mean-square value of $\bar{h}^2$ and probability density function $f_h(h)$. The instantaneous signal-to-noise ratio (SNR) at the receiver antenna can be expressed as $\gamma=|h|^2E_s/N_0$, where E_s is the energy of the signal accumulated over the observation period. It is well known that the probability density function of the decision variable Y can be expressed in terms of the central and non-central Chi-square distributions with $u=TW/2$ degrees of freedom, where W is the BPF bandwidth and TW is the time-bandwidth product. Based on the statistics of Y and given a fixed threshold λ, the conditional probabilities of false alarm $P_f=Pr(Y>\lambda|H_0)$ and detection $P_d=Pr(Y>\lambda|H_1)$ for a certain value of γ can be expressed as [7]:

$$P_f=\frac{\Gamma\left(u,\frac{\lambda}{2}\right)}{\Gamma(u)} \tag{2}$$

$$P_d=Q_u(\sqrt{2\gamma},\sqrt{\lambda}) \tag{3}$$

where $\Gamma(.)$ is the gamma function, $\Gamma(.,.)$ is the upper incomplete gamma function, and $Q_u(.,.)$ is the generalized Marcum Q-function [4, eq.(4.59)].

When the fading channel is characterized by the $\alpha-\mu$ generalized fading distribution, the envelope h of the fading signal has the following probability density function (PDF) $f_h(h)$ [5, eq.(1)]:

$$f_h(h)=\frac{\alpha\mu^{\mu}\,h^{\alpha\mu-1}}{\bar{h}^{\alpha\mu}\Gamma(\mu)}\;e^{-\mu\left(\frac{h}{\bar{h}}\right)^{\alpha}} \tag{4}$$

where α is a positive arbitrary parameter, and $\mu > 0$ is the inverse of the normalized variance of h^{α}, $\bar{h} = \sqrt[\alpha]{E(h^{\alpha})}$ is the α-root mean value of h, and $\Gamma(\mu)$ is the gamma function. By setting $\alpha = K$ and $\mu = 1$, the $\alpha - \mu$ distribution reduces to the Weibull distribution with parameter K. Setting $K = 1$ results in exponential distribution. In addition, Nakagami-m distribution can be obtained by setting $\alpha = 2$ and $\mu = m$, where m is Nakagami-m severity parameter. Furthermore, Rayleigh and one-sided Gaussian distributions are obtained from the Nakagami-m distribution by setting $m = 1$ and $m = 1/2$, respectively. Moreover, Gamma distribution is obtained by setting $\alpha = 1$ and $\mu = a$, where a is the parameter of Gamma distribution. Fig. 2 and Fig. 3 shows the PDF $f_x(x)$ of the normalized envelope $x = h/\bar{h}$ of the $\alpha - \mu$ general fading distribution for several values of α and μ, resulting in several well known fading distributions. Indeed, the $\alpha - \mu$ distribution is general, flexible and covers vast range of fading situations [5].

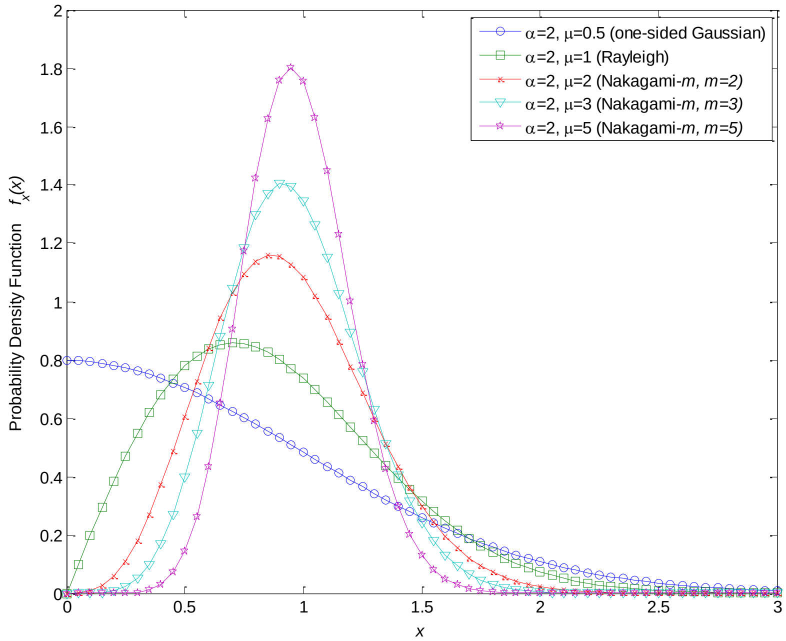

Fig. 2. The probability density function $f_x(x)$ of the $\alpha - \mu$ generalized fading distribution for $\alpha = 2$ with several values of μ.

