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An Iterative Technique for Solving Nonlinear Quadratic Optimal Control Problem Using Orthogonal Functions

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Dedication

To the late memory of my Father

Amjad Ahed Mustafa Majdalawi

Declaration:

I certify that this thesis submitted for the degree of Master, is the result of my own research, except where otherwise acknowledged, and that this study (or any part of the same) has not been submitted for a higher degree to any other university or institution.

Signed:……………………..

Amjad Ahed Mustafa Majdalawi

Date: /   /

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My deep thanks go to my wife for her support and encouragement. I also would like to thank my great children: Mohammad, Nada, and Jana for their continuous smiles which gave me a great support.

Thank you all.

Abstract

Over the recent years, many techniques and methods have been proposed to solve the difficult nonlinear optimal control problem. These methods can be classified as: direct and indirect methods. The proposed method in this work is classified as a direct method in which the optimal control problem is directly converted into a mathematical programming problem. As its name implies, direct methods are employed by directly substituting the control and state variables into the performance index.

Direct methods can be implemented using either discretization or parameterization. Parameterization can be implemented using one of three ways: (1) Control parameterization, (2) Control-state parameterization and (3) State parameterization. The proposed method in this work is based on state parameterization which is employed by parameterizing the system state variables by a finite length series Chebyshev or Legendre polynomials with unknown parameters.

The proposed method in this work is also based on the iteration technique which replaces the nonlinear state equations by an equivalent sequence of linear time-varying state equations. Then, state parameterization is applied on this sequence. By this, the original nonlinear quadratic optimal control problem is directly converted into quadratic linear programming problems, which are easier to solve.

To show the effectiveness of the proposed method, several optimal control problems free and subject to different types of constraints were solved, and the simulation results show that the proposed method gives good and comparable results with some other methods. Among the optimal control problems which were solved is the complex containers crane problem.

Contents

Chapter One

Introduction

1.1 Motivations

Optimal control problem is to find an optimal controller u^* that minimizes a certain cost function (specification) keeping at the same time the system state equations, initial condition, and any other constraints of the system satisfied. Examples of optimal control applications include environment, engineering, economics etc.

Unlike linear optimal control problem which has an analytical solution that is given as a closed loop feedback control law, nonlinear optimal control problem does not. This motivates many researchers to try to find a solution to this problem. In most cases, if not all, these solutions are numerical i.e. approximate or suboptimal solutions.

Generally, solution methods to optimal control problem are classified as direct and indirect methods. Indirect methods are usually employed by converting the optimal control problem into a two-point boundary value problem TPBVP and solving this new problem which is easier than the original problem or finding a solution that satisfies the Hamilton-Jacobi-Bellman equation. The main advantage of indirect methods is that the resulted solutions produce a closed loop or feedback control law. However, indirect methods suffer some drawbacks which are [8]: (1) There is no solution to the Hamilton-Jacobi-Bellman equation of general nonlinear optimal control problem. (2) The introduction of artificial costates. (3) The lack of robustness. (4) A deep knowledge of the system model (mathematical and physical) is required.

For those reasons and others direct methods were proposed to solve optimal control problems. Direct methods are employed by either discretization or parameterization of the state and/or the control variables. In discretization, many discrete points (samples) of the state and/or control variable are required in order to produce accurate results. This would ends up with a system of large dimension (curse of dimensionality). On the other hand, parameterization can be implemented by one of the three ways: (1) Control parameterization is employed by approximating the control variables by a finite series of known functions with unknown parameters, then the state variables are obtained as a function of the unknown parameters by integrating the system state equation. This process is computationally expensive [15]. (2) Control-state parameterization is employed by approximating both state and control variables by a finite series of known functions with unknown parameters. The resulted system would ends up with large unknown parameters. (3) State parameterization is the least used method compared with control parameterization and control-state parameterization. In state parameterization, only some state variables are directly approximated by a finite series of known functions with unknown parameters. The remaining state and control variables are obtained as a function of the unknown parameters directly from the state equation(s).

Though, state parameterization is not used extensively in optimal control. Our choice in this work is to use state parameterization because it has some advantages over both control parameterization and control-state parameterization. These advantages are: (1) There is no need as in control parameterization to integrate the system state equations. (2) The number of unknown parameters is smaller compared with control-state parameterization. (3) The state constraints can be handled directly.

