Al-Quds University
Jerusalem

Department Of Graduate Studies

One-Way and two-way Interaction Gas-Particulate Flow through Porous Media

By

Khaled M.M.Takatka

Main supervisor: Dr. Naji Qatanani
Co-supervisor : Dr. Fathi Allan

Thesis Submitted in Partial Fulfillment
of the Requirements for Degree of

Master of Science

in
Mathematics

at

Department of Mathematics

College of Science & Technology

Al-Quds University

Abu Dies-Jerusalem
Palestine, February 2001

Approved by:

Dr. Naji Qatanani (Main supervisor)

N. Qatanani

Dr. Fathi Allan (Co- supervisor)

Dr. Raymon Jadoan (External examiner)

Dr.Tahsin Mughrabi (Internal examinar)

Dedication

This thesis is respectfully dedicated to my beloved parents, my wife, my daughter, my beloved brothers and my sister for their help, support and encouragement.

Acknowledgment

First of all, I would like to express my most sincere thanks and appreciation to my main supervisor Dr. Naji Qatanani for his many valuable ideas, advice and excellent guidance.

My sincere thanks also to my second supervisor Dr. Fathi Allan of Birzeit University for his help and support, he provided me during this work.

To my external referee Dr. Raymon Jadoan for his useful comments in regard to this thesis.

To my internal referee Dr.Tahsin Mughrabi for his participation in refereeing this thesis.

My heartfelt gratitude goes to Professor Mohammad H. Hamdan of the University of New Brunswick, Canada, who introduced me to this subject during his stay at AL–Quds University.

My thanks also to the faculty members of the department of mathematics at Al–Quds University.

Special thanks to Mr.Mohammad M. Takatka and his family for their love, help, support and encouragement all through the years.

Last but not least my acknowledgment goes to the administration of Beit–Fajjar secondary school for their encouragement.

Abstract

Sets of partial differential equations describing the motion of a dusty fluid in porous media are developed in both cases of one–way and two–way interaction. The governing equations are derived using intrinsic volume averaging and are based on Saffman's dusty gas model. The effect of the porous microstructure on the flowing mixture is analyzed via the concept of representative unit cell (RUC) and distinction is made between flow in consolidated and granular porous media.

The equations governing the flow of a dusty fluid through different types of porous media, and the boundary conditions associated with each equation are investigated. Numerical simulation is carried out for the flow of a dusty fluid through naturally occurring porous media in a configuration of interest. Results indicate that the value of the Forchheimer drag coefficient C_d should be less than 0.75 .

Table of contents

Chapter one

Introduction

1.1 Importance of Gas – particle flow through porous media

Gas–particle flow, dusty fluid flow and the flow of suspension through porous media have received considerable attention due to the importance of these types of flow in studies associated with the design of industrial filters, liquid – dust separators and water purification plants.

Applications of the above types of flow in the environment are exhibited in solid – waste and pollutant dispersal and storage in the ground layers.

Fluid flow through porous media has become a topic of increasing importance due to its direct applicability in many physical situations including the prediction of oil reservoir behavior, groundwater flow, irrigation problems and the biophysical sciences where the human lungs, for example, are modeled as porous layers.

The above and many other applications emphasize two fundamental aspects of the study of gas particulate flow in porous media:

i) Models describing gas–particulate flow through porous media must accurately take into account the effect of the porous media quantities on the flow constitute on each other. This necessitates developing models that take into account the porous microstructure and its effects on the flowing phases.

ii) Solution of boundary and initial value flow problems that should accurately predict the flow patterns and the nature of the flow fields of the constituents involved.

1.2 Problem formulation and derivation of field equations

In this section a general derivation of the dusty fluid flow equations is presented, this developed general model which accounts for inertial effects and two - way interaction between the phases present is cast in different forms that take into account different porous media structures.

The model's equations that developed are based on the following physical assumptions on the porous medium and on the dusty fluid.