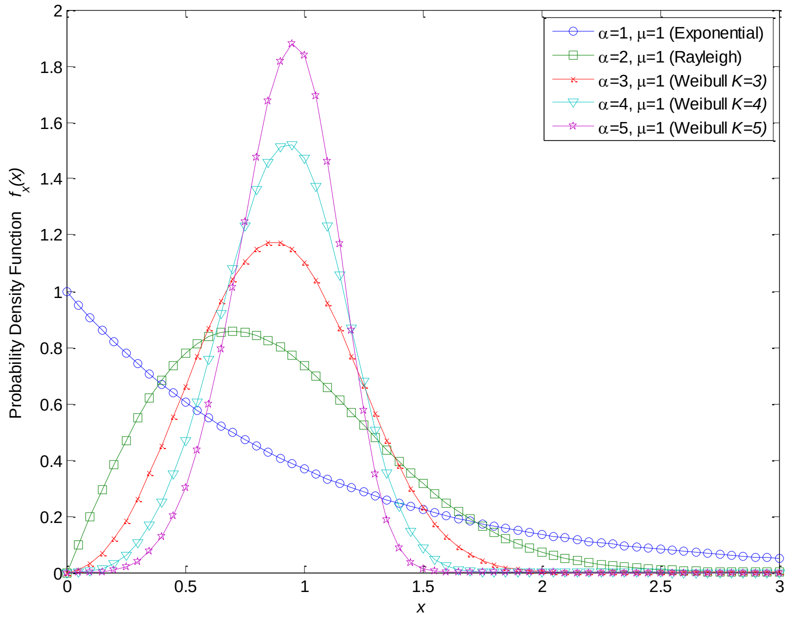

Fig. 3. The probability density function $f_x(x)$ of the $\alpha - \mu$ generalized fading distribution for $\mu = 1$ with several values of α.

To obtain the average probability of detection $\bar{P}_d$ when considering AWGN and $\alpha - \mu$ fading channel, equation (3) should be averaged over the PDF $f_\gamma(\gamma)$ of the output signal-to-noise ratio $\gamma = |h|^2 E_s/N_0$ as follows:

$$\bar{P}_d = \int_0^\infty P_d(\gamma)\, f_\gamma(\gamma) d\gamma \qquad (5)$$

where $f_\gamma(\gamma)$ is derived from $f_h(h)$ by simple change of variables as shown in [4, eq.(2.3)]:

$$f_\gamma(\gamma) = \frac{\alpha \mu^\mu \gamma^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\, \bar{\gamma}^{\frac{\alpha\mu}{2}}} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \qquad (6)$$

where $\bar{\gamma} = \bar{h}^2 E_s/N_0$ is the average SNR.

Note that the probability of false alarm given by (2) has no terms relating to fading channel parameters, and so, doesn't change.

III. Performance of Energy Detector With Selection Combining

In [9], the average probability of detection for the ED is obtained over the $\alpha - \mu$ general fading distribution. However, no diversity combining techniques are considered. In this section, the performance of the ED over the $\alpha - \mu$ fading channel is revisited to derive a mathematical expression for the average probability of detection when selection combining (SC) diversity technique is employed at the receiver. When considering SC diversity technique, the diversity branch with highest SNR is chosen by the selection combiner. The PDF of the instantaneous SNR for single branch $\alpha - \mu$ fading channel is given by (6). For L diversity branches, the instantaneous SNR of the SC would be equal to the maximum of $\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$, where γ_i is the i-th branch instantaneous SNR. Assuming that the average SNRs for all branches are equal, let's denote it by $\bar{\gamma}$, then for any single branch, the probability that its SNR γ_i is less than some value γ is given by:

$$Pr\,(\gamma_i \le \gamma) = \int_0^\gamma f_{\gamma_i}(\gamma_i) d\gamma_i = \int_0^\gamma \frac{\alpha \mu^\mu \gamma_i^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\, \bar{\gamma}^{\frac{\alpha\mu}{2}}} e^{-\mu\left(\frac{\gamma_i}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma_i \; = 1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)} \qquad (7)$$