State parameterization requires known functions for the approximation of the state variables. To simplify computation of the optimal control problem, the known functions are usually chosen to be orthogonal. In this work, we choose two orthogonal functions; Chebyshev and Legendre polynomials. These polynomials offer some advantages over other orthogonal functions. Fast convergence and good min-max properties [17] are only few advantages that both functions offer.

1.2 Thesis Goals

The main goal of this work is to apply the iteration technique developed by Banks [3-7] on the optimal control problem under consideration to directly obtain a numerical solution to this problem. As a result, state parameterization via Legendre or Chebyshev polynomials will be applied on the resulted optimal control problems of the iteration technique. In the application of state parameterization, we will follow Jaddu method [8].

1.3 Thesis Contribution

The main contribution of this work is the introduction of a new technique for solving the nonlinear quadratic optimal control problem, both free and subject to different types of constrains. As a result, other contribution can be stated as follows:

- Introducing a new Legendre polynomial property called the differentiation operational matrix. This matrix is used to approximate the derivatives of the state polynomials using Legendre polynomials.

- Introducing a new formula for the approximated performance index using Legendre polynomials.

- Introducing a new method for solving the linear quadratic optimal control problem using state parameterization via Legendre polynomials.

1.4 Thesis Organization

The remaining chapters of this thesis are organized as follows:

Chapter two reviews the optimal control problem in general and discusses some of the important previous works that are proposed to handle the optimal control problem. In this chapter, the computational techniques and methods used to handle optimal control problems are classified into direct and indirect methods.

Chapter three describes a method for solving the linear quadratic optimal control problems. In spite of the fact that this work is intended for nonlinear optimal control problems, it is necessary to solve linear optimal control problems, because as will be demonstrated in chapters four and five, the solution method for nonlinear optimal control problems is based on converting the nonlinear optimal control problem into a sequence of linear time-varying optimal control problems. All aspects of state parameterization via Legendre polynomials are discussed in this chapter. In addition, some of the important properties of Legendre polynomials are reviewed. One of them is a newly introduced property for the Legendre polynomials called the differentiation operational matrix. This property is used to approximate the derivative of the state variables. An explicit formula to approximate the quadratic performance index using Legendre polynomials is introduced. Finally, computational results of a standard example are introduced and the results are compared with some other methods.

Chapter four presents the main idea of this work, where a computational method for solving the nonlinear quadratic optimal control problem is introduced. In this chapter, the concept of the iteration technique is presented. Also introduced in this chapter is state parameterization via Chebyshev polynomials developed by Jaddu [8]. In this chapter, the steps of converting the nonlinear quadratic optimal control problem into a sequence of quadratic programming problems are introduced. To verify the proposed method, a standard example is solved for the purpose of comparison with other methods.

Chapter five is an extension of chapter four, where the optimal control problem under consideration is subject to different types of constraints. These constraints include: Terminal state constraints, State saturation constraints and Control saturation constraints. In this chapter, the constrained nonlinear quadratic optimal control problem is converted into a constrained sequence of standard quadratic programming problems solved using the active set method. To show the effectiveness of the proposed method, a typical Van der Pol problem subject to different types of constrained is solved and the results are compared with the results of other methods. Also introduced in this chapter is the complex problem of transferring containers from ships to trucks at the port of Kobe.

Finally, Chapter six presents some of the important conclusions of this work and a suggestion of the future work that can be built over this thesis.

Chapter two

Optimal Control Problem: Literature Review

2.1 Introduction

In this chapter, we present a review of the optimal control problem in general. We discuss some of the important previous works presented to handle the optimal control problem. Many textbooks [1-2] and survey papers [31-32] that handled optimal control problem were published.

Basically, the main objective of optimal control is to find a controller that can be an open loop (off- line) controller denoted as $u^*(t)$ or a closed loop (on-line) controller denoted as $u^*(x(t), t)$. This controller is optimal because when applied to the dynamic system, it minimizes (maximizes) a certain function called the cost function or performance index keeping at the same time the system physical constraints unviolated. The performance index or cost function can be considered as the desired specifications of the system.

