Deanship of Graduate Studies

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# Survey Of Fredholm Integral Equations And Applications

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M. Sc. Thesis

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# Abstract

Integral equations are one of the most useful mathematical tools in mechanical vibrations and the related fields of Engineering and mathematical physics. We focus our attention in this work on the Fredholm integral equation and its applications.

Chapter one introduces a historical introduction of integral equations and some important definitions, which we will use in the following chapters. Chapter two deals with the theory of Fredholm and solution of Fredholm integral equation with some specific kernels, where the solution is presented by absolutely and uniformly convergent series. Chapter three is devoted to the solution of integral equations with symmetric kernel by using Hilbert-Schmidt and Mercer's theorems. Cauchy, Hilbert and Carleman Types singular integral equations will be discussed in chapter four. Chapter five is wholly concerned with some applications of Fredholm integral equation in the field of mathematical physics.

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# Chapter One

# Introduction

This chapter contains a historical introduction to the integral equations and its important applications in the applied math and mathematical physics; also it is devoted to a discussion of some of the broad classes of integral equations to be discussed in more detail in later chapters. Some definitions for some kind of equations, functions and kernels are described; which we will use them more often in our research.

## 1.1 Historical Introduction

The theory of integral equations has closed contacts with many different areas of mathematics, foremost among these are differential equations and operator theory. Integral equations are one of the most useful mathematical tools in both pure and applied analysis. This is particularly true of problems in mechanical vibrations and the related fields of engineering and mathematical physics, where they are not only useful but often indispensable even for numerical computations. Suffice it to say that there is almost no area of applied mathematics and mathematical physics where integral equations do not play a role.

The integral equation is any equation involving unknown function $g(x)$ under the integral sign [14]. It was first introduced by Du Bois Reymond in 1888. However, the