The cumulative distribution function (CDF) of the output SNR of L independent and identically distributed (i.i.d) selection combiner branches is derived as follows:

$$CDF(\gamma) = Pr(\gamma_1 \le \gamma, \gamma_2 \le \gamma, \ldots, \gamma_L \le \gamma) = \prod_{i=1}^{L} \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma\,(\mu)}\right) = \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma\,(\mu)}\right)^L \qquad (8)$$

Now, the PDF of the combiner's output SNR, denoted by $f_{SC}(\gamma)$, is the derivative of $CDF(\gamma)$. It is calculated as:

$$f_{SC}(\gamma) = \frac{L\alpha\mu^\mu}{2\Gamma(\mu)\, \bar{\gamma}^{\frac{\alpha\mu}{2}}} \left(1 - \frac{\Gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \qquad (9)$$

Therefore, the average probability of detection for the SC can be evaluated by averaging (3) over (9) as follows:

$$\bar{P}_{d,\alpha\mu,sc} = \int_0^\infty Q_u\left(\sqrt{2\gamma},\sqrt{\lambda}\right) \times \frac{L\alpha\mu^\mu}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{\alpha\mu}{2}}}\left(1-\frac{\Gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1}\gamma^{\frac{\alpha\mu}{2}-1}e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\,d\gamma \qquad (10)$$

The generalized Marcum Q-function $Q_u\left(\sqrt{2\gamma},\sqrt{\lambda}\right)$ can be rewritten into series representation using [4, eq.(4.74)] as follows:

$$Q_u\left(\sqrt{2\gamma},\sqrt{\lambda}\right) = \sum_{n=0}^{\infty}\frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)}\gamma^n e^{-\gamma} \qquad (11)$$

Substituting (11) into (10) and using binomial and then multinomial expansion for the term $\left(1-\frac{\Gamma\left(\mu,\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(\mu)}\right)^{L-1}$, it follows after solving the integral in (10) based on the general Laplace transform [10, 2.2.1-22] that:

$$\bar{P}_{d,\alpha\mu,sc} = C\sum_{n=0}^{\infty}a_n\sum_{i=0}^{L-1}(-1)^i\binom{L-1}{i}\sum_{m=0}^{i(\mu-1)}\frac{\mu^m l^{\frac{\alpha m}{2}+n}}{\bar{\gamma}^{\frac{\alpha m}{2}}}\beta_{mi}(\mu)\,G_{l,k}^{k,l}\left(z;\begin{matrix}\Delta(l,-w)\\ \Delta(k,0)\end{matrix}\right) \qquad (12)$$

where $C = \frac{\alpha\mu^\mu L\sqrt{k}\,l^{\frac{1}{2}(\alpha\mu-1)}}{2\Gamma(\mu)\,\bar{\gamma}^{\frac{1}{2}(\alpha\mu)}(2\pi)^{\frac{k+l}{2}-1}}$, $a_n = \frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{n!\,\Gamma(n+u)}$, $z = l^l\left(\frac{(i+1)\mu}{k\bar{\gamma}^{\alpha/2}}\right)^k$, and $w = \frac{1}{2}\alpha(\mu+m)+n-1$. In addition, $G_{p,q}^{q,p}\left(z;\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ is the Meiger-*G* function [11, eq.(16.17.1)], $\Delta(k,a) = \frac{a}{k},\frac{a+1}{k},\dots,\frac{a+k-1}{k}$, l and k are some integers such that $\frac{l}{k}=\frac{\alpha}{2}$, $\beta_{mi}(\mu)$ is the multinomial expansion coefficient which can be computed recursively as illustrated in [4, eq.(9.124)]. To the best of authors' knowledge, (12) is new.