The basic optimal control problem consists of three elements:

1. Plant model: This is the system to be controlled. Mathematically, it is represented as a set of state equations which are a set of first order differential equations

$$\dot{x} = f(x(t), u(t), t) \quad , t \in [t_0, t_f] \tag{2.1}$$

   where $x \in R^n$ is the state vector, $u \in R^m$ is the control vector. f is assumed continuous differentiable function with respect to all its arguments.

2. Initial plant state: A set of initial conditions which indicate the system state values at initial time

$$x(t_0) = x_0 \tag{2.2}$$

   where $x_0 \in R^n$ represents a known initial condition vector.

3. Plant performance index (specifications): The desired specifications of the system that needs to be minimized (or maximized). Mathematically, the performance index is represented by a scalar function given by

$$J = h\big(x(t_f), t_f\big) + \int_{t_0}^{t_f} g(x(t), u(t), t)dt \tag{2.3}$$

where t_0 and t_f are the initial and final time; h and g are scalar functions. t_f may be specified or "free", depending on the problem statement. Figure (2.1) shows the elements of a basic optimal control problem.

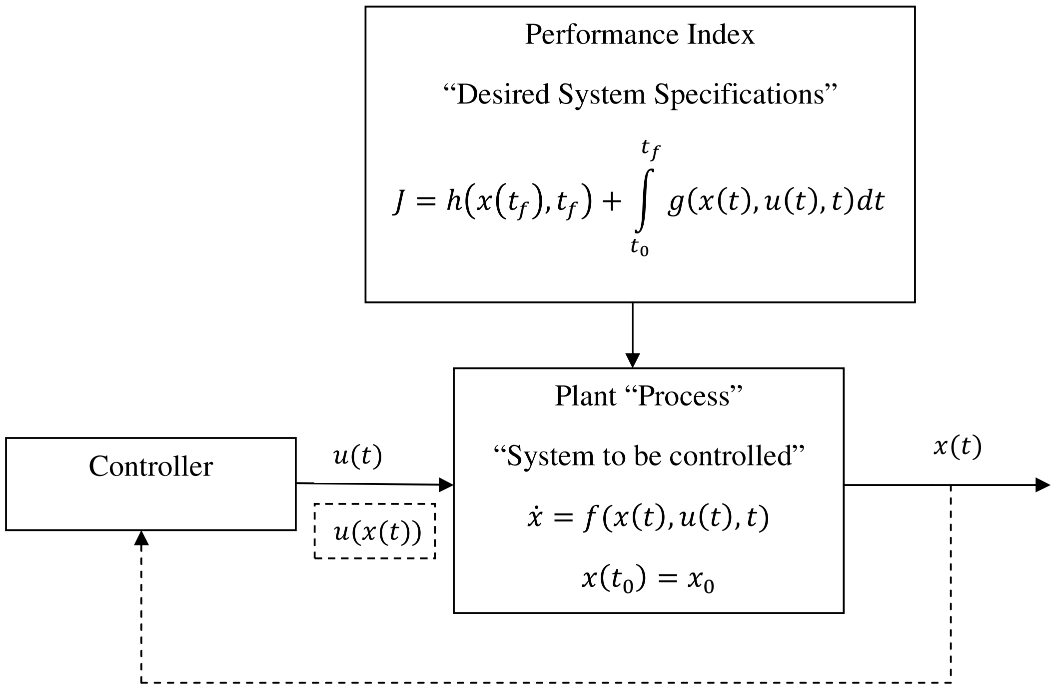

Figure (2.1) Elements of an optimal control problem

2.2 Problem Statement

The general unconstrained optimal control problem can be stated as follows:
Find an optimal controller, feedback $u(x(t), t)$ if possible, or if not an open loop $u(t)$ that minimizes the following performance index

$$J = h\big(x(t_f), t_f\big) + \int_{t_0}^{t_f} g(x(t), u(t), t)dt \tag{2.4}$$

subject to

$$\dot{x} = f(x(t), u(t), t) \qquad x(t_0) = x_0 \tag{2.5}$$

Many methods have been proposed to solve the problem (2.4)-(2.5). More or less, these methods can be categorized into one of the following tracks:

- Dynamic programming (Hamilton-Jacobi-Bellman HJB Equation).
- Calculus of Variation (Euler-Lagrange Equations).
- Parameterization or discretization (nonlinear mathematical programming).