1. It is assumed that the dusty fluid flows through a porous matrix consisting of a sparse distribution of particles fixed in space.
2. It is assumed that porosity change of the medium due to the retention of dust particles on possible retention site are negligible and that clogging of the pores does not occur and thus straining action is neglected.
3. The permeability to the fluid is assumed to be constant and the reduction in permeability is therefor insignificant.
4. The distribution of the solid grains comprising the solid matrix is uniform.
5. The suspended dust particles in the viscous fluid are uniform in shape and size with particle diameter large enough so that diffusion by Brownian motion can be neglected and small enough so that clogging of the pores can also be neglected.
6. Flow redistribution in the porous medium is assumed to be unaltered by the presence of captured particles.
7. It is assumed that the main mechanism of particles capture is the direct interception on the surface of the grains of the porous matrix .The effect of settling is ignored in this study by neglecting gravitational forces.

8. It is assumed that the dust behaves as a continuum and that the porous structure and dimensions allow for this behavior and allow for the possibility of volume averaging over a certain control volume.
9. The dust particles are assumed to be non - interacting chemically or otherwise, and the dust concentration by volume is very small.

To this end we consider the fluid flow through a porous medium having a constant porosity and a constant permeability to the fluid. The fluid saturating the chosen medium is assumed to consist of two phases, the fluid – phase and the dust-phase. When the fluid – phase and the porous medium compressibility effects are neglected, the averaged Navier – Stokes equations averaged over a control volume of the porous medium, take the following macroscopic form in the absence of momentum transfer between the phases involved [5]

$$\gamma_j \vec{\rho}_j \left[\frac{\partial \hat{\vec{u}}_j}{\partial t} + (\hat{\vec{u}}_j . \vec{\nabla}) \hat{\vec{u}}_j \right] = - \gamma_j \vec{\nabla} \vec{p}_j + \gamma_j \mu_j \nabla^2 \hat{\vec{u}}_j + \vec{f}_j \quad . \tag{1.1}$$

The macroscopic equation of continuity in the absence of sources and sinks is given by

$$\frac{\partial \gamma_j \vec{\rho}_j}{\partial t} + \vec{\nabla} . \gamma_j \vec{\rho}_j \hat{\vec{u}}_j = 0 \ . \tag{1.2}$$

Where γ_j is the volume fraction of the j^{th} phase in the system, $\vec{\rho}_j$ is the intrinsic volume – averaged density of phase j, $\vec{p}_j$ is the intrinsic volume – averaged pressure of phase j, $\hat{u}_j$ is the volume – averaged velocity vector of phase j, μ_j is the viscosity of phase j and $\vec{f}_j$ is the sum of external forces exerted on phase j.

The volume fraction γ_j satisfies the relation

$$\sum_j \gamma_j = 1 \tag{1.3}$$

with γ_j is defined as

$$\gamma_j = V_j / V. \tag{1.4}$$

Where V denotes the control volume and V_j is the volume occupied by phase j.

Since it is assumed that the dust–phase has a very small concentration by volume, it is reasonable to take the volume fraction of each of the phases to be constant so that the intrinsic volume - averaged quantities in equation (1.1) are related to the volume – averaged quantities by

$$\hat{\Gamma}_j = \gamma_j \vec{\Gamma}_j \quad . \tag{1.5}$$

Where $\hat{\Gamma}_j$ refers to a volume – averaged quantity and $\vec{\Gamma}_j$ refers to an intrinsic volume-averaged quantity.

In light of (1.5) equation (1.1) and (1.2) take the following forms respectively,

$$\hat{\rho}_j \left[\frac{\partial \hat{\vec{u}}_j}{\partial t} + (\hat{\vec{u}}_j \, . \, \vec{\nabla} \,) \, \hat{\vec{u}}_j \right] = - \vec{\nabla} \hat{p}_j + \mu_j \nabla^2 \hat{\vec{u}}_j + \vec{F}_j \tag{1.6}$$

$$\frac{\partial \vec{\rho}_j}{\partial t} + \nabla . \hat{\rho}_j \hat{u}_j = 0 \quad . \tag{1.7}$$

Where

$$\vec{F}_j = \vec{f}_j / \gamma_j \quad . \tag{1.8}$$

Dropping "^" from equations (1.6) and (1.7)and expressing them for the first and second phases we obtain

i) For the fluid – phase

The continuity equation is

$$\frac{\partial \rho_1}{\partial t} + \vec{\nabla} \, . \, \rho_1 \overrightarrow{u_1} = 0 \quad . \tag{1.9}$$