In the case of no diversity reception, i.e. single diversity branch $L=1$, the derived expression for the probability of detection of the ED with SC diversity reception in (12) reduces to a previously known result found in [9, eq.(8)], as follows:

$$\bar{P}_d = A\sum_{n=0}^{\infty}l^n\,a_n G_{l,k}^{k,l}\left(S;\begin{matrix}\Delta(l,-v)\\ \Delta(k,0)\end{matrix}\right) \qquad (13)$$

where $A = \frac{\alpha\mu^\mu\sqrt{k}\,l^{(\alpha\mu-1)/2}}{2\Gamma(\mu)\,\bar{\gamma}^{\left(\frac{\alpha\mu}{2}\right)}(2\pi)^{\frac{k+l}{2}-1}}$, $S = \left(\frac{\mu}{k\bar{\gamma}^{\alpha/2}}\right)^k l^l$, and $v = n+\frac{\alpha\mu}{2}-1$. Note that a typo in [9, eq.(8)] is fixed in (13) by adding the missing term l^n.

For the special case of Rayleigh fading channels, the probability of detection for the ED with SC diversity reception can be found from (12) by setting $\alpha=2$ and $\mu=1$:

$$\bar{P}_{d,Ray,SC,L} = \frac{L}{\bar{\gamma}}\sum_{n=0}^{\infty}\frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{\Gamma(n+u)}\sum_{i=0}^{L-1}(-1)^i\left(\frac{\bar{\gamma}}{1+i+\bar{\gamma}}\right)^{n+1}\binom{L-1}{i} \qquad (14)$$

It should be noted that (14) is an alternative form to the one derived in [12, eq.(30)]. For the no diversity case $L=1$, equation (14) reduces to:

$$\bar{P}_{d,Ray,L=1} = \frac{1}{1+\bar{\gamma}} \sum_{n=0}^{\infty} \frac{\Gamma\left(n+u,\frac{\lambda}{2}\right)}{\Gamma(n+u)} \left(\frac{\bar{\gamma}}{1+\bar{\gamma}}\right)^{n} \tag{15}$$

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, the performance of the ED is quantified by depicting the Receiver Operating Characteristics (ROC) ($\bar{P}_d$ versus P_f), or equivalently, complementary ROC (probability of miss detection $P_m = 1 - \bar{P}_d$ versus P_f) for different values of α, μ, and L. In the following examples, the degree of freedom u of both H_0 and H_1 distributions are set to $u = 5$.

Fig. 4 shows the complementary ROC of the ED over $\alpha - \mu$ fading channel with different values of α when L=1, 3. One can notice that the performance of the ED is greatly improved when increasing the number of SC diversity branches from L=1 to L=3. In addition, increasing the value of the nonlinearity parameter α improves the performance of the ED by decreasing the probability of miss detection. Indeed, increasing α enhances the tail under the PDF as illustrated in Fig. 3, and hence for a given fixed threshold it decreases the miss detection probability.

According to Fig. 5, the complementary ROC of the ED is greatly enhanced when L is increased from L=1 to L=3 with the several values of μ. In addition, increasing the value of μ increases the number of multipath clusters contributing to the envelope of the received signal, and hence increases the diversity gain resulting in lower miss detection probability.

Fig. 6 shows the complementary ROC of the ED for several well known fading distributions obtained from the generalized $\alpha - \mu$ fading distribution by selecting different values of α and μ. Exponential faded channel yields the worst performance while Nakagami-m faded channel with severity parameter m=5 results in the best performance.