Dynamic programming is based on methods that satisfy HJB equation. The optimal controller resulted from these methods is a closed loop or feedback controller $u(x(t))$. Methods that are based on the calculus of variation (Euler-Lagrange equations) convert the optimal control problem into a Two-Point Bounded Value Problem (TPBVP). The optimal controller resulted from using these methods would also produce a feedback or closed loop controller $u(x(t))$. Methods that are based on HJB equation or Euler-Lagrange equations are usually classified as indirect methods.

Methods that are based on parameterization or discretization are classified as direct methods. Direct methods usually produce an open loop optimal controller $u(t)$. Direct methods are based on solving the optimal control problem by converting it into a nonlinear programming problem. The proposed method in this work is classified as a direct method.

In the following sections, we discuss these methods and review some of the important papers that were published. Figure (2.2) shows a block diagram that illustrates the computational methods of optimal control problem.

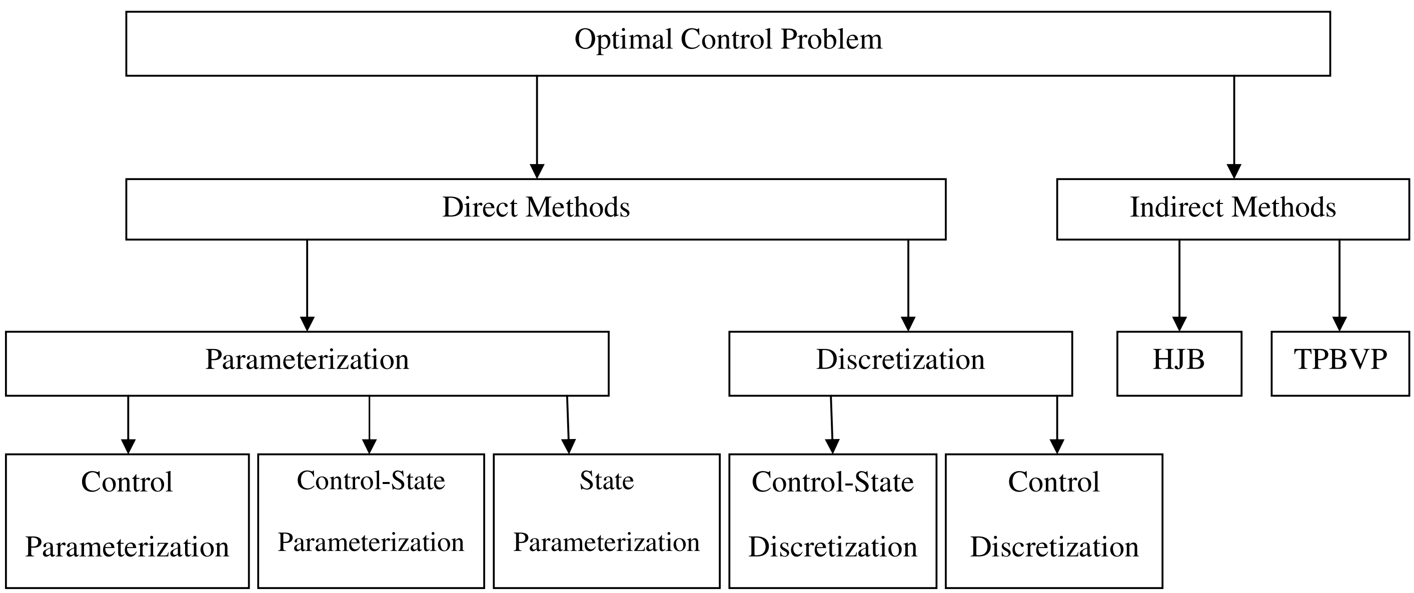

Figure (2.2) Computation methods of optimal control problem

2.3 Indirect Methods

In this section, we review some of the important methods that are classified as indirect method. As indicated earlier, these methods are based on solutions that satisfy the HJB equation or on solutions that convert the optimal control problem into a TPBVP. In what follows is a review of these methods:

1. Power series approach: This approach is based on finding an approximate solution to the Hamilton-Jacobi-Bellman equation or the nonlinear two-point boundary value