The momentum equation is

$$\rho_1 \left[\frac{\partial \overrightarrow{u_1}}{\partial t} + \left(\overrightarrow{u_1} \, . \, \overrightarrow{\nabla} \right) \overrightarrow{u_1} \right] = - \vec{\nabla} p_1 + \mu_1 \nabla^2 \overrightarrow{u_1} + \vec{F}_1 \quad . \tag{1.10}$$

ii) For the dust - phase

The continuity equation is

$$\frac{\partial \rho_2}{\partial t} + \vec{\nabla} \, . \, \rho_2 \, \vec{u}_2 = 0 \quad . \tag{1.11}$$

The momentum equation is

$$\rho_2 \left[\frac{\partial \vec{u}_2}{\partial t} + \left(\vec{u}_2 \cdot \vec{\nabla} \right) \vec{u}_2 \right] = - \vec{\nabla} p_2 + \mu_2 \nabla^2 \vec{u}_2 + \vec{F}_2 \quad . \tag{1.12}$$

Where $\vec{u}_1$ is the fluid – phase velocity vector, $\vec{u}_2$ is the dust – phase velocity vector, p_1 is the fluid – phase partial pressure, p_2 is the dust – phase partial pressure, μ_1 is the fluid – phase viscosity, μ_2 is the dust – phase viscosity, $\vec{F}_1$ is the sum of external forces exerted on a unit volume of the fluid – phase and $\vec{F}_2$ represents the sum of external forces exerted on a unit volume of dust – phase .Neglecting the dust–phase partial pressure and the dust –phase viscosity, and expressing the dust–phase density ρ_2 in terms of the macroscopic particle number density *N*, and the mass of a single dust particle *m*, then equation (1.12) takes the form

$$mN \left[\frac{\partial \vec{u}_2}{\partial t} + \left(\vec{u}_2 \cdot \vec{\nabla} \right) \vec{u}_2 \right] = \vec{F}_2 \quad . \tag{1.13}$$

The dust – phase continuity equation (1.11) can then be written in the form

$$\frac{\partial N}{\partial t} + \vec{\nabla} . N \overrightarrow{u_2} = 0 \tag{1.14}$$

where *N* is the particle number density.

1.2.1Nature of forces acting on the fluid-phase and dust-phase

i) Forces acting on the fluid-phase:

When the flow considered is that of a dusty fluid, then there are two forces acting on the fluid–phase, one of these is a frictional force due to the solid matrix of the medium, and the other is due to the influence of dust on the clean fluid.

Let $\vec{F}_{11}$ be the frictional force per unit volume on the fluid – phase. This friction force has to balance Darcy's pressure gradient in the medium. In the absence of body forces, Darcy's law is expressed in terms of the seepage type and takes the form

$$\vec{u}_1 - \vec{u}_2 = -\frac{k}{\mu_1}\vec{\nabla}\hat{p} \quad . \tag{1.15}$$

The expression for $\vec{F}_{11}$ can thus be obtained [7]

$$\vec{F}_{11} = -\frac{\mu_1}{k}(\vec{u}_1 - \vec{u}_2) \quad . \tag{1.16}$$

Let $\vec{F}_{12}$ denotes the force due to the influence of dust on the clean fluid. The assumption of a small concentration of dust by volume leads to the following expression for the effect of dust on the clean fluid

$$\vec{F}_{12} = C_r N(\overrightarrow{u_2} - \overrightarrow{u_1}) \quad . \tag{1.17}$$

Where C_r is the coefficient of resistance in the porous medium which is considered here to be constant under the assumption of uniform size and distribution of the dust particles and N is the macroscopic number density.