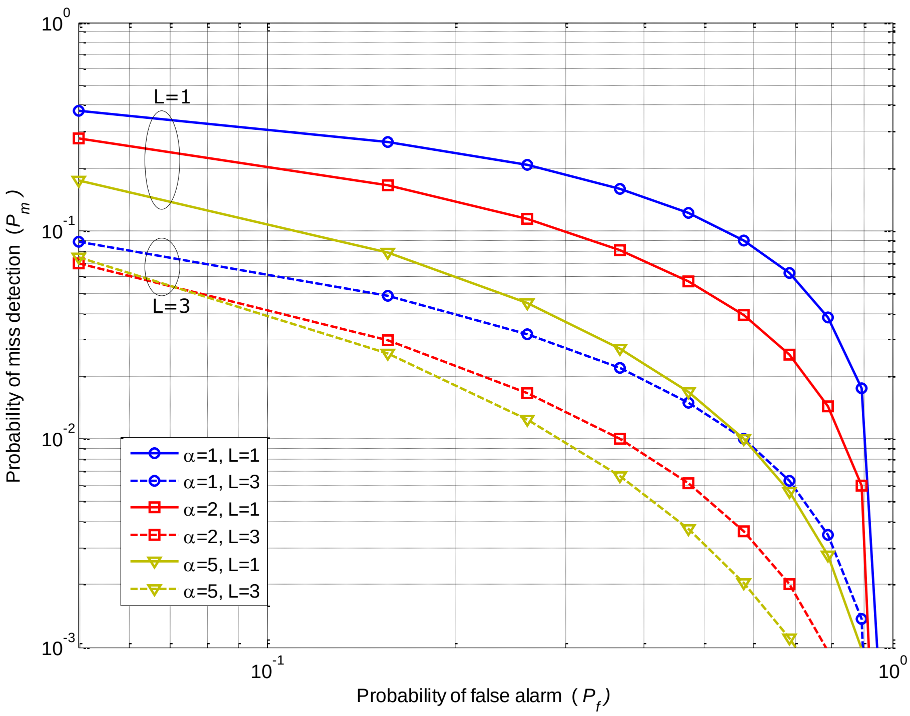

Fig. 4. Complementary ROC curves of the ED for $\alpha - \mu$ fading channel with SC and different values of α. $\mu = 2$ and $\bar{\gamma} = 10$dB.

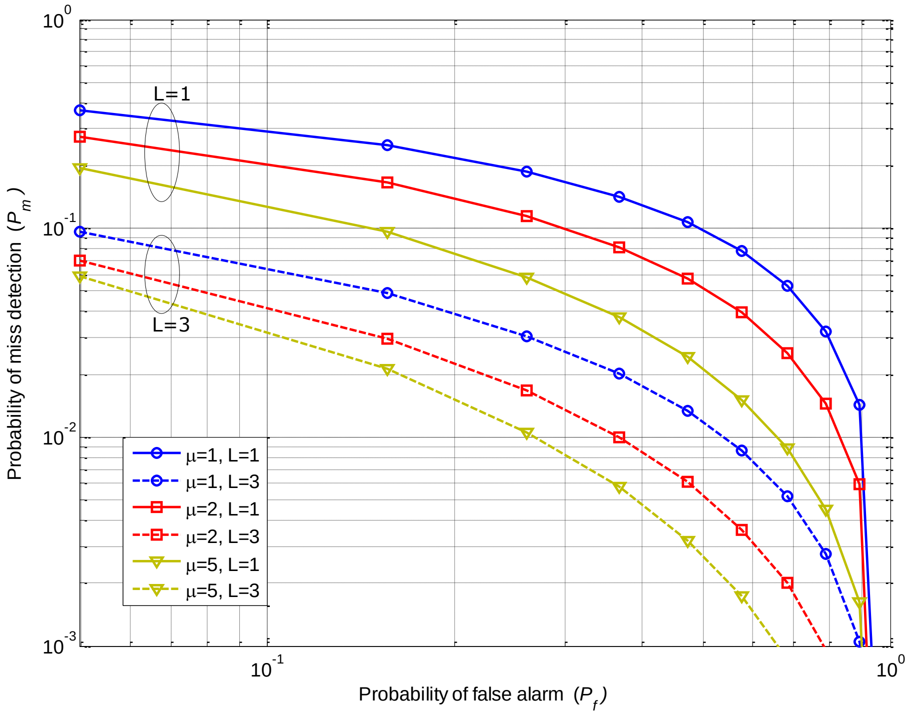

Fig. 5. Complementary ROC curves of the ED for $\alpha - \mu$ fading channel with SC and different values of μ. $\alpha = 2$ and $\bar{\gamma} = 10$dB.

