Deanship of Graduate Studies

Al – Quds University

Oscillation of Solutions of Third Order Linear Neutral Delay Differential Equations

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M. Sc. Thesis

Jerusalem – Palestine

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Oscillation of Solution of Third Order Linear Neutral Delay Differential Equation

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Dedication

To my mother,

my father,

my wife,

my daughter,

my sons,

my brothers,

and

my sisters,

I dedicate this work.

Declaration

The work provided in this thesis , unless otherwise references , is the researcher's own work , and has not been submitted elsewhere for any other degree or qualification .

Student's Name : Ayoub Hasan Ahmad Saleh

Signature :

Date: 10/ 5 /2015

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Index of Special Notations

$\mathbb{N}$ set of natural numbers

$\mathbb{R}$ set of real numbers

$\mathbb{R}^+$ set of positive real numbers

$f(t) \in C(D) = \{f : D \to \mathbb{R} :\ f \text{ is continuouse function}\}$

$f(t) \in C^1(D) = \{f : D \to \mathbb{R} :\ f \text{ is continuously differentiable function}\}$

$f(t)$ $f : [\,0\,,\infty) \to \mathbb{R}$ function of time t .

$y''(t)$ the usual second derivative $\dfrac{d^2y}{dt^2}$.

sup S the least upper bound, or the supremum of the set S.

$\int_a^b f(t)dt$ usual definite integral.

■ ends of proofs .

The form (A . B . C) A is the number of chapter .

B is the number of section .

C is the serial number.

Abstract

It is well-known that there are many types of differential equations and each type has its own applications and solution, one of these types known as neutral delay linear differential equations. This type of equations has solutions divided into three shapes: oscillatory , almost oscillatory , and nonoscillatory solutions. And to decide whether the solution is oscillatory or nonoscillatory, we have some necessary and sufficient conditions that must satisfied .

In this thesis , we study the oscillation of nontrivial real valued solutions $y(t)$ to the third order linear neutral delay differential equations of the form

$$\frac{d}{dt}\left(r_2(t)\,\frac{d}{dt}\left(r_1(t)\,\frac{d}{dt}\big(y(t)+p(t)y(t-\tau)\big)\right)\right)+f(t)y(t-\sigma)=0 \qquad (1N1-A)$$

$$\frac{d}{dt}\left(r_2(t)\,\frac{d}{dt}\left(r_1(t)\,\frac{d}{dt}\big(y(t)+p(t)y(\tau(t))\big)\right)\right)+f(t)y(\sigma(t))=0 \qquad (1N1-B)$$

$$\frac{d}{dt}\left(r(t)\,\frac{d^2}{dt^2}\left(\big(y(t)+p(t)y(\tau(t))\big)\right)\right)+f(t)y(\sigma(t))=0 \qquad (1N2-B)$$

$$\frac{d^2}{dt^2}\left(r(t)\,\frac{d}{dt}\big(y(t)+p(t)y(t-\tau)\big)\right)+f(t)y(t-\sigma)=0 \qquad (2N\,1-A)$$

$$\frac{d^2}{dt^2}\left(r(t)\,\frac{d}{dt}\left(y(t)+p(t)y\big(\tau(t)\big)\right)\right)+f(t)y\big(\sigma(t)\big)=0 \qquad (2N1-B)$$

where

$p(t)$, $f(t) \in C\,([t_0\,,\infty),\mathbb{R}\,)$, $f(t) \geq 0$, $r_1(t)$, $r_2(t)$, $r(t) \in C^1\,([t_0\,,\infty),\mathbb{R}^+\,)$

and τ , $\sigma \in [0\,,t\,)$.