By substituting the contributions to $\vec{F}_{11}$ given by equation (1.16) and (1.17) into Equ. (1.10), the fluid – phase momentum equation takes the form

$$\rho_1\left[\frac{\partial \overrightarrow{u_1}}{\partial t} + \left(\overrightarrow{u_1}.\vec{\nabla}\right)\overrightarrow{u_1}\right] = -\vec{\nabla} p_1 + \mu_1 \nabla^2 \overrightarrow{u_1} + C_r N(\vec{u}_2 - \overrightarrow{u_1}) + \frac{\mu_1}{k}(\vec{u}_2 - \overrightarrow{u_1}). \tag{1.18}$$

The fluid–phase continuity equation (1.9) is then expressed in the form

$$\vec{\nabla}.\vec{u}_1 = 0 \ . \tag{1.19}$$

ii) Forces acting on the dust–phase

The term $\vec{F}_2$ in equation (1.13) represents the sum of external forces exerted on a unit volume of the dust–phase. Although it might be possible to consider that $\vec{F}_2$ is composed of two forces $\vec{F}_{21}$ and $\vec{F}_{22}$, where the first force represents the effect of the fluid–phase on the dust–phase and the other due to the solid matrix which is smaller than the first and therefore $\vec{F}_{22}$ is negligible.

Hence the only contribution to the force $\vec{F}_2$ is due to the fluid–phase influence on the dust, thus $\vec{F}_2$ is given by

$$\vec{F}_2 = \vec{F}_{21} = C_r N(\vec{u}_1 - \vec{u}_2) \quad . \tag{1.20}$$

Substituting equation (1.20) into (1.13), the dust–phase momentum equation will always assume the form

$$mN\left[\frac{\partial \vec{u}_2}{\partial t}+(\vec{u}_2.\vec{\nabla})\,\vec{u}_2\right]=C_rN(\vec{u}_1-\vec{u}_2) \quad . \tag{1.21}$$

While the dust–phase continuity equation is given by (1.14) .

The general equations governing the flow of an incompressible dusty fluid in a homogeneous porous medium are given by equations (1.18) and (1.19) while the fluid–phase coupled with the dust–phase equations are given in equations (1.14) and (1.21) respectively.

1.3 Overview of previous work

A set of general averaged transport equations for a multiphase system consisting of an arbitrary number of phases, interfaces and contact lines is established. A structure for the system is proposed and hydrodynamic interaction between the phases, interfaces and contact lines is also structured [18].

The common features of dispersed two–phase flows from a continuum–mechanical approaches are examined [5].

In [21] currently used averaging theorems are extended to allow for averaging volumes, which vary in space and time.

A model describing the flow of a dusty gas in porous media was developed [12] and is based on the differential equations approach. It incorporates the factors affecting the gas–particulate mixture in the type of porous media where Brinkman's equation is applicable and thus inertial effects were ignored under the assumption of creeping motion in a high–porosity medium.

Dusty fluid flow through porous media with applications to deep filtration has been widely studied [14] via the empirical and semi–empirical approach, and takes into account the optimal design of filters, liquid–dust separation and clogging mechanisms of the pores. In the case of flow of suspensions through a deep porous bed [14] gave a review of the a available literature and outlined the mechanisms of deposition and the possible capturing processes which include sedimentation, inertial impacting, direct interception, hydrodynamic effects and diffusion by Brownian motion. In cases where the particle size is greater than one micro-meter the particle diffusion is negligible [18] while the effect of inertial impacting is negligible if the fluid–phase is liquid [21]. In addition, if the particles are spherical in shape then the hydrodynamic effects can be neglected [14]. This leaves the interception capture mechanism to be dominant and the particles are captured mainly on the surfaces of the media grains. Settling of particles by sedimentation is also possible due to the high density associated with the dust particles.

Equations governing flow of a dusty fluid between tow porous flat plates with suction and injection are developed and closed – form solutions for the velocity profiles, displacement thickness and skin friction coefficients for both phases are obtained. Graphical results of the exact solutions are presented and discussed [3].

The problem of gas–particulate flow through a two – dimensional porous channel bounded by curved boundaries is considered [13].

Entry conditions to a porous channel compatible with the equations governing the flow of a dusty fluid in porous media are derived [10].

The equations governing the flow of a dusty fluid through isotropic, granular porous media are developed [1]. A set of fluid–phase momentum equations, and the boundary conditions associated with each equation are investigated [10].

A derivation of time–dependent field equations that are postulated to govern the flow of a two-phase fluid, with one of the phases being an oil phase, through porous sediment are presented [22].