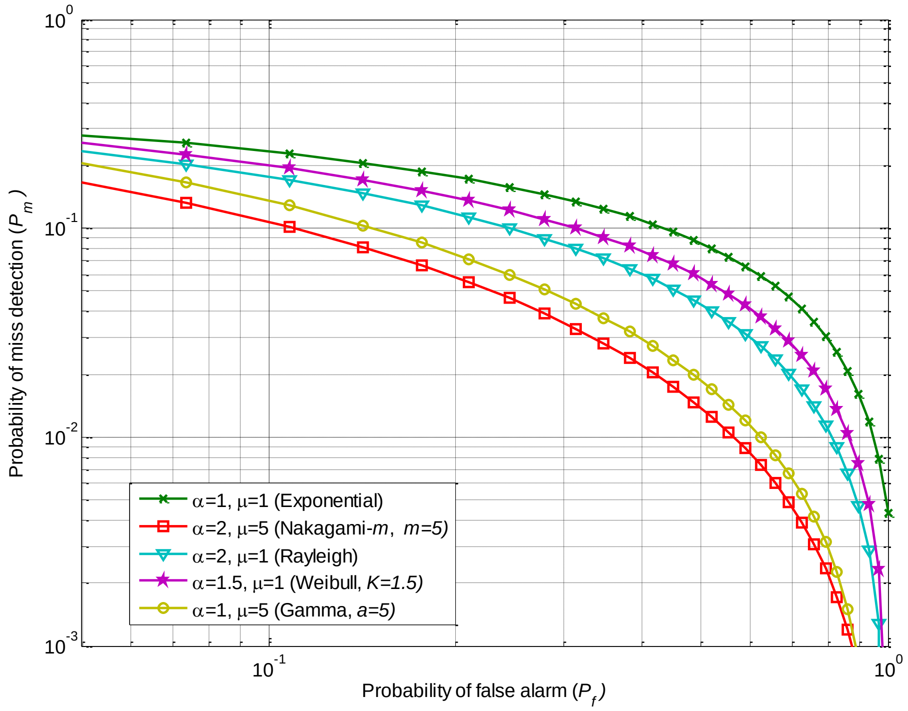

Fig. 6. Complementary ROC curves of the ED over different fading channels with dual SC (L=2). $\bar{\gamma} = 9$dB.

V. CONCLUSION

In this paper, a new closed-form mathematical expression is derived for the probability of detection of the ED over $\alpha-\mu$ generalized fading model with SC diversity reception. The derived expression covers several known fading distribution models as special cases. Complementary ROC curves were drawn for the ED over $\alpha-\mu$ fading channels where enhancement of the probability of detection was achieved by using SC diversity reception.

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تحليل أداء كاشف الطاقة المستشعر للطيف الكهرومغناطيسي عبر قنوات الاتصال المضمحلة العامة في شبكات الراديو الإدراكية

إعداد: حكمت يوسف محمد دراوشة

إشراف: د. علي جاموس

ملخص:

الراديو الإدراكي (CR) هو تكنولوجيا ناشئة اكتسبت اهتماماً واسعاً في العقود القليلة الماضية من أجل حل مشكلة الكفاءة المتدنية للطيف الكهرومغناطيسي الترددي. تم عرض العديد من تقنيات استخدام ترددات الطيف الكهرومغناطيسي لشبكات الراديو الإدراكية في الأبحاث العلمية المنشورة، ولكن كاشف الطاقة هو الأسلوب الأكثر استخداما على نطاق واسع في شبكات الراديو الإدراكية لتمكين الإستغلال الأمثل لترددات الطيف الكهرومغناطيسي. في هذه الأطروحة، سَنُعيد دراسة مشكلة كشف الطاقة للإشارات غير المعروفة عبر قنوات التلاشي والإضمحلال اللاسلكية، وبصورة أكثر تخصيصاً، سيتم اشتقاق معادلة رياضية جديدة لمتوسط احتمال الكشف عن الطاقة لجهاز كاشف الطاقة (ED) مع الأخذ بعين الإعتبار قنوات التلاشي العامة مثل (نموذج α-μ) والإستقبال المتنوع - اختيار الفرع ذي الطاقة الأعلى (SC). المعادلة المشتَقَّة هي معادلة عامة وتشمل العديد من الحالات الخاصة مثل ناكاجامي- م، ويبُل، جاما، رايلي والتوزيع الأُسِّي. هذه المعادلة مفيدة لقياس تحسُّن أداء كاشف الطاقة مع وجود الإستقبال المتنوع- اختيار الفرع ذي الطاقة الأعلى (SC).

تم مناقشة أثر المتغيرات المختلفة في المعادلة الرياضية على خصائص التشغيل التكميلية للمستقبل (ROC) لكاشف الطاقة. بالتحديد، تم دراسة تأثير عدد فروع الإستقبال المتنوع، ومتوسط نسبة الإشارة إلى الضوضاء (SNR)، والمُعامل غير الخطي للظروف البيئية (α)، وعدد التجمُّعات المتعددة (μ) على منحنيات الأداء لكاشف الطاقة.