The purpose of this thesis is to examine sufficient conditions established so that every solution to equations $(1N1-A)$, $(1N1-B)$, $(2N1-A)$, $(2N1-B)$, $(1N2-A)$, $(1N2-B)$ is either oscillatory or converge to zero . In particular , we extend the results that obtained in K. V. V. Seshagiri Rao to the equation

$$\frac{d}{dt}\left(r(t)\,\frac{d^2}{dt^2}\left(\big(y(t)+p(t)y(t-\tau)\big)\right)\right)+f(t)y(t-\sigma)=0 \qquad (1N2-A)$$

and follow the similar steps that used specially in Tongxing Li in studying equation $(1N1-B)$ to examine oscillatory properties that presented for the equation $(1N1-A)$ when the function $r_1(t)=1$. These criteria improve and complement those results in the literature . Moreover , we give the proof of the comparison lemma that appear in Seshagiri Rao , and some illustrating examples .

Introduction

Many important and significant problems in engineering, physical sciences, and social sciences when formulated in mathematical terms, require the determination of a function satisfying an equation that has one or more derivatives of an unknown function, which may be a function of time t . Such equations are called differential equations . Newton's second law of motion

$$m\frac{d^2u(t)}{dt^2} = F\left(t, u(t), \frac{du(t)}{dt}\right) \qquad (1)$$

for the position $u(t)$ of the particle acted on a force F is a good example, the position $u(t)$, and the velocity $\frac{d\,u(t)}{dt}$.

If the differential equation of the form

$$F\left(t\,, y(t)\,, y'(t)\,, y''(t), \ \ldots\ , \ y^{(n)}(t)\right) = 0 \qquad (2)$$

Or $$y^{(n)}(t) = f\left(t\,, y(t)\,, y'(t)\,, y''(t), \ \ldots\ , \ y^{(n-1)}(t)\right) \qquad (3)$$

$n \in \mathbb{N}$ is called nth order differential equation, where only the function y(t) and its derivatives are used in determining if the differential equation is linear, see [31] .

A solution of equation (3) on the interval $(\alpha\,,\beta)$ is a function $\emptyset$ such that $\emptyset'(t)\,, \emptyset''(t),\ \ldots\ ,\ \emptyset^{(n-1)}(t)$ exist and satisfy

$\emptyset^{(n)}(t) = f\left(t\,, \emptyset(t)\,, \emptyset'(t)\,, \emptyset''(t),\ \ldots\ ,\ \emptyset^{(n-1)}(t)\right)$ for every $t \in (\alpha\,,\beta)$.

We will assume that the function f is real valued function and we are interested in obtaining real valued solutions $y = \emptyset(t)$ in our work. Now, if the solution $y = \emptyset(t)$ has arbitrary large zeros on interval $(t_x\,,\infty)$ then this solution is said to be oscillatory; Otherwise, it is said to be nonoscillatory. If all solutions are oscillatory or converge to zero asymptotically then the differential equation is said to be almost oscillatory.

A type of differential equation in which the derivative of the unknown function at certain time (t) is given in terms of the values of the function at previous times $(t-\tau)$ are called delay equations . It is also called time-delay-systems , equation with deviating argument . The simplest constant delay has the form

$$y'(t) = f\big(t\,, y(t)\,, y(t-\tau_1)\,, y(t-\tau_2),\ \ldots\ ,\ y(t-\tau_n)\big) \qquad (4)$$

where the delays (lags) τ_j's are positive constants. More generally , state dependent delays may depend on the solution that is $\tau_j = \tau_j\big(t\,, y(t)\big)$.

Oscillation problems for first order ordinary differential equation with deviating arguments are interesting from a theoretical as well as the practical point of view. In fact , Bernoulli (1728), while studying the problem of sound in a tube with finite size , investigated the properties of solutions of first order ordinary differential equation with deviating argument , this was the first work in this area . Myskis investigated several oscillation problems of this type of equations, see [7].

Now a differential equation in which the highest order derivative of the unknown function appears both with and without delays is called Neutral Delay Differential Equation (NDDE). Concerning existence , uniqueness and continuous dependence for (NDDE), we refer to Driver [25 , 26] .

There has been great interest in studying the oscillatory behavior of differential equations , since there are many types of them not easy to solve and has many applications , see [7, 9 , 10 , 16 , 18] and the references cited therein . In fact oscillation theory of neutral delay differential equations has grown rapidly and has many interesting applications from the real world in many fields . Delay differential equations are important class of dynamical systems , so they often arise in either natural or technological control problems. In these systems, a controller monitors the state of the system, and makes adjustments to the system based on its observations. Since these adjustments can never be made instantaneously, a delay arises between the observation and the control action. They also have applications to electric networks containing lossless transmission lines. Such networks appear in high speed computers where lossless transmission lines are used to interconnect switching circuits. They also occur in problems dealing with vibrating masses attached to elastic bar and in some variational problems , see [12 , 22 , 28] . In addition to that, they now occupy a place of central importance in the biological applications since they give a better description of fluctuations, in population dynamics and epidemiology, see [1 , 8 , 21 , 23] for more applications . Lastly , they appear in dynamical economics as a delay differential equation models of cyclic economic behavior , and it is now known that a broad spectrum of dynamic behaviors can be found in nonlinear delay differential equations , see Saari [5], Mackey [17], Franke [27], and the references cited therein . In the last forty years , there has been many researches that study the oscillatory behavior of linear neutral delay differential equations of the form:

$$\frac{d^n}{dt^n}\left(y(t)+p(t)y(t-\tau)\right)+f(t)y(t-\sigma)=0 \qquad (5)$$

where $n \in \mathbb{N}$, $t - \tau \leq t$, $t - \sigma \leq t$, see [6] . For n = 1, equation (5) has been studied by Ladas and Sficas [6], Grammatikopoulos, Grove Ladas [19], and Zhang , Wang[32] , in these papers they established conditions for the oscillation of all solutions of first order linear neutral delay differential equations. For n=2 equation (5) has been studied by M. K. Grammatikopoulos , Grove Ladas and A. Meimaridou[20] and Philo[4], Dzurina , and Stavroulakis [11], Agarwal Shieh and Yeh [29] , Seshagiri Rao and Sai Kumar [14] in these papers they established conditions for the oscillation of all solutions of second order linear neutral delay differential equations. For n=3 , third order linear neutral delay differential equations have received less attention compared to first and second order . This research concerned with third order linear neutral delay differential equations of the form:

$$\frac{d}{dt}\left(r_2(t)\,\frac{d}{dt}\left(r_1(t)\,\frac{d}{dt}\big(y(t)+p(t)y(t-\tau)\big)\right)\right)+f(t)y(t-\sigma)=0 \qquad (1N1-A)$$

$$\frac{d}{dt}\left(r_2(t)\,\frac{d}{dt}\left(r_1(t)\,\frac{d}{dt}\big(y(t)+p(t)y(\tau(t))\big)\right)\right)+f(t)y(\sigma(t))=0 \qquad (1N1-B)$$

Some researches study equations $(1N1-A)$ and $(1N1-B)$ when $\mathrm{p(t)}=0$ and $r_1(t)=1$, for that we mention the works of Cemil [3] , Erbe [15] , and Paul [24]. And for $p(t)\neq 0$ equation $(1N1-A)$ was studied by Seshagiri Rao and Sai Kumar [13], and references therein. For $p(t)\neq 0$. Equation $(1N1-B)$ was studied by Tongxing Li , Chenghui Zhang , and Guojing Xing , see [30] and references cited therein .

From now , by solution of equations $(1N1-A)$ and $(1N1-B)$ we mean a nontrivial function $y(t)\in C\left(\left[t_y\,,\infty\right)\right)$, where $t_y\geq t_0$ which satisfies $(1N1-A)$ and $(1N1-B)$ on $\left[t_y\,,\infty\right)$. We consider only those solutions of $y(t)$ of $(1N1-A)$ and $(1N1-B)$ which satisfy $sup\{|y(t)|: t\geq T\}>0$ for all $T\geq t_y$ of $(1N1-A)$ and $(1N1-B)$ possesses such solution . Also when we write a functional inequality , it will be assumed to hold for sufficiently large t in our subsequent discussion .

The purpose of this research is to examine oscillatory behavior of third order linear neutral delay differential equations $(1N1-A)$ and $(1N1-B)$ and to